Free vibration analysis of orthotropic rectangular Mindlin plates with general elastic boundary conditions

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ABSTRACT

In this investigation, a modified Fourier solution based on the Mindlin plate theory is developed for the free vibration problems of orthotropic rectangular Mindlin plates subjected to general boundary supports. In this solution approach, regardless of the boundary conditions, the plate transverse deflection and rotation due to bending are invariantly expressed as a new form of trigonometric series expansions with a drastically improved convergence as compared with the conventional Fourier series. All the unknown coefficients are treated as the generalized coordinates and determined using the Raleigh-Ritz method. The change of the boundary conditions can be easily achieved by only varying the stiffness of the three sets of the boundary springs at the all boundaries of the orthotropic rectangular Mindlin plates without the need of making any change to the solutions. The excellent accuracy of the current result is validated by comparison with those obtained from other analytical approach as well as the Finite Element Method (FEM). Numerical results are presented to illustrate the current method is not only applied to the classical homogeneous boundary conditions but also other interesting and practically important boundary restraints on free vibrations of the orthotropic rectangular Mindlin plates with varying stiffness of boundary springs.

Keywords: orthotropic Mindlin plates, arbitrary boundary conditions, vibration, Fourier series I-INCE Classification of Subjects Number(s): 21.2.1

1. INTRODUCTION

As one of the most common type of fundamental structural elements, the orthotropic plates are plates are widely used in various engineering such as aeronautic, automotive and underwater structures and so on. The wide use of advanced composite laminates in industrial mainly because they exhibit properties which are more favorable than those of single layer and isotropic ones. Also, fiber enforced composite materials have high strength/weight and stiffness/weight ratios, relatively low cost [1], corrosion resistance and longer fatigue life [2]. The orthotropic behavior arises from the use of materials with such constitutive relations, and many composite plates can also be modeled analytically as orthotropic plates [3]. The ratio of in-plane Young's modulus to transverse shear modulus is relatively high for composite plates due to the great difference between elastic properties of fiber filament and matrix materials. This leads to using the thin plate theory, which neglects transverse shear deformation, is invalid for most composite plates, even those which are geometrically thin [4]. For dealing with complicated shear strain distribution, various shear deformation theories have been proposed. The Mindlin plate theory, generally referred to as the first order shear deformation theory and incorporated the effect of rotary inertial, is one of the most typical and used deformation theories

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for the analysis of composite laminates [5].

Mindlin [6] first uncoupled the equations by expressing the two rotations (Ψ_x, Ψ_y) and the deflection (w) in terms of three potentials. Then Mindlin et al. [7] presented exact solutions for simply supported rectangular plates and studied coupling of modes for the case of one pair of parallel edges free and the other pair simply supported. Special attention was given to the high modes and frequencies of vibration which were beyond the range of applicability of the classical theory of thin plates. Later, Brunelle [8] and Shahrokh and Arsanjani [9] derived the exact characteristic equations for elastic stability and free vibration, respectively, of a rectangular Mindlin plate with two parallel edges simply supported and the remaining two edges subjected to a variety of boundary conditions. The exact solutions for free vibrations of Mindlin plates are much more complex than those of thin plates. Recently, Xing and Liu [10,11] presented simplified characteristic equations, which were similar with those via Kirchhoff thin plate, for free vibrating Mindlin plates with any combinations of simply supported and clamped edges, involved free edges for plates with two simply supported opposite edges. However, are only available for simply supported plates so far [12]. For other classical boundary conditions, the application of an approximate method was regarded as unavoidable [13, 1, 4]. On this aspect, Liew et al. [14] have presented a comprehensive literature survey on the research works up to 1994 on vibrations of thick plates: 132 publications have been cited, attention has been mainly devoted to studies based on the vibration of thick laminated plates. Apparently the finite element technique [15] and the Rayleigh-Ritz technique [1] have been most widely used in free vibration analysis of orthotropic Mindlin rectangular plates. Other methods such as Galerkin technique [16], the superposition method [17], and the finite difference method [4] etc. have also been used to the free vibration analysis of orthotropic rectangular Mindlin plates. The state space concept has been used to develop Levy-type exact solutions for free vibration and buckling of laminated composite plates based on the first order and higher order theories [18, 19]. Beside the aforementioned studies on classical boundary conditions (namely free, simply supported and clamped), the vibration problem of orthotropic Mindlin rectangular plates with complicated edge supports are also considered by several researchers [20-24].

Motivated by the limitation of boundary conditions in the current studies of the vibration analysis of orthotropic Mindlin rectangular plates structures, in this paper, a modified Fourier solution based on the Mindlin plate theory is developed for the free vibration problems of orthotropic rectangular Mindlin plates subjected to general boundary supports. In this solution approach, regardless of the boundary conditions, the plate transverse deflection and rotation due to bending are invariantly expressed as a new form of trigonometric series expansions with a drastically improved convergence as compared with the conventional Fourier series. All the unknown coefficients are treated as the generalized coordinates and determined using the Raleigh-Ritz method. The change of the boundary conditions can be easily achieved by only varying the stiffness of the three sets of the boundary springs at the all boundaries of the orthotropic rectangular Mindlin plates without the need of making any change to the solutions. The excellent accuracy of the current result is validated by comparison with those obtained from other analytical approach as well as the Finite Element Method (FEM). Numerical results are presented to illustrate the current method is not only applied to the classical homogeneous boundary conditions but also other interesting and practically important boundary restraints on free vibrations of the orthotropic rectangular Mindlin plates with varying stiffness of boundary springs.

2. THEORETICAL FORMULATIONS

To obtain the free vibration information of orthotropic Mindlin plate structures under general boundary conditions, the combination of the artificial spring technique together with Rayleigh-Ritz method is feasible. Consider the orthotropic plate, with the dimension of $a \times b$, and the coordinate of the orthotropic Mindlin plate are depicted in Figure 1 Three groups of boundary restraining springs (translation, rotational and torsional springs) are arranged at the all sides of the plate to separately simulate the boundary force. By assigning the stiffness of the boundary springs with various values, it is equivalent to impose different boundary force on the mid-surface of the plate. For example, the clamped boundary condition can be readily obtained by setting the spring coefficients into infinity (a very large number in practical calculation) for the translation, rotations and torsional restraining springs along each edge.

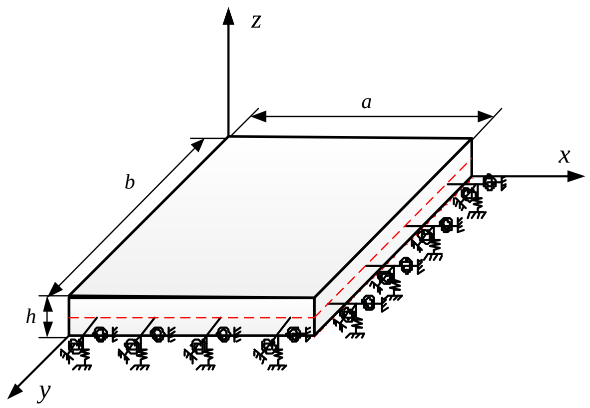


Figure 1 – The general elastic boundary conditions of orthotropic Mindlin plate

Based on the Mindlin plate theory, the displacements vectors with three directions are:

$$u(x,y,z,t) = z\psi_x(x,y,t) \tag{1}$$

$$v(x,y,z,t) = z\psi_y(x,y,t) \tag{2}$$

$$w(x,y,z,t) = w(x,y,t) \tag{3}$$

Where *u*, *v* and *w* are represents the *x*, *y* and *z* direction displacement function, the ψ_x and ψ_y are the slop due to bending along in the respective planes. The relationship w with the slop ψ_x and ψ_y are $\psi_x = - dw/dx$, $\psi_y = - dw/dx$.

For the orthotropic Mindlin plate, making use of the strain–stress relationship defined in elasticity theory, the normal, shear strains and transverse shear strains can be expressed as follows:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} z\partial\psi_x / \partial x \\ z\partial\psi_y / \partial y \\ z(\partial\psi_x / \partial y + \partial\psi_y / \partial x) \\ \psi_x + \partial w / \partial x \\ \psi_y + \partial w / \partial y \end{Bmatrix} \tag{4}$$

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \begin{bmatrix} E_x/1-\mu_x\mu_y & \mu_x E_x/1-\mu_x\mu_y & 0 & 0 & 0 \\ \mu_y E_x/1-\mu_x\mu_y & E_y/1-\mu_x\mu_y & 0 & 0 & 0 \\ 0 & 0 & G_{xy} & 0 & 0 \\ 0 & 0 & 0 & \kappa G_{xz} & 0 \\ 0 & 0 & 0 & 0 & \kappa G_{yz} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} \tag{5}$$

where ε_{xx}, ε_{yy} and γ_{xy} are the normal and shear strains in the *x, y, z* coordinate system. The transverse shear strains γ_{xz} and γ_{yz} are constant through the thickness. The σ_{xx} and σ_{yy} are the normal stresses in the *x*, *y* direction, τ_{xz}, τ_{yz} and τ_{xy} are shear stresses in the the *x, y, z* coordinate system. E_s and μ_s (*s*=*x* or *y*) are the Young's modulus and Poisson's ration in two different directions of the orthotropic rectangular plate; κ is the shear correction factor to account for the fact.

In terms of transverse displacements and slope, the bending and twisting moments, and the transverse shearing forces in plates can be expressed as:

$$M_x = -\left(D_{11}\frac{\partial\psi_x}{\partial x} + D_{12}\frac{\partial\psi_y}{\partial y}\right), M_y = -\left(D_{22}\frac{\partial\psi_y}{\partial y} + D_{21}\frac{\partial\psi_x}{\partial x}\right) \tag{6a}$$

$$M_{xy} = -D_{66}\left(\frac{\partial\psi_x}{\partial y} + \frac{\partial\psi_y}{\partial x}\right), Q_x = C_{44}\left(\frac{\partial w}{\partial y} - \psi_y\right), Q_y = C_{55}\left(\frac{\partial w}{\partial x} - \psi_x\right) \tag{6b}$$

where

$$D_{11} = \frac{E_x h^3}{12(1-\mu_x\mu_y)}, D_{12} = \frac{\mu_x E_x h^3}{12(1-\mu_x\mu_y)}, D_{22} = \frac{E_y h^3}{12(1-\mu_x\mu_y)} \tag{7a}$$

$$D_{21} = \frac{\mu_y E_y h^3}{12(1-\mu_x\mu_y)}, D_{66} = \frac{G_{xy}h^3}{12} = D_{xy}, C_{44} = \kappa G_{yz}h, C_{55} = \kappa G_{zx}h \tag{7b}$$

are the bending and shear rigidities, respectively. In view of the Betti Principle $\mu_x E_x = \mu_y E_y$, therefore, $D_{12}=D_{21}$. The above bending and shear rigidities are given for one layer. The formulations of rigidities of laminates can be found in text books or research papers, for example, [1].

The boundary conditions for a general supported orthotropic Mindlin plate can be expressed as the following forms based on the force equilibrium relationship on the four sides:

on $x=0$, $$k_{x0}w = -Q_x,\quad K_{x0}\psi_x = -M_{xx},\ K_{yx0}\psi_y = -M_{xy} \tag{8a}$$

on $x=a$, $$k_{xa}w = Q_x,\quad K_{xa}\psi_x = M_{xx},\ K_{yxa}\psi_y = M_{xy} \tag{8b}$$

on $y=0$, $$k_{y0}w = -Q_y,\quad K_{y0}\psi_y = -M_{yy},\ K_{xy0}\psi_x = -M_{xy} \tag{8c}$$

on $y=b$, $$k_{yb}w = Q_y,\quad K_{yb}\psi_y = M_{yy},\ K_{xyb}\psi_x = M_{xy} \tag{8d}$$

where k_{x0} and k_{xa} (k_{y0} and k_{yb}) are linear spring constants, K_{x0} and K_{x0} (K_{y0} and K_{yb}) are the rotational spring constants, and K_{yx0} and K_{yx0} (K_{xy0} and K_{xyb}) are the torsional spring constants at $x=0$ and a ($y=0$ and b), respectively. Therefore, arbitrary boundary conditions of the plate can be generated by assigning the linear springs、rotational springs and torsional springs at proper stiffnesses. For instance, a clamped boundary (C) is achieved by simply setting the stiffnesses of the entire springs equal to infinite (which is represented by a very large number, 10^{14}). Inversely, a free boundary (F) is gained by setting the stiffnesses of the entire springs equal to zero.

Thus the total potential energy of the spring restrained plate which is composed of two parts, namely the strain energy of the orthotropic Mindlin plates and the potential energy stored in the boundary springs, can be expressed as:

$$\begin{aligned} U = \int_0^a\int_0^b \Bigg\{ & D_{11}\left(\frac{\partial\psi_x}{\partial x}\right)^2 + D_{22}\left(\frac{\partial\psi_y}{\partial y}\right)^2 + \left(\mu_x D_{11} + \mu_y D_{22}\right)\frac{\partial\psi_x}{\partial x}\frac{\partial\psi_y}{\partial y} + \\ & D_{xy}\left(\frac{\partial\psi_x}{\partial y} + \frac{\partial\psi_y}{\partial x}\right)^2 + \kappa G_{xz}h\left(\psi_x + \frac{\partial w}{\partial x}\right)^2 + \kappa G_{yz}h\left(\psi_y + \frac{\partial w}{\partial y}\right)^2 \Bigg\} dxdy \\ & + \frac{1}{2}\int_0^a [(k_{y0}w^2 + K_{y0}\psi_y^2 + K_{xy0}\psi_x{}^2)\big|_{y=0} + (k_{yb}w^2 + K_{yb}\psi_y^2 + K_{xyb}\psi_x{}^2)\big|_{y=b}]dx \\ & + \frac{1}{2}\int_0^b [(k_{x0}w^2 + K_{x0}\psi_x^2 + K_{yx0}\psi_y{}^2)\big|_{x=0} + (k_{xa}w^2 + K_{xa}\psi_x^2 + K_{yxa}\psi_y{}^2)\big|_{x=a}]dy \end{aligned} \tag{9}$$

As the springs are considered with no mass while retaining certain stiffness, the total kinetic energy of the orthotropic Mindlin plates is :

$$T = \frac{\rho h\omega^2}{2}\int_0^b\int_0^a [w^2 + h^2(\psi_x^2 + \psi_y^2)/12]dxdy \tag{10}$$

In view of satisfying arbitrarily supported boundary conditions of the orthotropic Mindlin plates, the admissible functions expressed in a new form of trigonometric series expansions are introduced to remove the potential discontinuities with the functions and their derivatives. Thus, the orthotropic Mindlin plates displacements and rotation are expressed as:

$$\psi_x(x,y) = \sum_{m=-2}^{\infty}\sum_{n=-2}^{\infty} A_{mn}\varphi_m(x)\varphi_n(y) \tag{11}$$

$$\psi_y(x,y) = \sum_{m=-2}^{\infty}\sum_{n=-2}^{\infty} B_{mn}\varphi_m(x)\varphi_n(y) \tag{12}$$

$$w(x,y) = \sum_{m=-2}^{\infty}\sum_{n=-2}^{\infty} C_{mn}\varphi_m(x)\varphi_n(y) \tag{13}$$

where A_{mn}, B_{mn}, C_{mn} denotes the series expansion coefficients, and

$$\varphi_m(x) = \begin{cases} \cos\lambda_m x & m \ge 0 \\ \sin\lambda_m x & m < 0 \end{cases} \qquad \lambda_m = m\pi/a \tag{14}$$

The basis function $\varphi_n(y)$ in the y-direction is also given by equation (13) except for $\lambda_n = n\pi/b$. Mathematically, the series in the form of equations (11)-(13) are able to expand and uniformly converge to any function. Once the form of solution has been established, the remaining task is to find a suitable set of expansion coefficients that will ensure the series, as a whole, satisfies both the governing equations and the boundary conditions in some way. A solution can be obtained either in strong form by letting the series satisfy the relevant equations exactly on a point-wise basis, or in weak form by solving the series coefficients approximately using, for instance, the Rayleigh–Ritz technique. The weak form of solution will be sought here since it will be more attractive in modeling complex structures.

The Lagrangian for the orthotropic Mindlin plates can be eventually expressed as:

$$L = T - U \tag{15}$$

Then, the Lagrangian expression is minimized by taking its derivatives with respect to these coefficients:

$$\frac{\partial L}{\partial \vartheta} = 0,\ \vartheta = \{A_{mn}, B_{mn}, C_{mn}\} \tag{16}$$

Since the displacements and rotation components of the plate are chose as M and N to obtain the results with acceptable accuracy, a total of 3∗(M+1)∗(N+1)+6∗(M+N+2) equations are obtained.

They can be summed up in a matrix form:

$$\left(\mathbf{K} - \omega^2\mathbf{M}\right)\mathbf{E} = \mathbf{0} \tag{17}$$

The unknown coefficients in the displacement expressions can be expressed in the vector form as **E**. where

$$\mathbf{E} = \begin{Bmatrix} A_{-2,-2}, A_{-2,-1}, \cdots, A_{m',0}, A_{m',1}, \cdots, A_{m',n'}, \cdots, A_{M,N} \\ B_{-2,-2}, B_{-2,-1}, \cdots, B_{m',0}, B_{m',1}, \cdots, B_{m',n'}, \cdots, B_{M,N} \\ C_{-2,-2}, C_{-2,-1}, \cdots, C_{m',0}, C_{m',1}, \cdots, C_{m',n'}, \cdots, C_{M,N} \end{Bmatrix}^{\mathrm{T}} \tag{18}$$

In Eq.(17), the **K** is the stiffness matrix for the plate, and the **M** is the mass matrix. They can be expressed separately as:

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_{1,1} & \mathbf{K}_{1,2} & \mathbf{K}_{1,3} \\ \mathbf{K}_{1,2}^{\mathrm{T}} & \mathbf{K}_{2,2} & \mathbf{K}_{2,3} \\ \mathbf{K}_{1,3}^{\mathrm{T}} & \mathbf{K}_{2,3}^{\mathrm{T}} & \mathbf{K}_{3,3} \end{bmatrix}, \mathbf{M} = \begin{bmatrix} \mathbf{M}_{1\text{-}1} & \mathbf{M}_{1\text{-}2} & \mathbf{M}_{1\text{-}3} \\ \mathbf{M}_{2\text{-}1} & \mathbf{M}_{2\text{-}2} & \mathbf{M}_{2\text{-}3} \\ \mathbf{M}_{3\text{-}1} & \mathbf{M}_{3\text{-}2} & \mathbf{M}_{3\text{-}3} \end{bmatrix} \tag{19}$$

Obviously, the natural frequencies and eigenvectors can now be readily obtained by solving a standard matrix eigenproblem. Since the components of each eigenvector are actually the expansion coefficients of the modified Fourier series, the corresponding mode shape can be directly determined from Eq. (13). In the other word, once the coefficient eigenvector **E** is determined for a given frequency, the displacement functions of the plate can be determined by substituting the coefficients into the Eq. (13). When the forced vibration is involved, by adding the work done by external force in the Lagrangian energy function and summing the loading vector **F** on the right side of Eq. (17), thus, the characteristic equation for the forced vibration of the moderately thick rectangular plates is readily obtained.

3. NUMERICAL EXAMPLES AND DISCUSSION

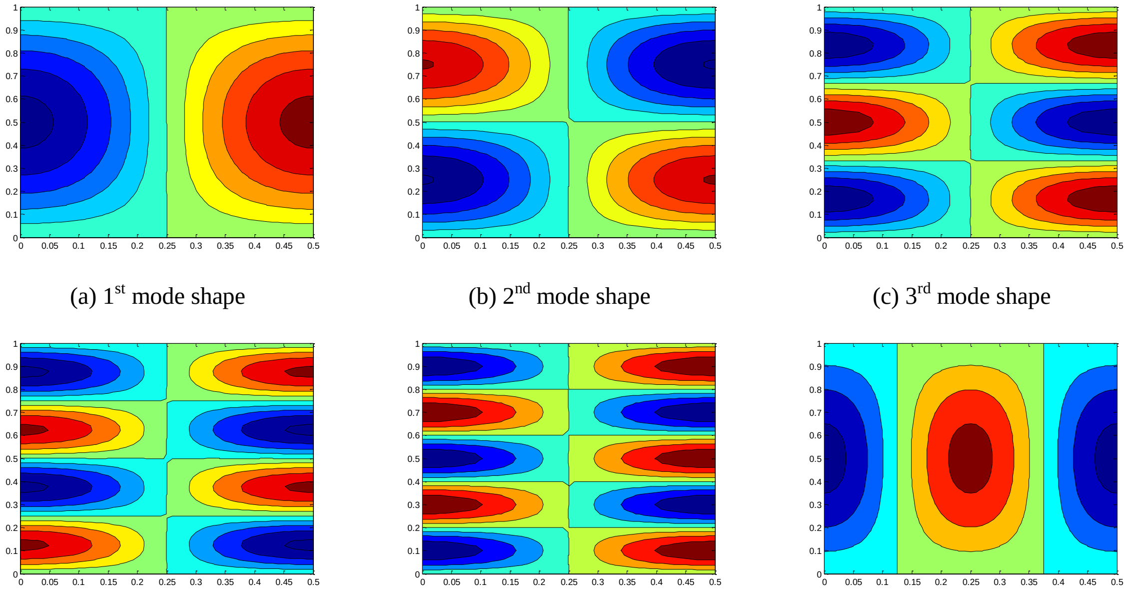
In this section, a systematic comparison between the current solutions and theoretical results published by other researchers or finite element method (FEM) results is carried out to validate the excellent accuracy, reliability and feasibility of the modified Fourier method. In the following calculation, μ_y=0.25, while μ_x can be immediately evaluated through the relationship of $\mu_x E_x = \mu_y E_y$. First, a S-S-S-S boundary can be considered as a special case when the stiffness constants for the rotational and torsional boundary springs become infinitely large (which is represented by a very large number, 10^{14}, in actual calculations) and the translation boundary spring become zeros. As mentioned earlier, the series expansions, Eqs. (11)-(13), will have to be truncated in numerical calculations. To examine the convergence of the solution, Table 1 compares the first eight frequency parameters $\Omega=(\omega a^2/\pi^2)(\rho h/D_{22})^{1/2}$ which are derived by using different numbers of term (represented by M and N) in the series expansions with a classical full simply-support boundary and different aspect ratio b/a. The Table shows the proposed method has fast convergence behavior. The maximum discrepancy in the worst case between the 5×5 truncated configuration and the 10×10 one is less than 0.57%. It is also shown that the results converge at M=N=15 for the given five-digit precision. Therefore, in the following calculations, all the Fourier series are truncated in into M=N=15.

Table 1 – Convergence and accuracy of the Frequency parameter $\Omega= (\omega a^2/\pi^2)\ (\rho h/D_{22})^{1/2}$ for full simply-support orthotropic Mindlin rectangular plates with different aspect ratios b/a.

a/b	M=N	Model sequence							
		Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6	Ω_7	Ω_8
0.5	5	13.3708	14.1679	16.1199	19.6214	24.4594	29.6808	30.0957	30.2710

	10	13.3422	14.1011	16.0830	19.5737	24.3366	29.6808	30.0149	30.0957
	12	13.3308	14.0879	16.0828	19.5725	24.3302	29.6808	29.9961	30.0957
	15	13.3308	14.0879	16.0827	19.5719	24.3301	29.6808	29.9882	30.0957
	Ref.[21]	13.3309	14.0880	16.0827	19.5717	24.3288	29.6810	-----	-----
1	5	5.16612	6.64529	10.0158	13.3708	14.1372	14.8897	16.1199	19.6214
	10	5.14388	6.62718	9.98902	13.3422	14.0920	14.8268	16.0831	19.5738
	12	5.14214	6.62529	9.98850	13.3310	14.0881	14.8245	16.0828	19.5725
	15	5.14212	6.62529	9.98848	13.3308	14.0880	14.8243	16.0828	19.5719
	Ref.[21]	5.14212	6.62529	9.98836	13.3309	14.0880	14.8241	16.0827	19.5717

Now, let us turn our attention to the free vibration analysis of orthotropic Mindlin plates with the uniform boundary conditions. The natural frequency for fully simply-support orthotropic Mindlin rectangular plate is recalculated and the results are compared with those obtained in Ref[21]. Tabulated in Tables 2-4 are the first six frequency $\Omega=(\omega a^2/\pi^2)(\rho h/D_{22})^{1/2}$ for the orthotropic Mindlin plate with different boundary conditions, various aspect ratio and thickness, in which the stiffness ratio $E_x/E_y = 40$ and the shear parameter is defined as $G_{xy}=G_{zx}=3E_y/5$. The first six mode shapes of orthotropic Mindlin rectangular plates with S-S-S-S boundary conditions, a/b=0.5 and h/b=0.1 is shown in Fig.2. It can be seen that current results agree well with those obtained in Ref [21].



(a) 1st mode shape (b) 2nd mode shape (c) 3rd mode shape

(d) 4th mode shape (e) 5th mode shape (f) 6th mode shape

Figure 2 – The first six mode shapes of S-S-S-S orthotropic Mindlin plates

Table 2 – The first six Frequency parameter $\Omega= (\omega a^2/\pi^2)\ (\rho h/D_{22})^{1/2}$ for orthotropic Mindlin rectangular plates with S-C-S-C boundary condition, different aspect ratios a/b and thickness.

a/b	h/b	Model sequence					
		Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6
0.5	0.05	19.8766	21.1325	23.9389	28.6594	35.2450	43.4135
		19.8765[a]	21.1322	23.9370	28.6552	35.2345	43.3955
	0.20	7.68757	8.74772	10.6647	13.2021	15.4630	16.0547
		7.68768[a]	8.74780	10.6647	13.2021	15.4633	16.0549
2	0.05	2.86663	6.48226	6.48544	9.00855	11.9607	12.5333

		2.86654[a]	6.48179	6.48536	9.00806	11.9588	12.5331
	0.20	2.31791	3.87040	4.67706	5.64948	5.75272	7.09121
		2.31791[a]	3.87040	4.67707	5.64949	5.75271	7.09120

[a] Results in parentheses are taken from Ref[21]

Thus far, the entire examples are confined to the classical boundary conditions and their combinations on the four edges. In the next example, we will account for the free vibration of orthotropic Mindlin plates with elastic edge supports. The model considered is a C-E_1-C-E_1 orthotropic plate elastically restrained at y=0 and y=b in the rotational direction; that is K_{y0}=K_{yb}=K, and all the other restraining springs are set to have an infinite stiffness (namely, represented by 10^{14} in numerical calculation). The first six frequencies $\Omega=(\omega a^2/\pi^2)(\rho h/D_{22})^{1/2}$ are given in Table 5 for several different restraining coefficient values. Since there is little published data for the free vibration of orthotropic Mindlin plate with elastic boundary conditions, the FEA results calculated using ABAQUS are also listed here as a reference. It can be clearly seen that the comparison is extremely good, which implies that the current method is able to make correct predictions for the transverse modal characteristics of orthotropic Mindlin plate with not only classical boundary conditions but also elastic edge restraints.

Table 3 – The first six Frequency parameter $\Omega= (\omega a^2/\pi^2) (\rho h/D_{22})^{1/2}$ for orthotropic Mindlin rectangular plates with S-C-S-F boundary condition, different aspect ratios *a/b* and thickness.

a/*b*	*h*/*b*	Model sequence					
		Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6
0.5	0.05	19.6085	20.0954	21.4783	24.3652	29.1366	35.7661
		19.6084[a]	20.0956	21.4787	24.3656	29.1366	35.7666
	0.20	7.42780	7.77757	8.92495	10.9484	13.5893	15.3242
		7.42788[a]	7.77759	8.92494	10.9483	13.5891	15.3241
2	0.05	1.66014	3.02462	5.92273	6.62654	6.74465	9.32276
		1.66010[a]	3.02461	5.92275	6.62651	6.74460	9.32273
	0.20	1.31655[a]	2.42304	3.34647	3.98179	4.90722	5.40681
		1.31658[a]	2.42302	3.34646	3.98173	4.90728	5.40686

[a] Results in parentheses are taken from Ref[21]

Table 4 – The first six Frequency parameter $\Omega= (\omega a^2/\pi^2) (\rho h/D_{22})^{1/2}$ for orthotropic Mindlin rectangular plates with S-F-S-F boundary condition, different aspect ratios *a/b* and thickness.

a/*b*	*h*/*b*	Model sequence					
		Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6
0.5	0.05	19.5515	19.7160	20.327	21.8168	24.7885	29.6114
		19.5579[a]	19.7126	20.3224	21.8161	24.7848	29.6133
	0.20	7.39972	7.46765	7.87991	9.10097	11.2570	13.9837
		7.39732[a]	7.46606	7.87859	9.10808	11.2555	13.9843
2	0.05	1.54958	1.74568	3.19566	5.84364	6.04243	6.76115
		1.54794[a]	1.74995	3.19852	5.84845	6.04228	6.76514
	0.20	1.22817	1.35760	2.58215	3.29680	3.39526	4.12341
		1.22206	1.35526	2.58500	3.29642	3.39040	4.12109

[a] Results in parentheses are taken from Ref[21]

Table 5 – The first six frequencies $\Omega=(\omega a^2/\pi^2)(\rho h/D_{22})^{1/2}$ for orthotropic Mindlin plates with several different restraining coefficient values (a/b=2, h/b=0.2).

K_r(Nm/rad)	Model sequence					
	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6
0	1.9878	3.6554	3.8149	4.9638	5.5888	6.5512
	1.9877[a]	3.6558	3.8155	4.9639	5.5890	6.5516
10^4	1.9880	3.6555	3.8153	4.9640	5.5889	6.5514
	1.9882[a]	3.6557	3.8157	4.9641	5.5891	6.5517
10^6	2.0119	3.6681	3.8487	4.9891	5.5968	6.5699
	2.0122[a]	3.6682	3.8489	4.9894	5.5966	6.5702
10^8	2.4617	3.9260	4.5690	5.5505	5.7657	6.9979
	2.4620[a]	3.9266	4.6594	5.5504	5.7660	6.9982
10^{10}	2.5699	3.9940	4.7709	5.7150	5.8118	7.1275
	2.5701[a]	3.9945	4.7712	5.7155	5.8121	7.1281
10^{12}	2.5712	3.9949	4.7734	5.7171	5.8124	7.1292
	2.5716[a]	3.9954	4.7738	5.7173	5.8128	7.1296
10^{14}	2.5712	3.9949	4.7735	5.7171	5.8124	7.1292
	2.5714[a]	3.9951	4.7735	5.7173	5.8128	7.1296

[a] Results in parentheses are calculated by using ABAQUS

4. CONCLUSIONS

In this paper, a modified Fourier method has been presented to study the free vibration behaviors of orthotropic rectangular Mindlin plates with arbitrary boundary conditions. The first-order shear deformation plate theory is adopted to formulate the theoretical model. The displacements and rotation components of the plate, regardless of boundary conditions, are invariantly expressed as a new form of trigonometric series expansions with a drastically improved convergence as compared with the conventional Fourier series to ensure and accelerate the convergence of the solution. At each edge of the plate, the general restraint condition are implemented by introducing one group of linear springs and two groups of rotational springs, which are continuously distributed and determined by the stiffnesses of these springs. Instead of seeking a solution in strong forms in the previous studies, all the Fourier coefficients will be treated equally and independently as the generalized coordinates and solved directly from the Rayleigh–Ritz technique. The change of the boundary conditions can be easily achieved by only varying the stiffness of the three sets of boundary springs along all edges of the rectangular plates without involving any change to the solution procedure. The convergence of the present solution is examined and the excellent accuracy is validated by comparison with existing results published in the literature and FEM data. Excellent agreements are obtained from these comparisons. Numerical results are presented to illustrate the current method is not only applied to the classical homogeneous boundary conditions but also other interesting and practically important boundary restraints on free vibrations of the orthotropic rectangular Mindlin plates with varying stiffness of boundary springs.

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