Realization of Air-borne Sound pressure unit with LDA technique by Spectrum and autocorrelation method in a travelling wave tube

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ABSTRACT

To realize the sound pressure unit directly, the method of sound pressure measurement based on acoustic particle velocity with LDA technique was described. In order to get a simple acoustic field, a travelling wave tube was designed. The sound pressure distribution obtained by microphone along the tube was measured. The result showed the acoustic field inside the tube could be considered as travelling wave. The air-borne sound pressure is equal to the product of the air density, sound speed and the particle velocity. The laser Doppler Anemometry was used to measure the particle velocity in the acoustic field. The modulated doppler signal was obtained by TSI laser doppler system and the doppler signal was processed by autocorrelation method and spectrum analysis method. With the spectral analysis of doppler signal, the particle vibration velocity was obtained with the Bessel function analysis and the velocity can also be calculated based on the time of characteristic values of the correlation function. The comparison of sound pressure measured by microphone and the value deduced from the velocity measured by laser Doppler system shows that they have good agreement and the deviation is less than 0.5 dB.

Keywords: Sound Pressure; Laser Doppler Anemometry; Particle Velocity; I-INCE Classification of Subjects Number(s): 71.1

1. INTRODUCTION

The realization of sound pressure unit Pascal is depending on the laboratory standard microphone and its reciprocity. With the pressure and free-field reciprocity method the sensitivity of the microphones could be obtained[1, 2]. This kind of unit realization is indirectly. In metrology, physical quantity always needs trace to a standard source directly, so acoustic scientists tried to get the sound pressure with other physical methods, including the laser piston-phone, and the Rayleigh Disk method. The former method is based on the ideal gas Equation and the sound pressure could be obtained by the volume change in a chamber. But this method is limited in low frequency due to the requirement of the ideal gas equation. While the Rayleigh Disk is used to measure the particle velocity in the sound field by mechanical principle, and the uncertainty is about 0.5 dB, much larger than the uncertainty of the reciprocity method.

With the development of optical technique, Laser Doppler Anemometry (LDA) becomes a new method to measure particle velocity in air and water. When two coherent laser beams are crossed, interference fringes appear. While particles moving through the fringes, they reflect the light into photomultiplier and lead to a Doppler frequency shift. The velocity of the particle could be calculated by measuring the Doppler frequency shift. LDA technique is a non-intrusive method which almost has no disturbance on the flow field. In recent decades, a few researchers focused on the particle velocity measurement in acoustic field by Laser Doppler technique. Taylor K J [3, 4] first described the method of measuring velocity by LDA in a standing wave tube. He also used LDA to measure velocity at a single frequency point in a travelling-wave tube and the result showed that the sound pressure measured by microphone and calculated with the velocity were conformable. Hann DB[5] used LDA to describe the sound fields. With the application of LDA technique in different fields, various ways of signal processing were applied to fluid measurements. Spectral analysis was used by Taylor K J, Davis [6] and MacGillivray [7] et al. Photon correlation method was used by Sharpe[8] and MacGillivray T J [9] et al. T. Koukoulas[10,11] reported the development of an optical system capable of measuring acoustic particle velocities in free-field conditions and agreement within less than 0.6 dB was obtained

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with standard microphone measurements.

To measure the acoustic pressure in free-field using optical method, the optical system is more complicated than in enclosed acoustic tubes. The research work so far has mostly been limited to standing wave tubes. With the comparison of pressure measured at the specific position by LDA and the pressure measured by microphone at the tube end, the agreement was studied. In this condition the position error of the optical probe is one of the uncertainty components. In this paper, a long tube was designed to provide travelling waves field. The characteristic of the acoustic field was analyzed in the frequency from 300 Hz to 1 kHz. Doppler signal provided by a commercial instrument, TSI LDA system, was demodulated using spectral analysis and Photon correlation method. The sound pressure was deduced from the velocity according to their linear relationship. With a laboratory standard microphone, the measurements were compared with the deduction result. And the influencing factors of the LDA method were discussed.

2. THEORY

2.1 Principles of LDA

The principle of LDA is described in Fig.1. Two coherent laser beams cross and an ellipsoidal probe volume is generated, which consisting of dark and bright fringes. The spacing of fringes d could be described as formula (1),

$$d = \frac{\lambda}{2\sin k} \tag{1}$$

Where λ is the optical wavelength, k is half angle between two laser beams. When seeding passes through the probe volume, the scattered light which consists of a frequency shift called doppler frequency is collected by receiving optics and converted into electrical signal. Doppler frequency f_d is related to the velocity v and the space d, so the particle velocity could be described as formula (2),

$$v = d * f_d \tag{2}$$

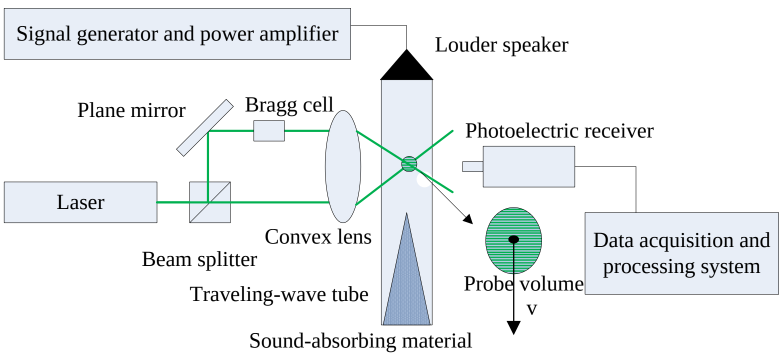


Fig.1. Schematic diagram of LDA

2.2 Spectrum of Doppler signal

Particle velocity has a linear relationship with the modulation coefficient,

$$v = m_f * \Omega * d \tag{3}$$

Where Ω is the modulation frequency, m_f is the modulation index. The Doppler signal is a frequency modulated signal. According to its characteristics in spectrum, the particle velocity can be written as[3,6],

$$\begin{aligned} V_f(t) = V_0\{J_0(m_f) + J_1(m_f)[\cos(w_0+\Omega)t + \cos(w_0-\Omega)t] \\ + J_2(m_f)[\cos(w_0+2\Omega)t) - \cos(w_0-2\Omega)t] + ...\} \end{aligned} \tag{4}$$

Where $V_f(t)$ is the particle velocity. V_0 is the mean velocity. $J_n(m_f)$ is the Bessel function of order n and w_0 is the carrier frequency. From equation (4) it can be derived that the wave spectrum has several side frequency components and each side frequency interval equal to the modulation frequency. If the modulation index is known, the ratio of amplitudes of any two side frequencies is equal to the ratio of corresponding Bessel functions. On the contrary, in the specific spectrum of the signal, the modulation index could be determined by the relative amplitude of side frequencies. Then the acoustic particle velocity could be calculated according to the formula (3).

2.3 Auto-correlation of Doppler signal

The autocorrelation function of Doppler signal with time interval τ can be expressed as[12],

$$R(\tau) \approx \frac{1}{2}\exp(-4\beta^2 u_0^{\ 2})\{1+\cos(Du_0\tau)J_0(Du_m\tau)\} \tag{5}$$

Where,

$$D = 4\pi\sin\theta/\lambda \tag{6}$$

u_0 is the average velocity and u_m is the particle vibration velocity, $\beta = \cos\theta/d'$, d' is the effective diameter of the focused laser beam and J_0 is zero order Bessel function. When there is no Bragg cell in the optical system,

$$u_m = 3.832/D\tau_{\min} \tag{7}$$

Where $\tau_{\min}$ is the time point when the autocorrelation function falls to the first lowest point. When the Bragg cell is used, the inherent frequency offset can be considered as the frequency shift induced by the mean velocity, and usually the real doppler frequency shift in acoustic field is far less than the frequency difference induced by the Bragg cell. So in essence $\cos(Du_0\tau)J_0(Du_m)$ is a modulation of $\cos(Du_0\tau)$ by zero order Bessel function. When the zero order Bessel function gets to zero, the corresponding products of amplitude is gradually reduced to zero. The first root of zero order Bessel function is 2.405, so when formula (5) goes to the first minimum, the following equations should meet,

$$Du_0\tau = n\cdot 2\pi \tag{8}$$

$$Du_m\tau = 2.405 \tag{9}$$

In order to increase the dynamic range of Doppler velocity measurement, the inherent frequency difference of dual beam always was set as high value, for example 40MHz. For data acquisition and signal processing, the downward mixing is always used to reduce the workload of data processing. At this time, the particle velocity can be expressed as,

$$u_m = 0.383 * V_{mix}/n \tag{10}$$

Where V_{mix} is the equivalent velocity of downward mixing, and n is the cycles number of periodic fluctuations when the Bessel function reduces to the first lowest point.

3. EXPERIMENTAL SET-UP

The experimental set-up consists of TSI LDA probe (wave length 488nm, focal length 363mm), signal generator, power amplifier, smoke generator and data acquisition and analysis system. A tube with the length of 3.2m and square inner diameter 100mm was used to get the travelling wave field, as Fig.2 shown. The loudspeaker was driven by sinusoidal function generator at one end of the tube and sound wave was absorbed by 1.2 m long fiber material at the other end. A travelling wave field was formed when acoustic waves spread over the tube. Seven holes on the surface of tube were prepared for 1/2 inch microphone mounting and the distance of adjacent holes was 50 mm. The microphone was LS2P microphone, B&K 4180, with the sensitivity calibrated by coupler reciprocity method. Temperature, static pressure and humidity were recorded at the measurement time.

Smoke generator made tracer particles entering into the tube, following with the medium movement. Sandalwood smoke was chosen to act as the tracer particles.Particles in measurement area moved in simple harmonic motion. Photoelectric device detected the scattering signals and converted it to electrical signals. After mixing and filtering, the particle vibration velocity in the acoustic field could be obtained with spectrum or autocorrelation method. For a plane wave sound field, sound pressure and particle velocity has a linear relationship, viz. p = ρCV, where p is the sound pressure (RMS), ρ is acoustic medium density, and C is the sound velocity, V is the particle vibration velocity. The experimental conditions was controlled at 23 ℃,and the characteristic impedance of air is around 404 Pa•s/m.

In order to ensure travelling wave field to be generated, equal sound pressure should distributed in the measurement region of the tube. Pressure distribution along the tube was shown in Fig. 3. The standard error of pressure along the measurement positions between 300 Hz and 1000 Hz was presented in table 1. From 300Hz to 1000Hz, the Pressure fluctuation in the measurement region is less than 0.5 dB and it could be nearly considered as a travelling wave tube.

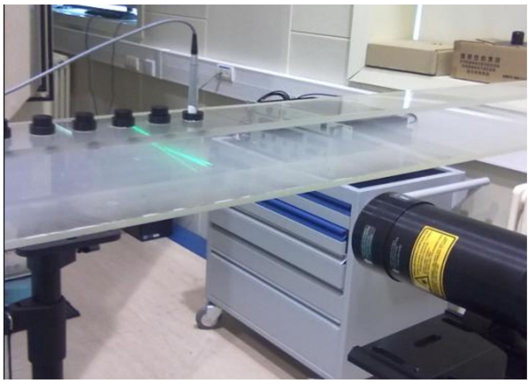


Fig.2 Map of LDV system

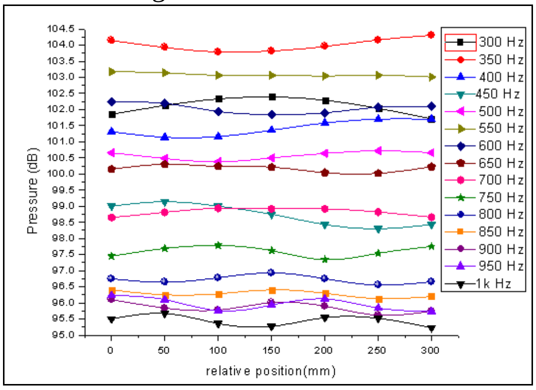


Fig.3 Acoustic pressure distribution along the tube

Table 1-The standard error of pressure measurement

Frequency (Hz)	300	350	400	450	500	550	600	650
Standard error (dB)	0.25	0.19	0.29	0.33	0.14	0.05	0.14	0.11
Frequency (Hz)	700	750	800	850	900	950	1000	
Standard error (dB)	0.12	0.16	0.12	0.10	0.16	0.19	0.16	

4. RESULTS AND DISCUSSIONS

The interference fringe spacing of TSI LDA system is 3.74 μm. In plane wave field from 94 dB to 110 dB SPL the corresponding particle vibration velocity is from 4 mm/s to 25mm/s and the equivalent doppler frequency shift is from 1070 Hz to 6685 Hz.

To realize the sound pressure unit by LDV technique in enclosed tube, the scattering light by particles should be received by the photoelectric device and the optic signal was transferred into electrical signals. The doppler signal after down-mixing is shown as Fig.4.

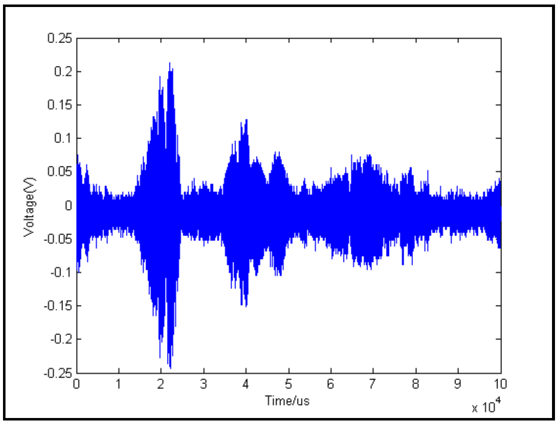


Fig.4 The Doppler signal after mixing

4.1 Spectrum method

The center frequency of original Doppler signal in the experiment is around 40 MHz induced by Bragg cell. To acquire the time domain signal and get the frequency domain signal more easily, the original Doppler signal was mixed by a frequency lower than 40MHz, for example 39.85MHz. In this section, the carrier frequency after mixing is 15 kHz. Then the time domain signal was dealt with Fourier transformation with 20 times average. To get a good signal to noise ratio, the time of smoke should last no less than 5s.

Fig.5 showed the spectrum characteristics of doppler signal with 400 Hz acoustic pressure. According to fomular (4) the particle velocity can be obtained as 8.23mm/s and the sound pressure can be calculated as 101.10 dB. If the noise is at high level or the tracer particle is not enough, the spectrum would be like Fig.6.

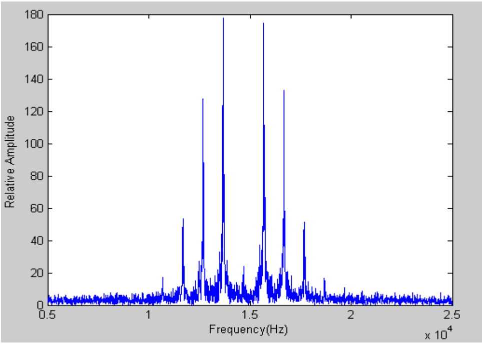


Fig.5 The spectrum characteristics of doppler signal

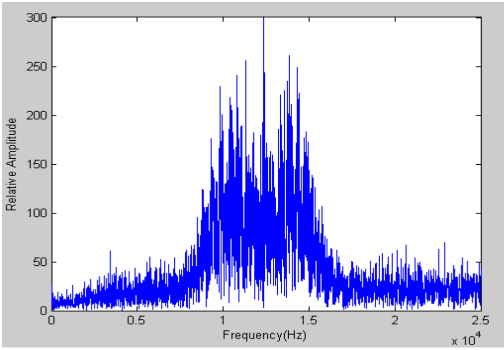


Fig.6 The Spectrum of signal with noise

The comparison of the microphone measurement and the LDV measurement at different frequency was shown in Table.2.The maximum deviation is 0.34 dB. Table 3 is the comparison of the microphone measurement and the LDV measurement at different SPL at 500 Hz. The sound pressure level was from 94 dB to 106 dB and the maximum deviation from the LDV measurement to microphone measurement is 0.22 dB. The main factor causing the deviation should be the non ideal of the travelling wave tube.

Talbe.2- comparison of microphone and LDV measurement at different frequency

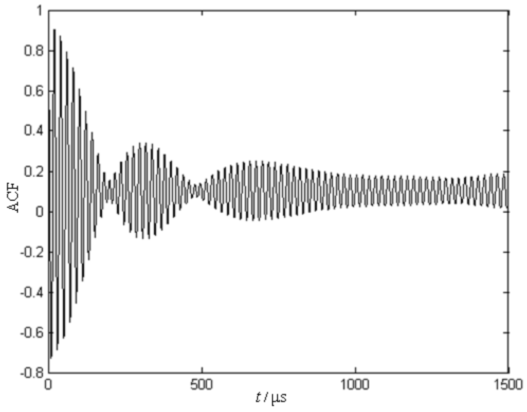
Frequency /Hz	SPL(Mic) /dB	Particle velocity /mm•s^{-1} (peak)	Sound pressure(LDV) /dB	Deviation /dB
300	101.93	8.63	101.83	0.10
400	101.10	8.23	101.40	-0.30
500	101.28	8.23	101.40	-0.12
550	101.37	7.92	101.07	0.30
650	101.35	8.02	101.18	0.17
700	104.72	12.30	104.90	0.18
750	103.61	10.38	103.42	0.19
800	102.89	9.87	102.99	0.10
850	102.72	9.85	102.98	0.26
900	102.34	9.09	102.27	0.07
950	102.16	8.60	101.78	0.34
1000	101.53	8.23	101.40	0.32

Talbe.3- comparison of microphone and LDV measurement at different SPL（500Hz）

SPL(Mic) /dB	Particle peak velocity /mm•s^{-1}	Sound pressure(LDV) /dB	Deviation /dB
94.08	3.46	93.88	0.20
98.55	5.98	98.64	-0.09
100.49	7.48	100.58	-0.09
102.08	8.98	102.16	-0.08
105.68	13.09	105.44	0.22

4.2 Autocorrelation method

In this part the doppler signal was mixed downward to 50kHz and 150 kHz respectively and the digital oscilloscope sampling rate was set as 1MHz.With the auto-correlation processing, the periods number of modulation signal could be obtained when the autocorrelation function (ACF) envelope down to the first minimum point. According to the period number and the mixing frequency, the particle vibration velocity could be calculated by formula (10). The smoke entering time is set as around 1s to 2 s. Fig.7 is the result of the particles acoustic velocity at 500Hz with ACF with the mixing frequency 50 kHz. From the point figure Fig.8 the value *n* in formula (10) was determined as 11.



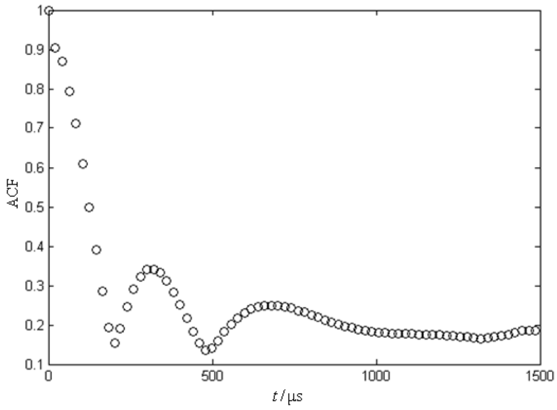
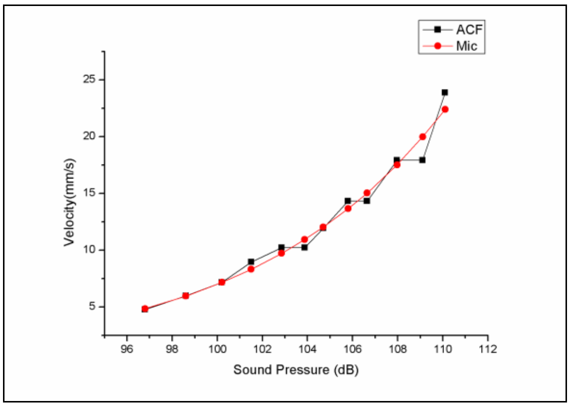


Fig.7 ACF result Fig.8 Point figure of fig.7 (one point means one period)

Different SPL could be obtained by adjusting the gain of power amplifier. In fig.9 the SPL was set as 96 dB to 110 dB, the corresponding particle vibration velocity is around 4 mm/s to 25 mm/s. The particle velocity increases with the SPL. From Fig.9 it could be concluded that the SPL measured by microphone and LDV with ACF has a good consistency. When the SPL was lower than 105 dB, the agreement is better than at the high SPL. When the SPL was 109 dB, the deviation is 0.7 dB.

In fig.10 the equivalent frequency shift corresponding to V_{mix} is 150 kHz, three times of the frequency in Fig.9. Comparing with Fig.9 and Fig.10, it seems that in the latter one the difference between microphone measurement and LDV ACF measurement is smaller. The reason is that at the same level of particle vibration velocity, the period number of larger equivalent shift frequency will be more and the error of the velocity will be less. At 110dB the difference was reduced to 0.5 dB. At the same time, from formula (10) it could derive that in stronger sound field, the velocity will be greater, and the equivalent frequency shift should be larger, to ensure that there is a certain period number from zero time to the time point that the ACF decreasing to the first lowest value.



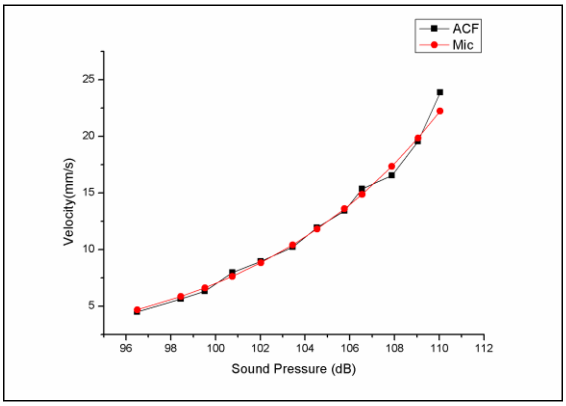


Fig.9 ACF with 50 kHz mixing Fig.10 ACF with 150 kHz mixing

4.3 Discussion

Both the spectrum and ACF method could obtain the particle vibration velocity in the traveling tube and the test result showed that it has good agreement with the microphone measurement.

For the spectrum method it needs more tracer particles and requires lower background noise. More tracer particles may change the property of the medium. It is a drawback of spectral analysis.

For the ACF method, it has good signal noise ratio at low seeding condition. In the calculation the particle mean velocity of medium was considered as zero, while it could be several millimeter per second due to the thermal motion of medium and the action of speaker. This factor is one of the main uncertainty components of ACF method.

It is known that velocity can be deduced from pressure p in the travelling wave tube, given by $p = \rho c v$. Besides the systematic errors, some factors which affected the accuracy of results should be analyzed according to experimental conditions.

Temperature. In order to calculate the programming velocity of sound wave in the air accurately, the pressure, temperature, and humidity should be measured, which direct affect the result in the experiment. The temperature is 23 degrees in the room, which have a deviation of 0.5 degree inside tube, corresponding to an error of 0.0086 dB in result.

Behavior of seeding particles. The size of particles is related to the signal-to-noise ratio. If the size of particle is large enough to overcome the poor SNR, it may lead to weak following behavior in the acoustic field. On the contrary, poor SNR may be caused. Therefore, appropriate size of particle is important for the experiment. Some authors had engaged in relevant work and drawn the conclusion that when the particle size is less than 0.35 times as the fringe spacing, both the SNR and the following behaviors of fluid are optimized. The fringe spacing is 3.74 um, the error of the angle of two laser beams is insignificant. In this paper white sandalwood was used as seeding particles, the exact size of the particle is still under investigation.

Minimum resolution of the spectrum. When the spectrum method was used, the frequency resolution is important. The rate of sampling is no less than twice of the signal frequency according to the Nyquist theorem. In this experiment, the rate of sampling is always ten times than the data length, corresponding to the minimum resolution of 10 Hz in frequency domain. In that case, the error caused by resolution is less than 0.04 dB at the velocities of 6.06mm/s-16.83mm/s from 300Hz to 1k Hz.

Density of medium. The density of air would be changed more or less because of adding particle into the tube. The density should be close to air after a period of time. Some work about uncertainty analysis of density of medium will be done in future.

Mean velocity of particles. When the spectrum was chosen, the influence of mean velocity could be neglected because the specific frequency value will be obtained and the process showed that the mean velocity will be eliminated by the ratios of the amplitude at different side frequency.

5. CONCLUSIONS

The results show that the commercial LDA probe with spectrum and ACF particle acoustic velocity demodulation method could be used to realize the sound pressure unit from 300 Hz to 1000 Hz in a travelling wave tube. The comparison with microphone measurement showed that the deviation could be less than 0.5 dB. The influencing factors of LDA technology include the non-ideal traveling wave field, the seeding particle size, the spectrum analysis resolution, the temperature, the air density measurement, the mean velocity of medium and so on. Effort will be committed to improve experimental environment including the acoustic field in further work, for example using active noise control technology to get better travelling wave filed, also to evaluate the uncertainty of the spectrum method and the ACF method. In the next stage, a specific LDA system will also be developed at NIM according to the acoustic tubes and the acoustic chamber.

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