



One-dimensional unidirectional acoustic boundary through active control method

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ABSTRACT

The unidirectional acoustic boundary has a broad application prospect. Besides the acoustic metamaterial or the strongly nonlinear acoustic medium, which were applied in the realization of the unidirectional acoustic boundary, the present paper proposes another approach through the active control method, where the control source has the property of single directivity. In order to validate its effectiveness in one-dimensional duct, experiment is implemented in two steps: 1. constructing the control source with single directivity by two monopoles; 2. constructing the unidirectional acoustic boundary through the control source in step 1. Results show that the approach to the unidirectional acoustic boundary in one-dimensional duct is feasible and effective, and this work has the potential to be achieved in broadband sound field, in three-dimensional case, and by the adaptive system.

Keywords: Unidirectional acoustic boundary, Single directivity, Active control method

I-INCE Classification of Subjects Number(s): 37.7

1. INTRODUCTION

The unidirectional acoustic boundary is the boundary that the sound wave can pass through from one side of the boundary to another side, but cannot propagate along the opposite direction. It has a broad application prospect. In the acoustic detection of the submarines and the underwater weapons, there are two main methods: 1. the passive mode based on collecting the acoustic radiation from the underwater structures; 2. the active mode based on receiving the sound scattered by the structures. When the unidirectional acoustic boundary is located around the underwater target, the acoustic stealth performance can be promoted through preventing the radiated and the scattered sound passing to the outside of the boundary. Furthermore, the special boundary is useful for extending the low frequency range of wedges for building anechoic rooms, and can also be applied in anti- interception systems, which restrict the conversation information in a given range.

Focusing on the function of the special boundary, i.e., protecting the objects inside from being detected, the current research on the acoustic cloak and the acoustic diode can be regarded as the unidirectional acoustic boundary, and it mainly pays attention on the acoustic metamaterial or the strongly nonlinear acoustic medium. In the aspect of the acoustic cloak, Greenleaf *et al* (1) provided the general transformation method in the acoustical context. Norris (2, 3) proposed more general types of ideal cloaks with finite mass by using anisotropic material. For special cases, Cummer and Schurig (4) showed that an analogy exists between 2D acoustic and electromagnetic anisotropic materials. Cummer *et al* (5) and Chen *et al* (6) derived the bulk modulus and the anisotropic mass density of a 3D acoustic cloak. For practical realization, the acoustic cloak made of a multilayered composite containing two types of isotropic materials has been considered by Torrent *et al* (7) and Cheng *et al* (8, 9), and Cai *et al* (10) investigated the 2D acoustic cloak with multilayered anisotropic materials. Progress was also made in the acoustic metamaterials experimentally (11-13), but until now there are still many problems confronted in the practical realization. In the aspect of the acoustic diode,

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Liang *et al* (14, 15) demonstrated a one-dimensional model of an acoustic diode formed by coupling a superlattice with a strongly nonlinear medium, and experiments were presented about a rectified energy flux of the acoustic waves. However, his current research is only effective at several frequencies.

On the other hand, the active control technology is applied widely in the sound field control. The research in the recent years also shows that the active control method was exploited effectively in the control of the acoustic impedance at object surface. Collet *et al* (16) aimed at a well controlled active skin that modifies the sound transmissibility or reflectivity of the corresponding homogeneous passive interface substantially. Yuan (17) proposed an absorber that has an absorption coefficient of 0.9 or above in a frequency range of 60–850 Hz, through the impedance matching technique. Howarth *et al* (18, 19) realized the impedance matching through the piezoelectric polymer sensors and a piezocomposite actuator encapsulated within a host polymer, and it can be adjusted in real-time according to the incident wave. Corsaro (20) developed an actuator—sensor tile for controlling the reflection and transmission characteristics of the generic underwater structures, which was 25 cm square and contained a full area actuator, acoustic pressure sensor, and (acoustic particle) velocity sensor. Hence it is seen that introducing the given control sources in the conventional acoustic medium can construct the special acoustic boundaries.

In this paper, the active control method is applied in the realization of the unidirectional acoustic boundary in one-dimensional duct. Theories and principles are described first to construct the unidirectional acoustic boundary, and then experiments are demonstrated to indicate its effectiveness.

2. THEORY

2.1 Unidirectional acoustic boundary

According to the Kirchhoff-Helmholtz equation (21), in the outside range surrounded by the boundary, the sound pressure originated by the boundary is expressed by

$$p(\vec{r}_e) = \int_S [G(\vec{r}_e | \vec{r}_0) \nabla p(\vec{r}_0) - p(\vec{r}_0) \nabla G(\vec{r}_e | \vec{r}_0)] \cdot \vec{n} dS \quad (1)$$

Where S is the acoustic boundary, \vec{n} is the unit vector perpendicular to S and towards to the inside, $G(\vec{r}_e | \vec{r}_0)$ is the Green Function in free field, \vec{r} is the point outside of the boundary, and $p(\vec{r}_0)$ is the sound pressure at \vec{r} on the boundary S . If the control sources with given strengths are positioned on the boundary to make the sound pressure and its normal gradient there satisfy

$$\int_S G(\vec{r}_e | \vec{r}_0) \nabla p(\vec{r}_0) \cdot \vec{n} dS = \int_S p(\vec{r}_0) \nabla G(\vec{r}_e | \vec{r}_0) \cdot \vec{n} dS \quad (2)$$

while the control sources don't radiate sound pressure to the inside, the sound pressure outside originated by the boundary will be zero, but the sound pressure inside is unchanged (i.e., the inside sound field will not influence the space outside, but the sound pressure outside can pass through the boundary without any restriction), thereby the unidirectional boundary is realized. In practice, it is difficult to obtain the source strengths directly, thus an equivalent prerequisite have to be explored. For simplicity, the one-dimensional case is investigated in the present research.

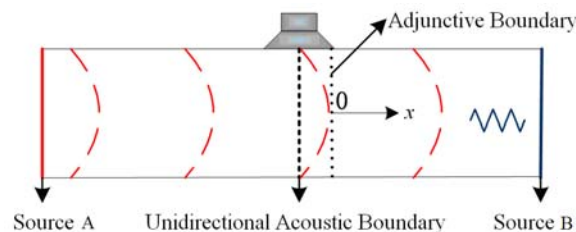


Figure 1 – Sketch map of the one-dimensional unidirectional acoustic boundary

Shown in Fig. 1, the property of the one-dimensional acoustic boundary is that the sound wave radiated from source A can pass through the acoustic boundary from left to right, while the sound wave from source B cannot get through the boundary. For the equation derivation, the origin of coordinate

$x = 0$ is set at right but not far from the unidirectional acoustic boundary, which is called the adjunctive boundary. In the one-dimensional duct, the sound pressure from left to right (in the forward direction) and the one from right to left (in the reverse direction) are written as

$$p_i = p_{ai} e^{j(\omega t - kx)}, \quad p_r = p_{ar} e^{j(\omega t + kx)} \quad (3)$$

respectively, thus the sound pressure and the particle velocity in the duct are

$$p = (p_{ai} e^{-jkx} + p_{ar} e^{jkx}) e^{j\omega t}, \quad v = (p_{ai} e^{-jkx} - p_{ar} e^{jkx}) e^{j\omega t} / \rho_0 c_0 \quad (4)$$

and the sound pressure and the particle velocity at the adjunctive boundary are

$$p_0 = (p_{ai} + p_{ar}) e^{j\omega t}, \quad v_0 = (p_{ai} - p_{ar}) e^{j\omega t} / \rho_0 c_0 \quad (5)$$

Then the amplitudes p_{ai} and p_{ar} are expressed as

$$p_{ai} = (p_0 + \rho_0 c_0 v_0) e^{-j\omega t} / 2, \quad p_{ar} = \frac{1}{2} (p_0 - \rho_0 c_0 v_0) e^{-j\omega t} / 2 \quad (6)$$

So the sound pressure propagating in the reverse direction is

$$p_r = (p_0 - \rho_0 c_0 v_0) e^{jkx} / 2 \quad (7)$$

where e^{jkx} denotes the time delay $\Delta t = -x / c_0$. In that way, the reverse-directional sound pressure can be predicted by the sound pressure and particle velocity at the adjunctive boundary. In the case that the reverse-directional sound pressure is known, a given control source, which radiates $-p_r$ to counteract the reverse-directional sound pressure in the left side of the boundary but never radiates sound wave to the right side, can be positioned at the acoustic boundary, and then the unidirectional acoustic boundary is realized.

In order to measure the sound pressure and particle velocity in the duct, the acoustic pressure signals are picked up by the two microphones p_1 and p_2 (22). If the distance d between the two microphones is small relative to the smallest wavelength of the sound, and for a plane wave, the pressure and the particle velocity at the midpoint are

$$p(t) = (p_1 + p_2) / 2 \quad (8a)$$

$$v(t) = \int_0^t (p_1 - p_2) dt / \rho d \quad (8b)$$

approximately. In the experimental realization of Eq. (8b), the integral is replaced by the summation,

$$\text{i.e. } v(t) = \sum_0^t (p_1 - p_2) \Delta t / \rho d.$$

From the analysis described above, it is known that the control source at the unidirectional acoustic boundary has to take on the property of the left directivity. Thus the construction of the special control source is the key point in the realization of the unidirectional acoustic boundary.

2.2 Single Directional Sound Source

It is shown that the control source with the property of left directivity plays an important role on the unidirectional acoustic boundary. In this section, attentions will be paid on the single directional sound source, which could be constructed by three or two loudspeakers.

2.2.1 Using Three Loudspeakers

As show in Fig. 2, the dipole source is constructed in the one-dimensional duct first, where the two sources have the same amplitude and the opposite phase, assumed as A and $Ae^{j\pi}$. The distance L between the two sources satisfies $kL < 1$, thus the sound pressure to the left is

$$p = jAkLe^{jkx} \quad (9)$$

and to the right is

$$p = -jAkLe^{-jkx} \quad (10)$$

where $k = \omega / c$, and c is the sound speed in the duct.

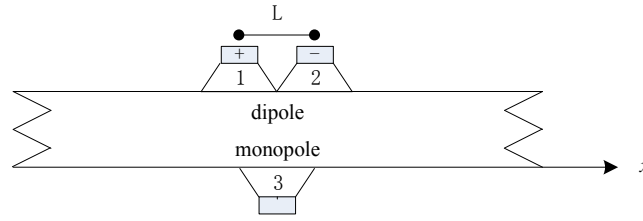


Figure 1 – Construction of the single directional source in the duct through three loudspeakers

After the dipole is constructed, another sound source (monopole) is added to counteract the sound pressure in the right sound field of the duct, i.e., to constitute the source with left directivity, where the source strength is $jAkL = AkLe^{j\pi/2}$, due to Eq. (10). In that way, the total sound pressure radiated by the three loudspeakers is $p = 2jkLe^{jkx}$ in the left sound field, and zero in the right sound field.

2.2.2 Using Two Loudspeakers

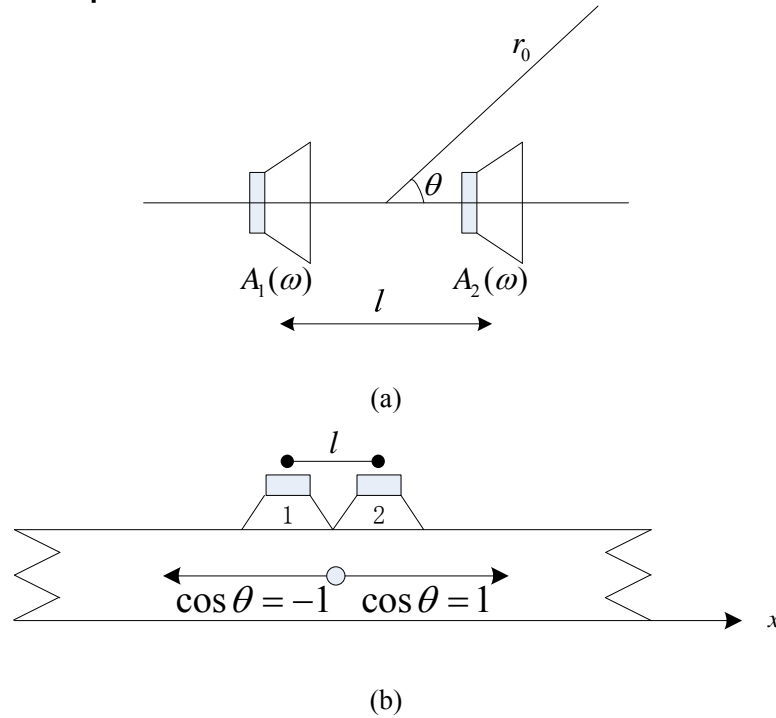


Figure 3 – Two closely spaced monopole loudspeaker fed with preprocessed audio signals A_1 and A_2 (a) in the two-dimensional sound field; (b) in the one-dimensional duct

We now show how the directivity pattern is realized by two closely spaced monopole loudspeakers. As shown in Fig. 3 (a), by feeding the two loudspeakers with signals $A_1(\omega)$ and $A_2(\omega)$ respectively, they produce the sound field (23)

$$p(r_0, \theta) = A_1(\omega)e^{-jk r_1} / r_1 + A_2(\omega)e^{-jk r_2} / r_2 \quad (11)$$

in two-dimensional sound field, where r_1 and r_2 are the distance from the loudspeakers to a far-field receiver position. r_1 and r_2 are further expressed by r_0 , which is the distance from the center of the two loudspeakers to the receiver position, and the angle θ is shown in Fig. 3 (a). For $l \ll r_0$, Eq. (11) is written as

$$\begin{aligned}
p(r_0, \theta) &= e^{-jk r_0} (A_1 e^{jk \Delta r} + A_2 e^{-jk \Delta r}) / r_0 \\
&= e^{-jk r_0} (A_1 \cos k \Delta r + j A_1 \sin k \Delta r + A_2 \cos k \Delta r - j A_2 \sin k \Delta r) / r_0
\end{aligned} \tag{12}$$

with $\Delta r = l / 2 \cos \theta$. If furthermore $k \Delta r \ll 1$, this expression is

$$p(r_0, \theta) = e^{-jk r_0} (A_1 + A_2 + jkl(A_1 - A_2) \cos \theta / 2) / r_0 \tag{13}$$

As shown in Fig. 3 (b), when the directivity pattern is realized in the one-dimensional duct, Eq. (13) for $\cos \theta = -1$ is reduced to

$$p(r_0, \theta) = e^{-jk r_0} (A_1 + A_2 - jkl(A_1 - A_2) / 2) \tag{14a}$$

and Eq. (13) for $\cos \theta = 1$ is reduced to

$$p(r_0, \theta) = e^{-jk r_0} (A_1 + A_2 + jkl(A_1 - A_2) / 2) \tag{14b}$$

In order to construct the control source with left directivity, $p(r_0, \theta)$ in Eq. (14b) should be zeros, that is, its real part and imaginary part are set to be zero, respectively. After derivations, the relationship between A_1 and A_2 is obtained

$$A_2 / A_1 = (1 + jkl / 2) / (1 - jkl / 2) \tag{15}$$

The relationships of the amplitude and the initial phase are also obtained between the two sources

$$A_{2amp} / A_{1amp} = 1, \quad A_{2pha} - A_{1pha} = \text{atg}[kl / (1 - kl^2 / 4)] \tag{16}$$

where A_{1amp} and A_{2amp} are the amplitude of loudspeaker 1 and 2, A_{1pha} and A_{2pha} are the initial phase of loudspeaker 1 and 2.

3. EXPERIMENTS

In the experiments, the single directional sound source is constructed first, and then it is applied in the realization of the unidirectional acoustic boundary. The Pulse 3560D is used to drive the sources with given amplitudes and initial phases, and record the sound pressure in the duct field. The cross section of the duct is $0.17 \times 0.17 m^2$, thus the cut-off frequency of the duct is 1 kHz.

3.1.1 Single Directional Sound Source

For simplicity, two sources are employed to construct the single directional sound source. The process is described as follows:

1. Measure the frequency response of the two loudspeakers, and then compensate their amplitude and phase in order to make them have the same frequency response at the frequency points of interest.
2. According to Eq. (16), the relative amplitude and the initial phase between the two sources are calculated. Because of the nonideal duct sound field, the relative amplitude and phase is modified in some sort, to obtain the optimal performance of the constructed single directional sound source.

Taking the frequency point of 400Hz as an example, the measured ratio of the sound pressure radiated to the left and that to the right is 0.078, shown in Fig. 4.

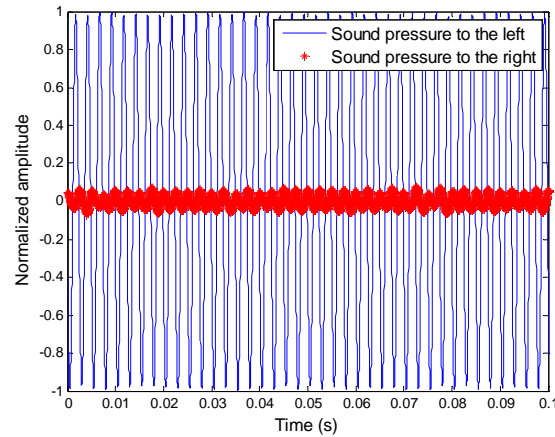
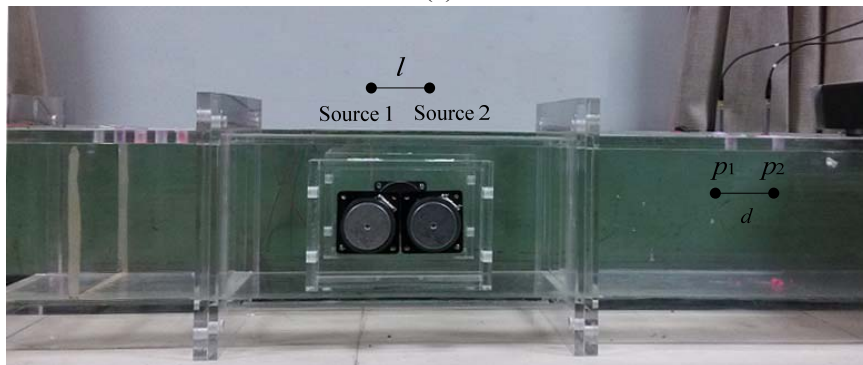
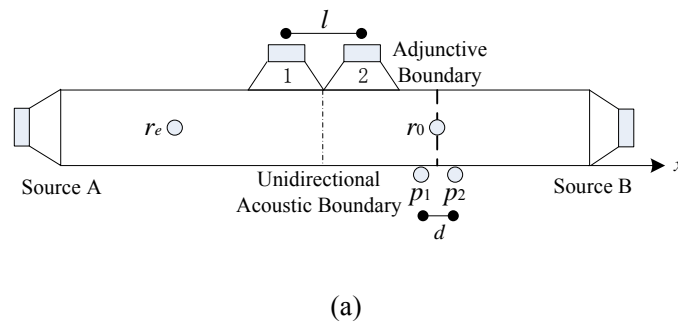


Figure 4 – Performance of the control source with left directivity

3.1.2 Unidirectional Acoustic Boundary

The experiment setup of the unidirectional acoustic boundary is shown in Fig. 5, where the performance of the unidirectional acoustic boundary is displayed in two patterns: 1. attenuating the reflected wave of Source A in the left side of the unidirectional acoustic boundary; 2. attenuating the incident wave from Source B in the left side of the acoustic unidirectional boundary. In both cases, two microphones are located around the adjunctive boundary with d distance, i.e., p_1 and p_2 . Their recorded sound pressure are used to calculate the sound pressure and particle velocity at the adjunctive boundary, and further predict the reverse-directional sound pressure at r_e .

Figure 1 – Experimental setup of the unidirectional acoustic boundary
(a) the schematic diagram; (b) the photograph

According to Section 2A, the process to implement the unidirectional acoustic boundary is described as follows:

1. Measure the frequency responses from the primary source (Source A or Source B) to the sound

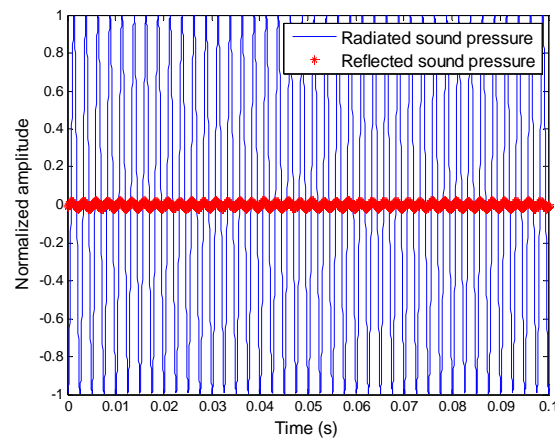
pressures at p_1 and p_2 , and then the frequency responses from the primary source to the sound pressure and the particle velocity at the adjunctive boundary are calculated through Eq. (8), thus the frequency response H_s from the primary source to the reverse-directional sound pressure at r_e is obtained according to Eq. (7).

2. Measure the frequency responses from source 1 and 2 of the control source to the sound pressures at r_e , when the duct ends are anechoic.

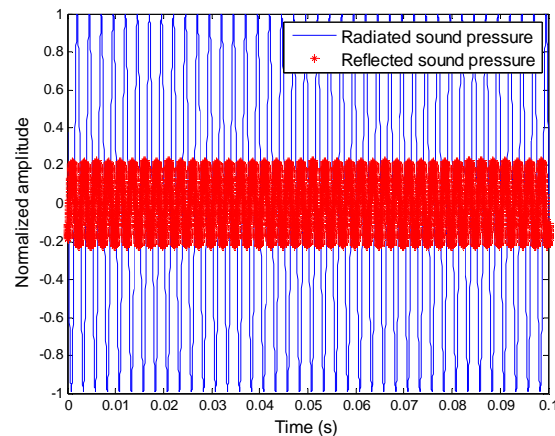
3. According to the relative amplitudes and initial phases between the source 1 and 2 of the control source, the frequency response H_c from the control source to the sound pressure at r_e is calculated.

4. Set the control source to have the amplitude A_{amp} and the initial phase A_{pha} , in order to have the relationship $A_{amp}H_c e^{jA_{pha}} = -H_s$. In that way, the unidirectional acoustic boundary is realized.

400Hz is chosen as the frequency point of interest. The performance of the unidirectional acoustic boundary corresponding to attenuating the reflected wave of Source A is shown in Fig. 6 (a), where the ratio of the sound pressure radiated by source A and its reflected sound pressure in the left sound field is 0.051, while the ratio is 0.24 when the unidirectional acoustic boundary is removed, as in Fig. 6 (b).



(a)



(b)

Figure 6 – Corresponding to attenuating the reflected wave of Source A, the radiated and reflected sound pressure in the left part of the duct (a) with the unidirectional acoustic boundary; (b) without the unidirectional acoustic boundary

The performance of the unidirectional acoustic boundary corresponding to attenuating the incident wave from Source B is shown in Fig. 7, where the ratio of the sound pressure in the left side and in the

right side of the boundary is 0.097.

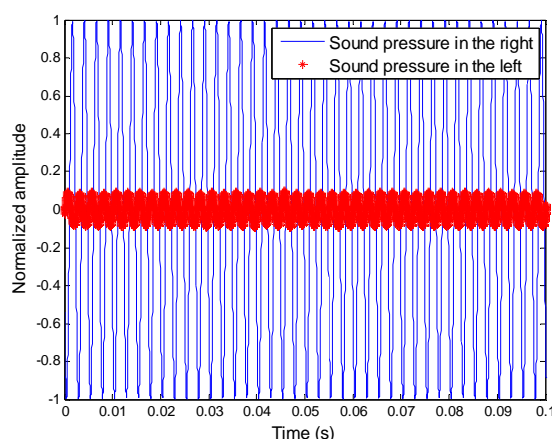


Figure 7 – Corresponding to attenuating the incident wave from Source B, the sound pressure in the right and left part of the duct

Experiments above validate the effectiveness of the proposed unidirectional acoustic boundary based on the active control method. However, because of the nonideal left directivity of the control source, there is still a little sound pressure radiated to the right side of the boundary, which further influences the performance of the unidirectional acoustic boundary. In practical realization, the left directivity of the control source can be optimized more ideally by modifying the relative amplitude and initial phase between the two sources of the control source more carefully, according to the required precision.

4. CONCLUSIONS

In this paper, the unidirectional acoustic boundary is implemented in one-dimensional sound field through the active control method. Compared with the acoustic cloak and the acoustic diode, the proposed realization method is much simpler and more convenient to apply. Although the current experiments focus on the single frequency, the theory of the unidirectional acoustic boundary doesn't restrict it in the single frequency points, thus it can be extended to the broadband sound field. Furthermore, based on the existing adaptive algorithm, it has the potential to be achieved through the adaptive system, which is more practical and stable.

Based on the inspiration of the one-dimensional case, further work will pay attention on the three-dimensional sound field, where multi-channel control sources should be adopted and the optimized positions of the control sources may be the main problems.

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