The modeling and free vibration analysis of coupled plates of various types

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ABSTRACT
This paper is concerned with the modeling and free vibration analysis of coupled plates of various types, which includes T-shape plates, cross shape plates and a panel-linked double-panel structure. The in-plane vibration and bending vibration are considered in coupled structures. The vibration problems are solved using an improved Fourier series method in which the in-plane displacement and bending displacement are expressed as the superposition of a double Fourier cosine series and several supplementary functions to ensure (improve) the uniform convergence (rate) of the Fourier series expansion. The dynamic responses of the coupled plates are obtained using the Rayleigh-Ritz procedure based on the energy expressions for the coupled system. The accuracy and effectiveness of the proposed method are validated through numerical examples and comparison with results obtained by the finite element analysis.

Keywords: T-shape plates, cross shape plates, panel-linked double-panel structure

1. INTRODUCTION
The coupled plates including T-shape plates, cross shape plates and a panel-linked double-panel structure exist in various engineering occasions, such as ship hulls, aircraft cabins and building structures. The modeling and free vibration analysis of coupled plates capture the growing attentions from researchers and design engineers. This kind of problem can be solved by the finite element method (FEM), lacking of flexibility with the changing parameter in the analysis.

The analysis of vibration characteristics were investigated by researchers with the well-known numerical methods. In the existing investigations on coupled plates, considerable contributions (1-3) were made by researchers. Wang et al. (4) adopted a substructure approach to study the vibration of L-shaped plates and investigate the power flow characteristics of L-shaped plates. Bercin (5) studied the effects of in-plane vibrations the energy flow between coupled plates. Cheng et al. (6) made a research on energy transmission in a mechanically-linked double-wall structure coupled to acoustic enclosure. Chen and Jin (7) investigated the vibration behaviors of a box-type structure built up by plates. A combination of a traveling wave and modal solution was used by Kessissoglou (8) to describe the flexural and in-plane displacements functions of the plates in L-shaped plates. Cuschieri (9) used a mobility approach to investigate structural power-flow of an L-shaped plate, then Cuschieri and McCollum (10) used the mobility power flow approach to analyze in-plane and out-of-plane waves power transmission through L-plate junction. Dimitriadis and Pierce (11) proposed an analytical solution for the power exchange between strongly coupled plates under random excitation. The concept of receptance (a numerical approach) proposed by Azimi et al. (12) were used by Farag and Pan (13) to analyze effects of the coupling angle for the coupling edge, which were extended by Kim et al. (14) to investigate the interactions of transmission of bending waves in coupling conjunction for rectangular plates.

In the existing techniques, some assumptions are used. Modeling the real-life plate structures is more of practical significance. In this paper, the modified Fourier series method proposed by Li (15) is
applied for the modeling and the free vibration analysis for coupled plates. The Rayleigh’s method is applied to calculate the coefficients of the modified Fourier series. The boundary conditions and coupling conditions can be simulated by four types boundary springs at edges and four types coupling spring at coupling conjunctions. The convergence and accuracy of the current method are proved by comparing the natural frequencies and mode shapes with the finite element analysis results. The effect of dimensional parameters and the coupling conditions are investigated.

2. THEORETICAL FORMULATIONS

2.1 Description of the models

The Analytical models for the coupled T-shape plates structure, cross shape plates structure and panel-linked double-panel structure under investigation, which are composed of elastic plates, are shown in Fig.2.1(a), (b) and (c), respectively. The geometry and the coordinate systems for these plates structure were showed in Fig. 2.1, in which, four types of uniform boundary spring are introduced to complete model the general restrained boundary condition. Similarly, along the structural conjunction, four types of uniform coupling spring, which are \( k_1, k_2, k_3 \) and \( k_4 \), are introduced to completely model the coupling effect, and \( \theta \) is the coupling angle of coupled plates structure. The displacements of the elastic plates with respect to this coordinate system are described by \( u, v \) and \( w \) in the \( x, y \) and \( z \) directions, respectively.

![Diagram showing analytical models for coupled plates structure](image)

(a) – T-shape plates structure  
(b) – cross shape plates structure  
(c) – the panel-linked double-panel structure

Fig. 2.1 – Analytical models for coupled plates structure

2.2 Theory for bending and in-plate vibration of plates

The partial differential equations for the bending and in-plate displacement functions of plate 1 are depicted as:

\[
D \nabla^4 w_i(x_1, y_1, t) + \rho_i h_i \frac{\partial^2 w_i(x_1, y_1, t)}{\partial t^2} = 0
\]
\[
\frac{\partial^2 u}{\partial x^2} + \frac{1}{2} (1 - \mu_i) \frac{\partial^2 u}{\partial y^2} + \frac{1}{2} (1 + \mu_i) \frac{\partial^2 v_i}{\partial x \partial y} + \frac{1}{c_{l1}^2} \omega^2 u_i = 0
\]

(2)

\[
\frac{\partial^2 v_i}{\partial y^2} + \frac{1}{2} (1 - \mu_i) \frac{\partial^2 v_i}{\partial x^2} + \frac{1}{2} (1 + \mu_i) \frac{\partial^2 u_i}{\partial x \partial y} + \frac{1}{c_{l1}^2} \omega^2 v_i = 0
\]

(3)

where \( \rho_i \) and \( h_i \), respectively, denote the mass density and the thickness of plate 1, \( D_i = E_i h_i^3 / (12(1-\mu_i^2)) \) is the bending rigidity of plate 1, \( E_i \) and \( \mu_i \), respectively, are the Young's modulus and the Poisson ratio of the plate 1, \( \nabla \) is the Laplace operator, \( c_{l1} = \sqrt{E_i / \rho_i (1-\mu_i)} \) is the p-wave speed in plate 1 structure and \( \omega \) is the angular frequency of plate.

The boundary conditions for the elastic plate are:

The boundary conditions for the elastic plate considering in-plate direction are:

on \( x_i = 0 \),

\[
k_{m10} w_i = -D_i \left( \frac{\partial^3 w_i}{\partial x^2 \partial y} + (2 - \mu_i) \frac{\partial^3 w_i}{\partial x \partial y^2} \right), \quad K_{m10} \frac{\partial w_i}{\partial x} = D_i \left( \frac{\partial^2 w_i}{\partial x^2} + \mu_i \frac{\partial^2 w_i}{\partial y^2} \right)
\]

(4)

\[
k_m u_i = -\frac{\partial u_i}{\partial x}, \quad k_{m1} v_i = -\frac{\partial v_i}{\partial y},
\]

(5)

on \( x_i = Lx_i \),

\[
k_{n10} w_i = D_i \left( \frac{\partial^3 w_i}{\partial y^2} + (2 - \mu_i) \frac{\partial^3 w_i}{\partial x \partial y^2} \right), \quad K_{n10} \frac{\partial w_i}{\partial y} = D_i \left( \frac{\partial^2 w_i}{\partial y^2} + \mu_i \frac{\partial^2 w_i}{\partial x^2} \right),
\]

(6)

\[
k_{n1} u_i = -\frac{\partial u_i}{\partial x} - \mu_i \frac{\partial v_i}{\partial y}, \quad k_{n1} v_i = -\frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y},
\]

(7)

on \( y_i = 0 \),

\[
k_{n01} w_i = -D_i \left( \frac{\partial^3 w_i}{\partial x^2 \partial y} + (2 - \mu_i) \frac{\partial^3 w_i}{\partial x \partial y^2} \right), \quad K_{n01} \frac{\partial w_i}{\partial x} = D_i \left( \frac{\partial^2 w_i}{\partial x^2} + \mu_i \frac{\partial^2 w_i}{\partial y^2} \right),
\]

(8)

\[
k_{n0} u_i = -\frac{\partial u_i}{\partial x} - \mu_i \frac{\partial v_i}{\partial y}, \quad k_{n0} v_i = -\frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y},
\]

(9)

on \( y_i = Ly_i \),

\[
k_{n01} w_i = -D_i \left( \frac{\partial^3 w_i}{\partial x^2 \partial y} + (2 - \mu_i) \frac{\partial^3 w_i}{\partial x \partial y^2} \right), \quad K_{n01} \frac{\partial w_i}{\partial x} = D_i \left( \frac{\partial^2 w_i}{\partial x^2} + \mu_i \frac{\partial^2 w_i}{\partial y^2} \right),
\]

(10)

\[
k_{n0} u_i = -\frac{\partial u_i}{\partial x} - \mu_i \frac{\partial v_i}{\partial y}, \quad k_{n0} v_i = -\frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y},
\]

(11)

where \( \mu_i \) is the Poisson ratio of plate 1, \( k \) and \( K \), respectively, denote the stiffness of the translational restraining spring and the rotational restraining spring. The effect and the position are represented by the subscripts.

### 2.2.1 Improved series representations of the displacement functions

The displacement functions can be described by a two-dimensional Fourier cosine series method. However, discontinuity problems would be encountered in the displacement partial differential along the edges by used such a traditional Fourier cosine. To overcome the difficulty, the bending displacement function is depicted by an improved Fourier series method, which is the superposition of a two-dimensional Fourier cosine series and eight supplementary functions. Therefore, The displacement components of plate 1, \( w_i \), \( u_i \) and \( v_i \) can be described as:

\[
w_i(x_i, y_i) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} A_{m,n} \cos \lambda_{m,Lx1} x_i \cos \lambda_{n,Ly1} y_i
\]

(12)

\[
u_i(x_i, y_i) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} B_{m,n} \cos \lambda_{m,Lx1} x_i \cos \lambda_{n,Ly1} y_i
\]

(13)

\[
+ \sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} h_{m,n} \xi_{m,Lx1} (x_i) \cos \lambda_{n,Ly1} y_i + \sum_{m=0}^{\infty} \sum_{n=-\infty}^{\infty} h_{m,n} \xi_{n,Ly1} (y_i) \cos \lambda_{m,Lx1} x_i
\]
\[ \psi_i(x_i, y_i) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} C_{mn} \cos \lambda_{m,Lx} x_i \cos \lambda_{n,Ly} y_i + \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} C_{nk} \cos \lambda_{k,Lx} x_i \cos \lambda_{l,Ly} y_i \]  \tag{14}

where \( \lambda_{m,Lx} = m \pi / Lx \), \( \lambda_{n,Ly} = n \pi / Ly \), \( n \) and \( m \) are all integers, describing the spatial characteristic of a particular mode, \( A_{mn} \), \( B_{mn} \), \( C_{mn} \), \( A_{ln} \), \( a_{ln} \), \( b_{ln} \), \( c_{ln} \) and \( \tilde{c}_{ln} \) are the modal amplitude constants for mode \((m, n)\). The expressions for the supplementary functions relating to \( x_i \) are defined as:

\[
\begin{align*}
\zeta_{12,1i}(x_i) &= \frac{9Lx_1}{4\pi} \sin \left( \frac{\pi x_i}{2Lx_1} \right) - \frac{Lx_1}{2\pi} \sin \left( \frac{3\pi x_i}{2Lx_1} \right), \\
\zeta_{22,2i}(x_i) &= -\frac{9Lx_1}{4\pi} \cos \left( \frac{\pi x_i}{2Lx_1} \right) + \frac{Lx_1}{2\pi} \cos \left( \frac{3\pi x_i}{2Lx_1} \right) \\
\xi_{12,1i}(x_i) &= Lx_1 \zeta_{12,1i}(x_i), \\
\xi_{22,2i}(x_i) &= Lx_1 \zeta_{22,2i}(x_i) 
\end{align*} \tag{15}
\]

It is uncomplicated to prove that

\[
\zeta_{12,1i}'(0) = \xi_{12,1i}'(0) = \zeta_{22,2i}'(Lx_1) = \xi_{22,2i}'(Lx_1) = 1, \tag{17}
\]

\[
\zeta_{12,1i}''(0) = \xi_{12,1i}''(0) = \zeta_{22,2i}''(Lx_1) = \xi_{22,2i}''(Lx_1) = 0, \tag{18}
\]

other related partial derivative supplementary functions are zero. The expressions for the supplementary functions relating to \( y \) can be obtained by substituting \( x \) with \( y \) in Eq. (15). These conditions make related derivative of the displacement function smooth and continuous in the whole solving domain.

### 2.3 Solution procedure of coupled plates

In this work, the unknown expansion coefficients of the displacement functions for the coupled plates structure system are calculated by using the Rayleigh–Ritz procedure, which is actually equivalent to solve the governing equations, the boundary conditions and coupling conditions directly.

The Lagrange function for the coupled T-shape plates structure, cross shape plates structure and the panel-linked double-panel structure, respectively, were written as:

\[
\begin{align*}
L_i &= U_i - T_i, \\
L_c &= U_c - T_c, \\
L_g &= U_g - T_g
\end{align*} \tag{19}
\]

where \( U \) and \( T \), respectively, are the total potential energy, the total kinetic energy of coupled plates structure system, the subscript \( i \), \( c \) and \( g \) represent the coupled T-shape plate structure, cross shape plates structure and the panel-linked double-panel structure, respectively.

\[
\begin{align*}
U_i &= U_{\text{bending}} + U_{\text{in-plate}} + U_{\text{in-panel}} + U_{\text{coupling}} + U_{\text{in-plate}} \\
T_i &= T_{\text{bending}} + T_{\text{in-panel}} + T_{\text{in-panel}} \\
U_c &= U_{\text{bending}} + U_{\text{in-plate}} + U_{\text{in-panel}} + U_{\text{in-panel}} + U_{\text{in-panel}} + U_{\text{in-panel}} \\
T_c &= T_{\text{bending}} + T_{\text{in-panel}} + T_{\text{in-panel}} + T_{\text{in-panel}} + T_{\text{in-panel}} \\
U_g &= U_{\text{bending}} + U_{\text{in-plate}} + U_{\text{in-panel}} + U_{\text{in-panel}} + U_{\text{in-panel}} + U_{\text{in-panel}} + U_{\text{in-panel}} \\
T_g &= T_{\text{bending}} + T_{\text{in-panel}} + T_{\text{in-panel}} + T_{\text{in-panel}} + T_{\text{in-panel}} \tag{20}
\end{align*}
\]

where \( U_{\text{bending}} \) and \( U_{\text{in-plate}} \), respectively, are the total bending potential energy including the total potential energy stored in the elastic springs at edges of the plate 1 and the total in-plane potential energy of the plate 1, the subscript 2 represents plate 2, \( U_{\text{coupling}} \) is the total coupling potential energy of the coupled T-shape plate system.

The total potential energy and kinetic energy of the elastic plate 1 are, respectively, expressed as...
where $G_i = E_i h_i / (1 - \mu_i^2)$ is the tension stiffness of plate 1, the total potential energy and kinetic energy of the elastic plate 2 or plate 3 can be obtained by substituting subscript 1 with subscript 2 or subscript 3 in the above equations. Because the functions: $U_{\text{coupling}}^c$, $U_{\text{coupling}}^g$ and $U_{\text{coupling}}^e$ resemble the function $U_{\text{coupling}},$ the description of $U_{\text{coupling}}$ will be given in this paper as:

\[
U_{\text{coupling}} = \frac{1}{2} \int_0^{L_1} \left[ k_n u_i + k_{n+1} v_i \right] dx_i + \frac{1}{2} \int_0^{L_1} \left[ k_{n+1} u_i + k_{n+2} v_i \right] dx_i
\]

(26)

where $G_i$ is the rotational coupling stiffness at coupling conjunction, $k_1$, $k_2$ and $k_3$ are the linear coupling stiffnesses at coupling conjunction in $z_i$ direction, $x_i$ direction and $y_i$ direction, respectively. $L_{\text{xc}}$ is the coupling conjunction of plate 1.

By substituting Eqs.(12)-(14) into Eq. (19) and then applying Rayleigh-Ritz procedure to solve, the linear algebraic equations about the unknown coefficients for coupled T-shape plates structure, cross shape plates structure and the panel-linked double-panel structure can be obtained, respectively.

\[
E_i - \omega^2 M_i E_i = 0,
\]

\[
E_c - \omega^2 M_c E_c = 0,
\]

\[
E_g - \omega^2 M_g E_g = F_g
\]

(31)

where $K$ and $M$, respectively, are stiffness and mass matrices of the coupled plates structure, the natural frequencies and eigenvectors of coupled T-shape plates structure, cross shape plates structure or the panel-linked double-panel structure can be obtained by solving Eq. (31). If the eigenvectors
are calculated, the dynamic response on the coupled plates can be determined by Eqs.(12) - (14).

3. NUMERICAL RESULTS AND DISCUSSIONS

3.1 Numerical results and discussions of coupled plates

3.1.1 Validation

In this section, the material parameters and dimensional parameters of the plates belonged the coupled plates structure are identical: $E = 71 \text{ GPa}$, $\mu = 0.3$, $\rho = 2700 \text{ Kg/m}^3$, the thickness $h = 0.002 \text{m}$ and $L_x \times L_y = 1 \text{ m} \times 1 \text{ m}$. Coupling conditions are: $K_1 = \infty$, $K_2 = \infty$, $K_3 = \infty$ and $K_4 = \infty$. Boundary conditions of plates are clamped-supported. Table 3.1 shows the first six natural frequencies of the coupled plates structure.

Table 3.1 – Natural frequencies (Hz) for the coupled coupled plates structure system

<table>
<thead>
<tr>
<th>Coupled types</th>
<th>Mode number</th>
<th>$M\times N$</th>
<th>FEA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>4×4</td>
<td>5×5</td>
</tr>
<tr>
<td>T-shape plates structure</td>
<td>1</td>
<td>17.284</td>
<td>17.227</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>33.095</td>
<td>33.036</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>36.012</td>
<td>35.960</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>42.630</td>
<td>42.228</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>49.264</td>
<td>49.264</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>51.449</td>
<td>51.092</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>17.777</td>
<td>17.777</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>32.515</td>
<td>32.462</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>35.856</td>
<td>35.787</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>31.208</td>
<td>31.077</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>35.785</td>
<td>35.690</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>44.571</td>
<td>43.889</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>49.262</td>
<td>49.262</td>
</tr>
</tbody>
</table>

In this paper, it is noted that the stiffness of springs varies from extremely large $(5 \times 10^{11})$ to extremely small (0). In the finite element program ANSYS, the coupling systems are meshed by SHELL63 elements, whose size is 0.02. Figs. 3.1, 3.2 and 3.3, respectively, show some comparisons of the mode shapes for coupled T-shape plates structure, cross shape plates structure and the panel-linked double-panel structure. The corresponding mode shapes obtained from the current method agree well with that obtained from ANSYS.
Fig. 3.1 – Mode shapes for the coupled T-shape plates structure system

Fig. 3.2 – Mode shapes for the coupled cross shape plates

Fig. 3.3 – Mode shapes for the panel-linked double-panel structure
3.1.2 The effect of dimensional parameters of the coupled plates structure

The structural parameters play a crucial role in the vibration behavior of coupled plates structure system. The effect of some dimensional parameters on the natural frequencies for the panel-linked double-panel structure would be discussed in this section. Boundary conditions, coupling conditions and material parameters are the same as ones described in the subsection 3.1.1. Table 3.2 and Table 3.3 show the natural frequencies for the panel-linked double-panel structures with different dimensions \((h_1 = 0.002\text{m}, \ h_2 = 0.002\text{m}, \ h_3 = 0.002\text{m}, \ Lx1 \times Ly1 = 1\text{m} \times 1\text{m}, \ Lx2 \times Ly2 = 1\text{m} \times 1\text{m}, \ Ly3 = 1\text{m})\), various \(Lx3\) and different thickness \((h_1 = 0.002\text{m}, \ h_2 = 0.002\text{m}, \) various \(h_3, \ Lx1 \times Ly1 = 1\text{m} \times 1\text{m}, \) \(Lx2 \times Ly2 = 1\text{m} \times 1\text{m}\) and \(Lx3 \times Ly3 = 1\text{m} \times 1\text{m})\), respectively. The truncation numbers are set as \(M=N=9\).

Table 3.2 –Natural frequencies (Hz) for the panel-linked double-panel structure with various \(Lx3\) (m).

<table>
<thead>
<tr>
<th>(Lx3)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>44.870</td>
<td>48.413</td>
<td>48.713</td>
<td>49.041</td>
<td>59.704</td>
<td>63.005</td>
<td>63.096</td>
</tr>
<tr>
<td>0.10</td>
<td>42.035</td>
<td>46.566</td>
<td>48.608</td>
<td>48.712</td>
<td>57.595</td>
<td>61.107</td>
<td>63.062</td>
</tr>
<tr>
<td>0.15</td>
<td>40.600</td>
<td>44.880</td>
<td>48.652</td>
<td>48.712</td>
<td>56.642</td>
<td>59.747</td>
<td>63.076</td>
</tr>
<tr>
<td>0.20</td>
<td>39.677</td>
<td>43.669</td>
<td>48.671</td>
<td>48.712</td>
<td>56.071</td>
<td>58.832</td>
<td>63.081</td>
</tr>
<tr>
<td>0.25</td>
<td>38.949</td>
<td>42.754</td>
<td>48.681</td>
<td>48.711</td>
<td>55.629</td>
<td>58.178</td>
<td>63.084</td>
</tr>
<tr>
<td>0.30</td>
<td>38.252</td>
<td>42.032</td>
<td>48.687</td>
<td>48.711</td>
<td>55.194</td>
<td>57.684</td>
<td>63.086</td>
</tr>
<tr>
<td>0.35</td>
<td>37.466</td>
<td>41.437</td>
<td>48.691</td>
<td>48.711</td>
<td>54.675</td>
<td>57.293</td>
<td>63.088</td>
</tr>
<tr>
<td>0.40</td>
<td>36.460</td>
<td>40.926</td>
<td>48.694</td>
<td>48.711</td>
<td>53.956</td>
<td>56.967</td>
<td>63.088</td>
</tr>
<tr>
<td>0.45</td>
<td>35.073</td>
<td>40.467</td>
<td>48.696</td>
<td>48.710</td>
<td>52.856</td>
<td>56.577</td>
<td>56.681</td>
</tr>
<tr>
<td>0.50</td>
<td>33.167</td>
<td>40.036</td>
<td>48.697</td>
<td>48.710</td>
<td>50.439</td>
<td>51.144</td>
<td>56.416</td>
</tr>
</tbody>
</table>

Table 3.3 –Natural frequencies (Hz) for the panel-linked double-panel structure with various \(h_3\).

<table>
<thead>
<tr>
<th>(h_3)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0010</td>
<td>8.799</td>
<td>17.799</td>
<td>18.084</td>
<td>26.553</td>
<td>31.236</td>
<td>32.557</td>
<td>36.677</td>
</tr>
<tr>
<td>0.0020</td>
<td>16.646</td>
<td>30.963</td>
<td>35.616</td>
<td>39.079</td>
<td>43.344</td>
<td>48.704</td>
<td>48.708</td>
</tr>
<tr>
<td>0.0025</td>
<td>20.020</td>
<td>34.961</td>
<td>41.927</td>
<td>43.907</td>
<td>48.706</td>
<td>48.709</td>
<td>48.715</td>
</tr>
<tr>
<td>0.0030</td>
<td>23.196</td>
<td>38.434</td>
<td>44.741</td>
<td>48.707</td>
<td>48.710</td>
<td>51.694</td>
<td>53.766</td>
</tr>
<tr>
<td>0.0035</td>
<td>26.317</td>
<td>41.618</td>
<td>47.139</td>
<td>48.708</td>
<td>48.710</td>
<td>58.168</td>
<td>58.320</td>
</tr>
<tr>
<td>0.0040</td>
<td>29.453</td>
<td>44.374</td>
<td>48.708</td>
<td>48.710</td>
<td>49.004</td>
<td>61.743</td>
<td>62.587</td>
</tr>
<tr>
<td>0.0045</td>
<td>32.622</td>
<td>46.570</td>
<td>48.709</td>
<td>48.711</td>
<td>50.405</td>
<td>63.094</td>
<td>63.094</td>
</tr>
<tr>
<td>0.0050</td>
<td>35.808</td>
<td>48.209</td>
<td>48.709</td>
<td>48.711</td>
<td>51.469</td>
<td>63.094</td>
<td>63.094</td>
</tr>
</tbody>
</table>

From Table 3.2 and Table 3.3, it is seen that the natural frequencies for the panel-linked double-panel structure decrease as \(Lx3\) increases except for the third and four modes whose natural frequencies almost remain unchanged. The natural frequencies for the panel-linked double-panel structure increase as \(h_3\) increases.
3.1.3 The effect of coupling conditions of the coupled plates structure

As an important factor of the coupled plates structure system, the effect of coupling conditions should be discussed. Some cases for the panel-linked double-panel structure with varied coupling conditions would be discussed to demonstrate the effect of the coupling conditions. The geometries, material parameters and boundary conditions are the same as the ones described in the subsection 3.1.1. Table 3.4 shows the natural frequencies for the panel-linked double-panel structure with different values of coupling springs \( \{ k_{c1} = \infty, k_{c2} = \infty, k_{c3} = \infty \} \) and the coupling springs: \( K_c = \infty, \) various \( k_{c1}, k_{c2} = \infty, k_{c3} = \infty; \) the coupling springs: \( K_c = \infty, \) various \( k_{c1}, k_{c2}, k_{c3} = \infty; \) and the coupling springs: \( K_c = \infty, \) various \( k_{c1}, k_{c2}, k_{c3} = \infty. \) The truncation numbers are \( M=N=9. \)

Table 3.4 –Natural frequencies (Hz) for the panel-linked double-panel structure with one pair of stiffness-variable coupling springs as the stiffness values of the others are extremely large \((5 \times 10^{11})\)

<table>
<thead>
<tr>
<th>Variable Stiffness</th>
<th>Value</th>
<th>Mode number</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_c )</td>
<td></td>
<td>1 2 3 4 5 6 7 8</td>
</tr>
<tr>
<td>( 10^2 )</td>
<td>14.809</td>
<td>28.988 34.471</td>
</tr>
<tr>
<td>( 10^3 )</td>
<td>16.027</td>
<td>30.223 35.157</td>
</tr>
<tr>
<td>( 10^4 )</td>
<td>16.565</td>
<td>30.879 35.550</td>
</tr>
<tr>
<td>( 10^5 )</td>
<td>16.637</td>
<td>30.954 35.609</td>
</tr>
<tr>
<td>( 10^6 )</td>
<td>16.645</td>
<td>30.962 35.616</td>
</tr>
<tr>
<td>( 10^7 )</td>
<td>16.646</td>
<td>30.963 35.616</td>
</tr>
<tr>
<td>( k_{c1} )</td>
<td></td>
<td>1 2 3 4 5 6 7 8</td>
</tr>
<tr>
<td>( 10^2 )</td>
<td>16.646</td>
<td>17.808 17.808</td>
</tr>
<tr>
<td>( 10^4 )</td>
<td>16.646</td>
<td>20.694 20.694</td>
</tr>
<tr>
<td>( 10^5 )</td>
<td>16.646</td>
<td>34.123 34.123</td>
</tr>
<tr>
<td>( 10^6 )</td>
<td>16.646</td>
<td>35.616 35.616</td>
</tr>
<tr>
<td>( 10^7 )</td>
<td>16.646</td>
<td>35.616 35.616</td>
</tr>
<tr>
<td>( k_{c2} )</td>
<td></td>
<td>1 2 3 4 5 6 7 8</td>
</tr>
<tr>
<td>( 10^2 )</td>
<td>11.082</td>
<td>14.144 25.536</td>
</tr>
<tr>
<td>( 10^3 )</td>
<td>11.450</td>
<td>14.766 25.926</td>
</tr>
<tr>
<td>( 10^5 )</td>
<td>16.118</td>
<td>28.752 34.561</td>
</tr>
<tr>
<td>( 10^6 )</td>
<td>16.587</td>
<td>30.749 35.490</td>
</tr>
<tr>
<td>( 10^7 )</td>
<td>16.639</td>
<td>30.941 35.603</td>
</tr>
</tbody>
</table>

It can be seen that the stiffness of coupling springs \( k_{c3} \) have very little impact on the natural frequencies for the panel-linked double-panel structure. \( k_{c2} \) and \( k_{c3} \) have larger influence on the natural frequencies than other stiffness of coupling springs for the panel-linked double-panel structure.

4. CONCLUSIONS

In this paper, the modified Fourier series method is applied to the modeling and the analysis of coupled plates of various types, which includes T-shape plates, cross shape plates and a panel-linked double-panel structure. The boundary conditions and coupling conditions are simulated by a set of elastic springs of arbitrary stiffness. All the displacements are expressed by the superposition of a two-dimensional Fourier series and several supplementary functions. Because related derivative of the displacement functions are smooth and continuous in the whole solving domain, the coefficients can be solved by using the Rayleigh-Ritz procedure based on energy principle. The reliability and accuracy of current method are verified by comparing the natural frequencies and mode shapes with ANSYS program. Some numerical examples are also conducted in order to illustrate the effect of geometry parameters and coupling conditions on the natural behavior of coupled plates. And the results show that: (a) The natural frequencies for coupled plates increase as the thickness of plate increases; (b) The natural frequencies decrease as the length of plate increases; (c) The natural
frequencies are sensitive to the coupling boundaries: $k_{c_1}$ and $k_{c_2}$.

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REFERENCES