



Expanding the horizon of machinery noise source control via a dedicated short course on gear dynamics and noise

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ABSTRACT

The dedicated 4-day short course on gear dynamics and noise (taught by the author and his colleague Donald Houser) is intended for designers and engineers involved in the analysis, manufacture, design specification, or utilization of gear systems. Over the past 35 years more than 1800 engineers from over 355 companies around the world have attended this short course. Industries that have found this course helpful include the automotive, transportation, wind-energy, process machinery, aircraft, appliance, and general manufacturing. The course covers the fundamentals of gearing kinematics, gear dynamics, source mechanisms governing major gear whine and rattle noise problems, structural paths with focus on bearings, noise and vibration measurements and diagnostics. A popular feature of this course is the interspersing of demonstrations with lectures; this makes the course appealing to both gear designers and noise (or measurement) specialists. Yet another novel approach is the workshop on "real life" gear noise and dynamics problems. This workshop allows the instructors and participants to interact and discuss case histories presented by attendees. A round table discussion on the last day is utilized to foster interactive problem solving, modeling and measurement discussions. Finally, issues posed by attendees have led to fundamental studies and graduate theses.

Keywords: Noise Sources, Machinery Noise, Vehicle Noise I-INCE Classification of Subjects Number: 07

1. INTRODUCTION

The purpose of the annual Gear Dynamics and Gear Noise Short Course is to provide a better understanding of the mechanisms of gear noise generation, the methods by which gear noise is measured and predicted, and the techniques employed in gear noise and vibration reduction. This course (under the Gear Noise title) was first taught in 1979 as a 3-day course. It has been offered annually since then, though now it is a 4-day course; an advanced course was also taught for a decade but it was discontinued as the materials covered demanded a higher level mathematical background. It has been taught in a few off-campus locations (such as in France and Australia) as well as at many corporate sites in several countries over the past two decades. The course material is covered in such a way that the fundamentals of gearing, gear dynamics, noise analysis and measurements are covered first. This makes the course appropriate to the gear designer with minimal knowledge of noise and vibration analysis as well as to the noise specialist with little knowledge of gears. Over the years, many practicing engineers inquired about the physics of noise sources and paths, and those questions have led to fundamental and applied research projects. This article focuses on noise and vibration control aspects (and not on gear design considerations) to reflect the typical audience of the Inter-Noise Congress. Course organizational issues are briefly discussed, and much of the article introduces unique aspects of gear noise. Some references will be cited to guide the reader to more advanced materials.

Given the vast nature of topics that must be addressed to examine gear dynamics and gear noise problems, only certain issues (of interest to designers and noise control engineers) are covered in the short course. The attendees are typically informed of the extensive literature on this topic, including Handbook chapters [1-5]; for example, more than 1000 articles can be found, though some of them

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simply mention whine noise as a key issue.

2. AN OVERVIEW OF MAJOR NOISE PROBLEMS

The short course covers major gear noise problems and identifies their sources. Table 1 provides an overview of two common gear noise problems: gear whine and gear rattle. Sometimes the gear noise term is synonymous with whine as this type of noise is characterized at the gear mesh frequencies and their side bands; thus the course spends about 80% of its time on this problem. Typically whine levels depend on the mean torques (T) and speeds (Ω). Many examples are considered to fully illustrate its nature and source-path-receiver characterization. At very low loads, rattle noise is introduced. Vibro-impacts, induced by backlash between meshing gears, lead to excessive vibration, noise, and dynamic loads in many geared rotating systems such as automotive transmissions, machine tools, and appliances. Excessive backlash between gears could enhance noise while insufficient backlash could create interference problems, and even the lubrication oil could be trapped. The gear rattle problem is more pronounced in essentially unloaded meshes. Therefore, the rattle problem is illustrated via vibro-impacts in the manual transmission of a ground vehicle.

Table 1 - An overview of two common gear noise problems

Issue	Whine	Rattle
Nature	<ul style="list-style-type: none"> • Steady state vibrations of gear pair(s) • Modulated tones (gear mesh frequencies and side bands) 	<ul style="list-style-type: none"> • Backlash-induced single- and double-sided impacts and tooth separations • Cyclic transients
Source(s)	Gear mesh interface primarily (transmission error, mesh stiffness variations, sliding friction, etc.)	Torque pulsations primarily from source (or load)
Mean Torque Load	At all loads, though high noise levels are seen at higher loads	At low or zero mean loads
Key Factors	<ul style="list-style-type: none"> • Manufacturing errors • Gear contact mechanics • Tooth modification 	<ul style="list-style-type: none"> • Torsional system issues including drag torque and inertial distribution • Interactions among nonlinear elements

3. COVERAGE OF TOPICS

On the first day, the lecturers discuss why even perfect gears make noise. They present in both qualitative and quantitative terms how gear design parameters and manufacturing errors affect noise. The concept of gear transmission error, one of the major contributors to gear noise, is developed, and methods of predicting transmission errors from design and manufacturing data are presented. Participants get a clear physical insight into the problems they face and how they may apply course knowledge to help solve their gear noise problems. A popular feature of this course is the interspersing of demonstrations with lectures. The extensive measurement and computer software capabilities of the OSU's Gear and Power Transmission Research Laboratory <gearlab.org> allow the lecturers to do this in a simple and non-commercial manner.

On the second day, lecturers concentrate on gear system dynamics and acoustics, transmission error calculations, and advanced signal processing. The third day's lectures briefly discuss the sources and simulation models of gear rattle and the activities of the Gear and Power Transmission Research Laboratory as well as spending several hours in the case history workshop. Throughout the course,

laboratory and computer software demonstrations are used to illustrate gear noise measurement and analysis techniques. The facilities of the Gear and Power Transmission Research Laboratory <gearlab.org> and the Acoustics and Dynamics Laboratory <autonvh.org> are used for these demonstrations. The round table discussions on Day 4 are intended to foster interactive problem solving discussions on the following topics. 1. Application of basic concepts covered in the lectures to practical problems attendees may have. 2. Advanced computer modeling methods used for transmission errors, geared system dynamics, gear rattle etc. 3. Experimental methods for casing dynamics, and acoustics, advanced signal processing, transmission error measurement, etc. 4. Discussion of pertinent literature and prior approaches utilized to address difficult problems. Several parallel sessions are organized to suit the needs of attendees. The latest agenda can be found on <nvhgear.org>.

4. SOURCE-PATH-RECEIVER CONCEPTS FOR GEAR WHINE

The source-path-receiver concept of Fig. 1 is employed to predict gear whine noise excited by both the static transmission error (STE) and sliding friction. These two excitations are inputs to a linear 8 degree-of-freedom (DOF) model, which is characterized by natural frequencies (ω_r) and mode shapes (ϕ_r). This study focuses on the prediction of dynamic bearing forces in both the line-of-action (LOA) and off-line-of-action (OLOA) directions. These forces are coupled at the bearings with housing structures which cause the out-of-plane vibrations of housing panels. The structural velocity of the housing is radiated as sound pressure, where it is perceived by the receiver. A simple model utilizing measured acoustic-structural transfer functions (pressure/acceleration or p/a) may then be employed to predict the sound pressure level at the gear mesh harmonics. The transfer functions may be measured (on a test rig) or calculated [6,7].

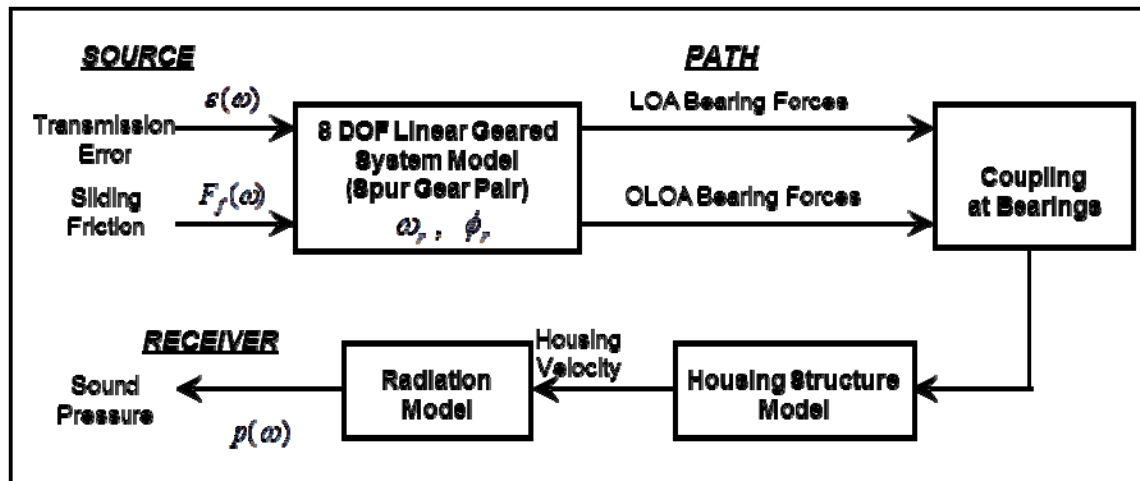


Figure 1 - Conceptual description of the vibro-acoustics of a single-mesh geared system with two excitations, given linear system assumption

The schematic for an 8 DOF gear pair system (as displayed by a block in Fig.1) is shown in Fig. 2. The pinion (subscript p) and gear (subscript g) each have one vibratory angular motion θ as well as two translational motions x and y corresponding to the LOA and OLOA motions. The base radius and inertia are denoted by R and J with averaged mesh stiffness represented by k_m . Symbol m represents the mass of the pinion/gear along with contributions from the respective shafts. The inertias of the motor and load are denoted by J_d and J_L , and the torsional input and output shaft stiffness (viscous damping coefficients) are given as k_{Td} and k_{TL} (c_{Td} and c_{TL}). The input and output torques at the motor and load are T_d and T_L . Here, both shafts are modeled as simply supported beams with the effective shaft-bearing stiffness elements designated by k_x and k_y in the LOA (x) and OLOA (y) directions, respectively. The corresponding viscous damping coefficients of similar notation are also included. Besides the loaded STE displacement excitation $\varepsilon(t)$ at the gear mesh in the LOA direction, the friction force excitation $F_f(t)$ is assumed to act externally at gear mesh in the OLOA direction.

Further, rating indices for gear whine could be computed based on the frequency-domain objective descriptors that employ physical vibro-acoustic measurements [8]. Implementation of such a rating scheme requires several weighting functions and parameters which must be determined specifically for each product or machine considered. Once these weighting functions and parameters are found, the method provides a useful and cost effective quality rating tool which can be used to assess various engineering changes on the reduction of objectionable gear whine during the product development phase. Further refinements in the proposed method are possible based on extensive applications to specific gear noise problems [8]. Variations of this new rating method may be applicable to other types of quasi-steady state machinery noise problems which exhibit similar noise signatures.

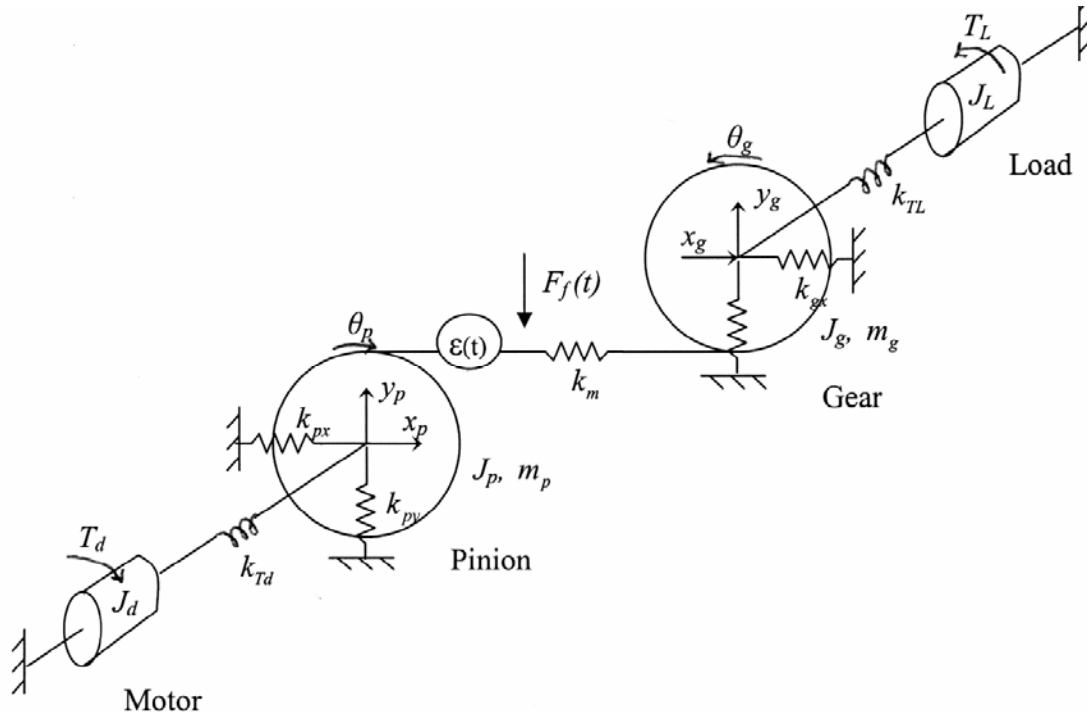


Figure 2 - Schematic of the whine source (8 DOF geared system) model with two gear noise sources for a spur gear pair (damping elements not shown).

5. ENGINEERING SOLUTIONS TO GEAR WHINE

The first step in the noise control strategy is the diagnosis of the problem. This implies detailed noise and vibration experiments over a range of applicable torques and speeds including frequency spectra, waterfall plots, and order tracks. Gear mesh frequencies, ghost frequencies (if any), resonances, and other peaks must be properly identified given the geared system kinematics, vibration modes, and forced responses under the real operating conditions. Correlation between static (or dynamic) transmission error measurements, casing accelerations, and sound pressure levels are desirable. For the whine problem, gear profile and lead measurements must be made to assess profile errors due to manufacturing processes along with a determination of the contact patterns under a specific torque T (for instance). Next, the gear contact mechanics codes (such as Ohio State's Load Distribution Program [9,10]) may be utilized to calculate $e(t)$ and $k_m(t)$ as well as their spectral contents at mf_m . At this stage, the effects of gear design parameters, including profile modifications, may be studied. The intent is to minimize the excitations at a given T . Refer to Houser and Singh [11] for some examples. Also, examine the natural frequencies and torsional-flexural modes of the internal (rotating) system. Based on the frequency and forced responses of the linear system (with no tooth separation), an assessment of the dynamic forces at the gear mesh interfaces and bearings can be made. These excitation forces can then be related to the radiated sound pressures via analytical or computational models for transmissibility across the bearings, casing, and supporting structure motions and radiation mechanisms. Overall, the whine noise should be viewed as the geared system

noise problem though the vibrational sources originate at the gear interface(s). In the case of an open geared system, direct radiation from the gear meshes may occur. Thus, one must examine strategies to reduce noise by focusing at one or more of the following: gear design and kinematics, profile and lead modification, surface finish and lubrication considerations, torsional-flexural modes of the internal system, modes of the gear blanks, stiffness of shafts and bearings, stiffness matrix of the rolling element bearings, misalignments and eccentricities casing dynamics, casing mounts and supporting structures, and finally the radiation properties of the casing and other surfaces. Damping treatments either on the gear blanks or the casing structure would work only when resonances are involved. Given strong dynamic interactions between source(s) and structure-borne noise paths, caution must be exercised to ensure that overly compliant paths or mounts do not alter the sources at the meshes. Multi-disciplinary skills are often needed to solve difficult gear whine problems.

A summary of key factors that lead to noise reduction is given below, though the user must exercise caution: 1. Lower mesh frequencies f_m (by decreasing the number of teeth or the speed) to reduce L_p at lower frequencies (say below 1 kHz). 2. In the absence of strong resonances, L_p is proportional to $20 \log_{10} |e|$ where $|e|$ is the magnitude of static transmission error. 3. Tip relief and other profile modifications reduce sound, generally at the design loads. 4. Helical gears are generally quieter over spur gears. 5. Gears with high contact ratios typically reduce the excitation. 6. Mobility mismatches in internal and external systems may yield some benefits provided the source level does not change. 7. Bearing type and its installation could affect the system noise levels by up to 6 dB. Similarly, many other assertions may be made though many of the reported claims are rather application specific and depend on the gear accuracies and operational conditions [1-5,11].

6. GEAR RATTLE PROBLEM

For the rattle problem in vehicle (manual) transmission, a mathematical formulation generates an interesting criterion that is related to the dynamic torque transmissibility across the clutch damper (R_T). For the rattle-free case, the following mathematical condition is derived based on several assumptions [11-12]: $R_T < \left\{ \left(c_4 \Omega_e / |T_p| \right) \left[I_1 + I_2 / I_4 \right] \right\}$. Here, I_1 is the flywheel inertia, I_2 is the clutch hub inertia, I_4 is the output/input counter gear inertia, Ω_e is the engine speed, c_4 is the drag torque coefficient, T_{d4} is the drag torque (which is $c_4 \Omega_e$), and $|T_p|$ is the pulsating torque. The following design solutions emerge: (i) higher drag torque on output gear (T_{d4}), or high c_4 , Ω_e , (ii) very small excitation $|T_p|$, and (iii) large $(I_1 + I_2)$ compared to I_4 . From a typical R_T curve for a single degree of freedom linear system, one knows that $r = \omega_p / \omega_2$ should be greater than $\sqrt{2}$, and ζ_2 should be kept as low as possible. The natural frequency ω_2 can be easily altered by changing clutch stiffness k_{c1} ; for low transmissibility across the clutch, k_{c1} should be as low as possible. Additionally, the following desirable conditions for the rattle-free case emerge: (i) higher c_4 and engine idling speed Ω_e to yield large drag torque T_{d4} ; (ii) very small $|T_p|$ - this is intuitively obvious; and (iii) very large flywheel and clutch inertia $(I_1 + I_2)$ compared to the intermediate gear I_4 - this is in agreement with experimental results [11-12]. Thus, one finds that the formulation yields several design guidelines which are consistent with experimental and numerical findings [11-12]. See Table 2 for a summary.

7. CASE HISTORY WORKSHOP

A novel approach to discussing "real life" gear noise and dynamics problems has been used in this course since its inception. The workshop, which has been lauded by past attendees for its practical flavor, takes place on the third day of the course. The purpose of this workshop is to allow the course instructors and participants to interact and discuss gear noise and dynamics case histories presented by course attendees. They are asked to present a brief synopsis of problems they have encountered or of a procedure they have used for gear noise analysis and reduction. Possible approaches to solve each problem will be discussed.

One example of workshop type problems is given below. The sources of rattle or periodic vibro-impacts in mechanical systems are clearance non-linearities, which include backlashes, multi-valued springs, hysteresis, and the like. Table 3 lists some key parameters that influence conditions for single sided or double sided impacts in a vehicle transmission. The following should be

recognized: (i) the rattle problem is generally a dynamic system problem, i.e., it is not a gear dynamic issue (such as whine); (ii) a suitable nonlinear torsional model of the system is often needed to understand the basic characteristics, to find optimal design solutions, and to examine signal processing and sound perception issues; and (iii) rattle problems are best analyzed in time domain even though we can use other domains to obtain some fundamental properties of a vibro-impact oscillator. For details, refer to Singh et al. [11-12] and other literature cited in these articles.

Table 2 - Effect of vehicle transmission design trends on gear rattle

Design Change	Parameter	Rattle Level or Likelihood of Rattle Occurring
<ul style="list-style-type: none"> • Fewer number of engine cylinders • Diesel engines in place of gasoline engines • Turbo-charging 	$ T_p \uparrow$	\uparrow
<ul style="list-style-type: none"> • Reduced flywheel inertia 	$I_4 \downarrow$	\uparrow
<ul style="list-style-type: none"> • Synthetic lubricants or higher temperatures 	Lower viscosity	\uparrow
<ul style="list-style-type: none"> • Addition of 5th speed to a 4 speed transmission 	I_4 increase but T_{d4} remains the same	\uparrow
<ul style="list-style-type: none"> • High system load 	$\Omega_e \downarrow$	\uparrow

Table 3 - Factors influencing rattle levels in vehicle transmissions (manual type); these are often brought up by course attendees

- | |
|---|
| <ul style="list-style-type: none"> • Engine type and torque pulsations (gasoline, diesel, turbo-charged) • Mean torque or preload • Flywheel and inertial distribution within system • Clutch dampers [spring rates, friction, hysteresis], damper, etc. • Backlash or clearances (gears, synchronizers, hub splines, bearing) • Drag torque, oil level and oil viscosity, temperature, etc. • Impact damping mechanism • System load affecting torsional dynamics • System resonances • Structure-borne or airborne path(s) • Other factors unique to the application |
|---|

8. ROUND TABLE PROBLEM SOLVING SESSIONS

The round table discussions on Day 4 are intended to foster interactive problem solving discussions, based on the needs of attendees. Examples include the following: 1. Application of basic concepts covered in the lectures to practical problems attendees may have. 2. Advanced computer modeling methods used for transmission errors, geared system dynamics, gear rattle, etc. 3. Experimental methods for casing dynamics and acoustics, advanced signal processing, transmission error measurement, etc. 4. Discussion of pertinent literature and prior approaches utilized to address difficult problems.

For the sake of illustration, consider the noise radiated by the casing. Analytical vibro-acoustic models of a simple geared system are developed and experimentally validated. An eight

degree-of-freedom model incorporating the off-line-of-action direction is analytically formulated for a spur gear pair. Next, a model of the gearbox with embedded bearing stiffness matrices is developed to characterize the structural paths and to calculate the surface velocity distributions. Predictions are first validated by comparing with structural modal tests and transfer function measurements from gear mesh to the housing plates. Radiated noise is then estimated by using two approximate methods, namely the Rayleigh integral method and a substitute source technique. Fig. 3 compares the sound pressure measured at the microphone (6 inches above the top plate) to predictions over a range of pinion torque given $\Omega_p = 4875$ RPM and 140 °F. Notice that the second mesh harmonic, which is most susceptible to the sliding friction, becomes increasingly more dominant at higher torques for this example case. The proposed Rayleigh integral method and substitute source technique are capable of calculating the acoustic field and quantifying the frictional noise. Next, Fig. 3 compares the sound pressure level predicted under $T_p = 500$ lb-in (close to the “optimal” load where transmission error is minimized) and under high torque with $T_p = 800$ lb-in. At each gear mesh frequency, the individual contributions of transmission error (via the LOA path) and frictional effects (via the OLOA path) are compared to the overall whine noise. The proposed formulation provides an efficient analytical and computational tool to quantify the relative contribution of sliding friction to the structure-borne noise, which is found to be significant when the transmission error is minimized say via tooth modifications [7,11].

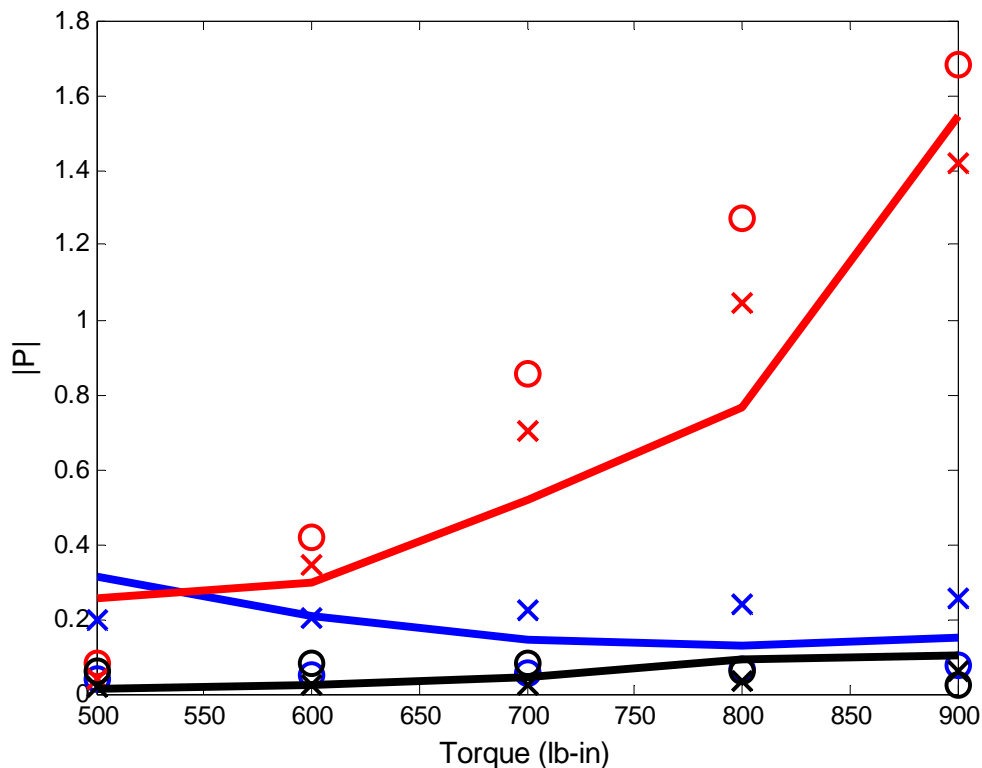


Figure 3: Illustrative sound pressures (Pa) for a spur gear set at the first three gear mesh frequencies (with speed $\Omega_p = 4875$ RPM) over a range of torque T_p at 140 °F. Key: —, measurements (6 inches above the top plate); \circ , Rayleigh integral predictions; \times , substitute source predictions. Color code: Blue, gear mesh frequency harmonic $m = 1$; red, $m = 2$; black, $m = 3$. Note that this formulation utilizes analytical descriptions of sources, paths, and radiators.

9. CONCLUSION

Integrated models of a gearbox and its internal components are essential for accurate predictions of gear noise, and most studies on gearbox system dynamics rely on a combination of detailed finite element, boundary element and semi-analytical methods. However, finite and boundary element methods with high resolution may require much computational time for parametric studies. In such

cases, simpler models for the gearbox are more desirable. Further, this article has presented some aspects of gear rattle. Current research and future plans focus on the development of new or improved semi-analytical and computational methods, impact damping mechanisms, sound perception metrics, and optimization of driveline parameters for rattle-free conditions [13-24].

Some important gear whine noise topics have not been addressed by this article, primarily due to space limitations. These include transmission error models and measurements, gear dynamics and resulting linear and nonlinear behavior, spectral modulations, gear blank modes, transmissibility across the bearings, casing dynamics, role of damping in controlling structure-borne noise and sound radiation, casing mounts and struts, sound radiation mechanisms, airborne sound control, failure diagnostics, and finally, active control of gear noise [13,18,21-24]. Likewise, the nonlinear characterization of various torsional sub-systems that control gear rattle have not been addressed; also, a related problem (clunk) has not been covered [19-20]. All of these topics are addressed by the literature cited, as well as by gear dynamics and gear noise courses that are regularly taught by the author and his colleague (Dr. D. Houser) [11,24]. Potential attendees should go to the <nvhgear.org> site for future offerings and coverage of topics.

Some of the benefits of the course include the training of noise control engineers to better understand the mechanisms of gear noise generation, methods by which gear noise is measured and predicted, and techniques employed in gear noise and vibration reduction. This particular short course has acted as an incubator for fundamental and applied research in gear dynamics, noise tribology, design, and measurement methods. It has led to a successful Gear Lab <gearlab.org> which provides a world class gear research facility along with a venue for training graduate students. Therefore this course has benefited both industry and academia.

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