



# Numerical and experimental analysis of the effectiveness of material composition of piezoelectric elements with chosen shapes on plate vibration reduction

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## ABSTRACT

This paper presents numerical analyses of efficiency of different shapes and with a step change of material properties of piezoelectric actuators used for vibration and structural noise reduction. Analytical models representing a plate clamped on all sides with two attached piezoelectric elements were created. For each model, one element had the same shape and composition and was used for plate excitation, and the other one, used for vibration reduction, had different shape (circular, square, triangular) and location of the inner part of a two-part piezoactuator. Analyses of results were performed with ANSYS software. Finally, results of numerical simulations concerning effectiveness in vibration reduction of piezoactuators constructed this way are presented and are compared with experimental results.

Keywords: vibration reduction, piezoactuators I-INCE Classification of Subjects Number(s): 38.3

## 1. INTRODUCTION

For more than 20 years piezoelements have been used for active methods. Starting with works of Dimitriadis, Fuller, Rogers (1) these materials are typically used for beams (2-4) and plate structures (5-13). Using piezoelements analytical models (2, 5, 9, 10), numerical ones (3, 8, 10) or physical experiments (6-7) show significant vibration or noise reduction. Most of papers are devoted to homogeneous piezoactuators. Very few works deal with different composition of piezoelements. These works use Functionally Graded Materials (FGM) (3, 9-10) where material properties change along the thickness of the element. This work concentrates on using two-part elements with a step change in composition. A kind of element that could be manufactured without too much difficulty.

## 2. DESCRIPTION OF ANALYSIS

The subject of this work is to determine whether two-part piezoactuator can be an improvement over a standard homogeneous actuator. This was done in two steps. First, numerical models consisting of a steel plate (400x400x2 mm) with attached piezoelements and acoustic volume (Fig. 1) were created using ANSYS software. For each model one of the piezoelements was always a square based homogeneous element and was used for plates excitation. The other element, used for vibration reduction had different shape or composition (Fig. 2). The considered shapes and composition were:

- square based homogeneous element with  $a = 40$  mm, and  $h = 1$  mm;
- disk based homogeneous element with  $r = 40/\pi$  mm, and  $h = 1$  mm;
- right-angled triangle based homogeneous element with  $a = 40\sqrt{2}$  mm, and  $h = 1$  mm;
- square based two-part element with  $a = 40$  mm, and  $h = 1$  mm (the inner part area is 1/4 of the whole element area);
- disk based two-part element with  $r = 40/\pi$  mm, and  $h = 1$  mm (the inner part area is 1/4 of the whole element area);

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- right-angled triangle based two-part element with  $a = 40\sqrt{2}$  mm and  $h=1$  mm (the inner part area is 1/4 of the whole element area).
- Therefore all actuators had the same base area  $A=1600 \text{ mm}^2$ , which in case of two-part elements is divided between the inner part and the outer part.

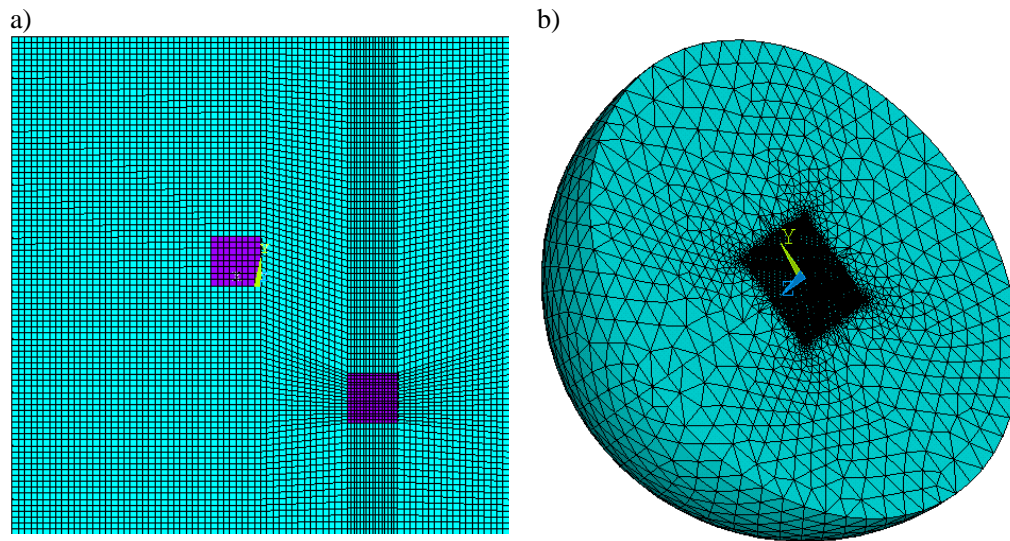


Figure 1 – Created model a) without acoustic volume, b) with acoustic volume

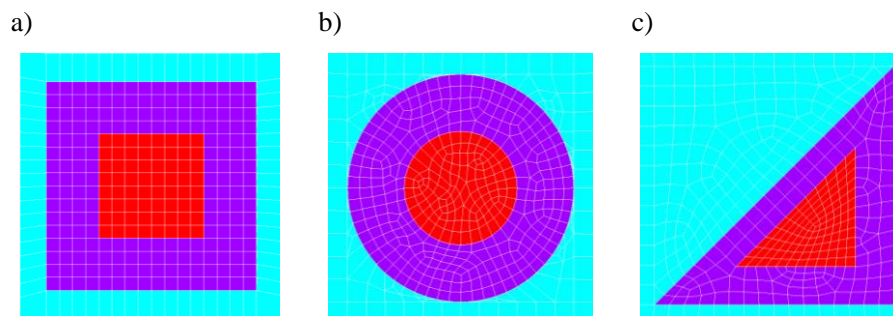


Figure 2 – Modelled two-part piezoactuators: a) square based, b) disk based, c) right-angled triangle based

For structural sound radiation analyses a half ball of air with  $R=1$  m was attached to the front area of the plate. The outer areas of the modelled acoustic volume had an attenuation coefficient equal to 1.

Table 1 shows what elements were used for modelling plate and piezoelements and what material properties were given to them.

Table 1 – Physical parameters of liquids

Structural elements	Element used for modelling	Material properties
Plate	SOLSH190	$E = 1.93 \cdot 10^{11} \text{ Pa}$ , $\nu = 0.29$ $\rho = 7800 \text{ kg/m}^3$
Piezoelement used for plates excitation	SOLID226	Properties of PZ 28
Actuators	SOLID226	Combination of properties of PZ 28 and PZ 29
Air (sound medium)	FLUID30	$c = 343 \text{ m/s}$ , $\rho = 1.2 \text{ kg/m}^3$ $p_0 = 2 \cdot 10^{-5} \text{ Pa}$

Properties of PZ 28:  $\rho = 7.70 \cdot 10^3 \text{ kgm}^{-3}$ ,  $d_{31} = -1.14 \cdot 10^{-10} \text{ mV}^{-1}$ ,  $d_{33} = 2.75 \cdot 10^{-10} \text{ mV}^{-1}$ ,  $d_{15} = 4.03 \cdot 10^{-10} \text{ mV}^{-1}$ ,  $s_{11}^E = 1.26 \cdot 10^{-11} \text{ pa}^{-1}$ ,  $s_{33}^E = 1.83 \cdot 10^{-11} \text{ pa}^{-1}$ ,  $s_{12}^E = -3.71 \cdot 10^{-12} \text{ pa}^{-1}$ ,  $s_{13}^E = 6.60 \cdot 10^{-12} \text{ pa}^{-1}$ ,  $s_{44}^E = 3.77 \cdot 10^{-11} \text{ pa}^{-1}$ ,  $s_{66}^E = 3.26 \cdot 10^{-11} \text{ pa}^{-1}$ ,  $e_{11}/e_0 = 1.22 \cdot 10^3$ ,  $e_{33}/e_0 = 9.90 \cdot 10^2$ . Properties PZ 29:  $\rho = 7.46 \cdot 10^3 \text{ kgm}^{-3}$ ,

$d_{31}=-2.43\text{E-}10 \text{ mV}^{-1}$ ,  $d_{33}=5.74\text{E-}10 \text{ mV}^{-1}$ ,  $d_{15}=7.24\text{E-}10 \text{ mV}^{-1}$ ,  $s_{11}^E=1.70\text{E-}11 \text{ pa}^{-1}$ ,  $s_{33}^E=2.29\text{E-}11 \text{ pa}^{-1}$ ,  $s_{12}^E=-5.78\text{E-}12 \text{ pa}^{-1}$ ,  $s_{13}^E=8.79\text{E-}12 \text{ pa}^{-1}$ ,  $s_{44}^E=3.80\text{E-}11 \text{ pa}^{-1}$ ,  $s_{66}^E=4.56\text{E-}11 \text{ pa}^{-1}$ ,  $e_{11}^S/e_0=2.44\text{E+}03$ ,  $e_{33}^S/e_0=2.87\text{E+}03$ .

Modal analyses were performed for the first six mode shapes after which first and fifth modes were chosen for further analyses. The amplitude of voltage applied to the element used for plates excitation was  $V_e=100 \text{ [V]}$ . The reduction of plates vibration was obtained using internal ANSYS optimization procedures. The goal function took the form of:

$$J = \min \sum_{i=1}^n \vec{X}(i)_{sum} \quad (1)$$

where:

$\vec{X}(i)_{sum}$  - displacement vector sum in  $i$  node;

$n$  – number of nodes (in this case all nodes making up the base of the plate).

The design variables for the procedures were:  $V_a$ – amplitude of voltage applied to the actuator. The number of procedure steps was 30. After each completion of optimization procedure the procedure was repeated with the ranges of the amplitude being narrowed, with the final ranges being 5 [V]. The phase angle of applied voltages were set for  $180^\circ$  (for first mode) and  $0^\circ$  (for fifth mode).

The second step was conducting a physical experiment to verify numerical results. As two-part element are not normally manufactured the samples were created in the Department of Material Science at Faculty of Computer Science and Materials Science of University of Silesia. The object was a steel plate  $500 \times 500 \times 2 \text{ mm}$  clamped on all sides with piezoelements attached. One of those piezoelements was used for plate excitation, a pair was used as sensors, and two pairs were used as actuators. Sensors and element used for excitation were square based  $20 \times 20 \times 1 \text{ mm}$  piezoelements made of PZT 4D material. Actuators consisted of:

- rectangle based homogeneous element with  $a=20 \text{ mm}$ ,  $b=30 \text{ mm}$ , and  $h=2 \text{ mm}$  (made of either PZ 45 or PZ 54);
- disk based homogeneous element with  $d=10 \text{ mm}$  and  $h=1 \text{ mm}$  (made of either PZ 45 or PZ 54);
- rectangle based two-part element with  $a=20 \text{ mm}$ ,  $b=20 \text{ mm}$ , and  $h=2 \text{ mm}$  with inner part having 1/4 area size of the whole element (made of combination of PZ 45 and PZ 54);
- disk based two-part element with  $d=10 \text{ mm}$ , and  $h=1 \text{ mm}$  with inner part having 1/4 area size of the whole element (made of combination of PZ 45 and PZ 54).

The placement of piezoelements attached to the plate is shown on Figure 3.

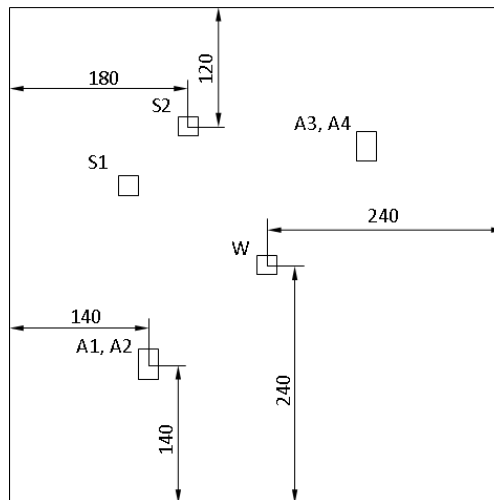


Figure 3 –Piezoelements placement: S1, S2 – sensors; A1-A4 – actuators (pairs attached on both sides of the plate; W – element used for excitation

The laboratory stand consisted of steel plate clamped on all sides, a laptop computer with labVIEW software installed, a voltage amplifier, a NI 9215 and NI 9263 modules for signal generation and acquisition. This set-up was used in order to minimize any outside electric interference.

Resonance frequencies were obtained by first exciting plate with white noise generated from one

piezoelement followed by a check with sine signal with frequency rounded to the nearest 10.

The plate was excited by sine signal of a given resonance frequency from range 100-1200 Hz using element W. The voltage applied for excitation was 25 V. Reduction was carried out using each actuator one at a time first minimizing the vibrations on S1 then on S2. The upper limit of voltage for actuators was 350 V. In the case when the limit was reached without achieving sufficient reduction the amplitude of excitation was reduced to 15 V.

### 3. RESULTS

Results of performed simulations are presented on Tables 2-4. First column in each table describes the material used for piezoactuator and in the case of two-part actuators the first name refers to the inner layer, second one to the outer layer. Second column relates to the mode shape. In the third is the amplitude of the voltage applied to the actuator. Next column is the phase angle of said voltage. Second to the last is the vibration reduction obtained in the analysis. Finally, the last column is difference between the max level of SPL of sound radiated from the plate before and after vibration reduction.

A vibration reduction about 41 dB for the 1st mode and about 35 dB for the 5th mode for square based elements was achieved (Table 2). For disk based elements the obtained reduction was less about 0.5dB than the square based elements, for the 1st mode and about 0,5 greater than the square based elements for the 5th mode (Table 3). For the right-angled triangle based piezoactuators the obtained reduction was greater about 3 dB than the square based elements, for the 1st mode and about 0,3 greater than square based elements for the 5th mode (Table 4).

Table 2 – Results for the square based piezoactuators: material – material used for inner/outer part of actuator; mode – mode shape;  $V_a$  – amplitude of voltage applied to element;  $\varphi_a$  – phase angle of voltage applied to element;  $L_{red}$  – vibration reduction level obtained from the simulation;  $\Delta L_{maxSPL}$  – difference between max SPL levels before and after vibration reduction

Material, in./out.	Mode	$V_a$ , V	$\varphi_a$ , °	$L_{red}$ , dB	$\Delta L_{maxSPL}$ dB
PZ 28	1	365.29	180.00	41.4	41.3
PZ 29/PZ 28	1	294.88	180.00	41.4	41.3
PZ 29	1	198.29	180.00	41.0	40.8
PZ 28/PZ 29	1	228.29	180.00	41.4	41.2
PZ 28	5	159.95	360.00	34.6	33.9
PZ 29/PZ 28	5	129.1	360.00	35.0	33.1
PZ 29	5	86.93	360.00	34.6	33.7
PZ 28/PZ 29	5	100.45	360.00	34.4	33.7

Table 3 – Results for the disk based piezoactuators: material – material used for inner/outer part of actuator; mode – mode shape;  $V_a$  – amplitude of voltage applied to element;  $\varphi_a$  – phase angle of voltage applied to element;  $L_{red}$  – vibration reduction level obtained from the simulation;  $\Delta L_{maxSPL}$  – difference between max SPL levels before and after vibration reduction

Material, in./out.	Mode	$V_a$ , V	$\varphi_a$ , °	$L_{red}$ , dB	$\Delta L_{maxSPL}$ dB
PZ 28	1	371.94	180.00	40.0	41.2
PZ 29/PZ 28	1	300.24	180.00	39.8	41.1
PZ 29	1	200.24	180.00	39.8	41.2
PZ 28/PZ 29	1	230.81	180.00	39.8	41.3
PZ 28	5	161.98	360.00	35.2	34.1
PZ 29/PZ 28	5	131.24	360.00	34.8	33.5
PZ 29	5	87.29	360.00	35.2	34.0
PZ 28/PZ 29	5	101.24	360.00	35.4	33.6

Table 4 – Results for the right-angled triangle based piezoactuators: material – material used for inner/outer part of actuator; mode – mode shape;  $V_a$  – amplitude of voltage applied to element;  $\varphi_a$  – phase angle of voltage applied to element;  $L_{red}$  – vibration reduction level obtained from the simulation;  $\Delta L_{maxSPL}$  – difference between max SPL levels before and after vibration reduction

Material, in./out.	Mode	$V_a$ , V	$\varphi_a$ , °	$L_{red}$ , dB	$\Delta L_{maxSPL}$ dB
PZ 28	1	255.02	180.00	43.6	44.3
PZ 29/PZ 28	1	212.02	180.00	43.0	44.0
PZ 29	1	145.55	180.00	43.2	43.9
PZ 28/PZ 29	1	165.55	180.00	43.2	44.1
PZ 28	5	201.24	360.00	35.4	26.9
PZ 29/PZ 28	5	156.94	360.00	35.0	28.3
PZ 29	5	109.24	360.00	35.0	25.4
PZ 28/PZ 29	5	127.69	360.00	43.6	26.1

Numerical results of vibration reduction are presented on Figures 4 and 5. Although it can be seen that shape of the actuator can be important, there are no significant differences between different material composition of used actuators (less than 1 dB). Also the voltage needed for obtained reduction is proportional to the amount of “stronger” material (in this case PZ 29).

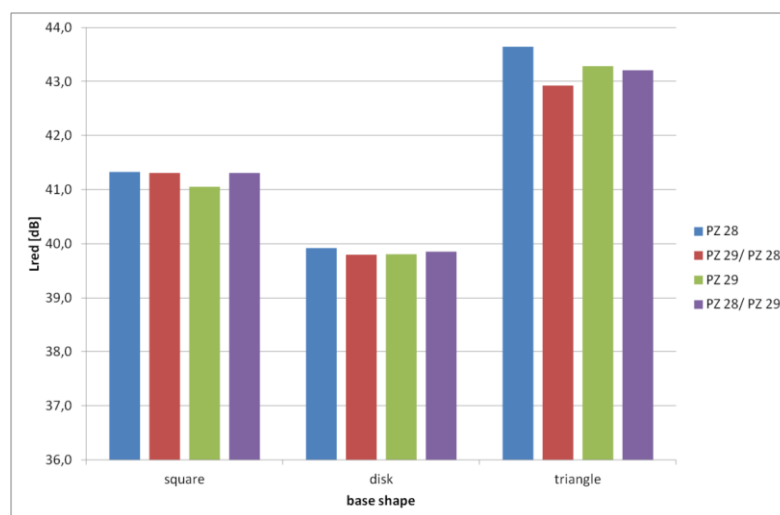


Figure 4 – Reduction obtained for first mode

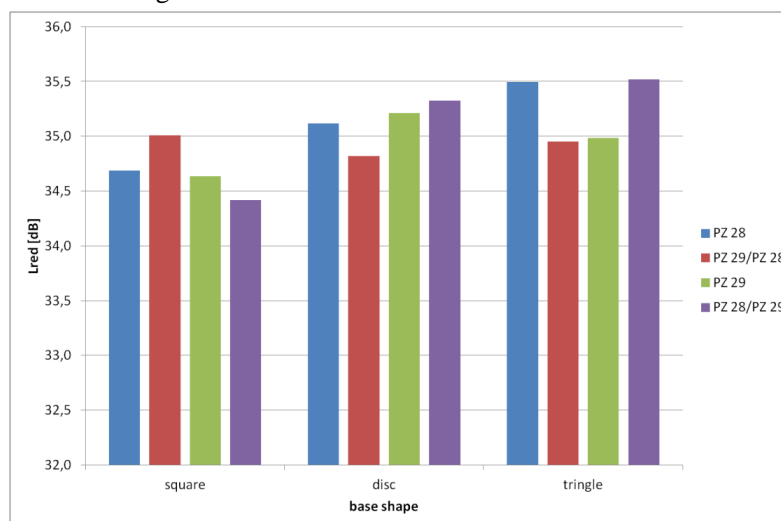


Figure 5 – Reduction obtained for fifth mode

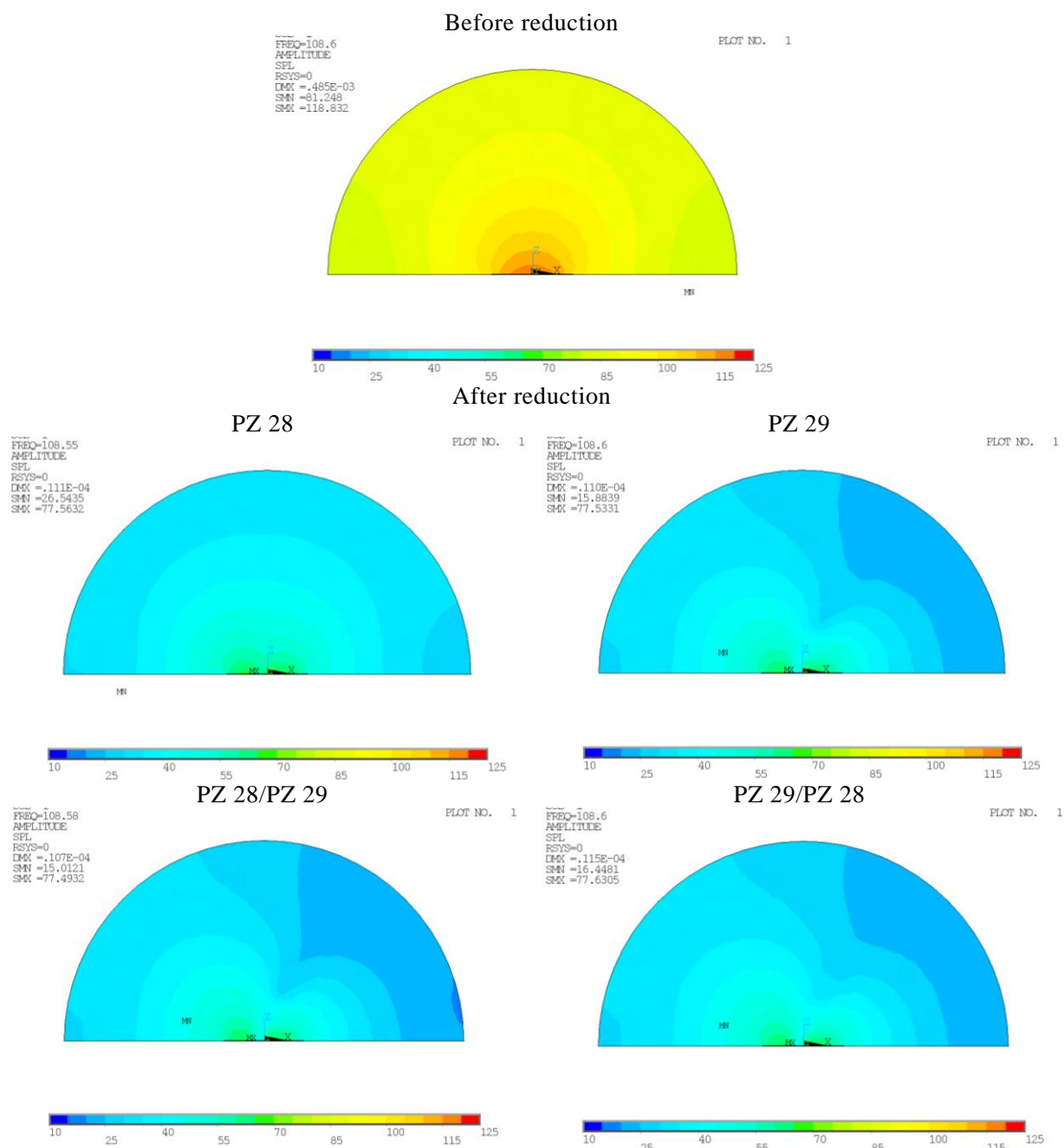


Fig. 6. Sound radiation for square based piezoactuators (first mode)

Figure 6 shows the sound radiation in the model for square based elements. Although the differences in levels of reduction for both vibration and sound pressure are really small (less than 0.5 and 0.6 dB, respectively) the distribution of sound pressure differs quite significantly. Another interesting thing is relatively small reduction of max SPL for the fifth mode for triangle based elements (in comparison to the vibration level reduction and max SPL level reduction for different shapes of actuators).

Results of experiment with a rectangle based homogeneous piezoactuators are presented on Tables 5 and 6. A significant vibration reduction was obtained in all analyzed cases.

Table 5 – Results of experiment with a rectangle based homogeneous piezoactuators made of PZ 45. f, Hz - resonance frequency,  $A_w$ , V;  $A_1$ , V;  $A_2$ , V - amplitude of voltages applied to element used for excitation and actuators,  $\varphi_1$ , °,  $\varphi_2$ , ° - phase angles of voltage applied to actuators,  $L_{redS1}$ , dB,  $L_{redS2}$ , dB; - obtained reductions for S1 and S2

f, Hz	$A_w$ , V	$A_1$ , V	$\varphi_1$ , °	$A_2$ , V	$\varphi_2$ , °	$L_{redS1}$ , dB	$L_{redS2}$ , dB
130	25	100	141	0	0	35,7	
130	25	60	253	0	0		23,5
130	25	0	0	240	141	31,8	
130	25	0	0	115	258		29,5
230	25	135	102	0	0	37,6	
230	25	100	245	0	0		34,9
230	25	0	0	250	102	35,3	
230	25	0	0	215	246		34,9
370	25	130	281	0	0	36,6	
370	25	90	255	0	0		32,5
370	25	0	0	225	291	36,9	
370	25	0	0	180	261		31,7
540	25	95	289	0	0	28,7	
540	25	175	275	0	0		33,7
540	25	0	0	235	292	35,4	

Table 6 – Results of experiment with a rectangle based homogeneous piezoactuators made of PZ 54. f, Hz - resonance frequency,  $A_w$ , V;  $A_1$ , V;  $A_2$ , V - amplitude of voltages applied to element used for excitation and actuators,  $\varphi_1$ , °,  $\varphi_2$ , ° - phase angles of voltage applied to actuators,  $L_{redS1}$ , dB,  $L_{redS2}$ , dB; - obtained reductions for S1 and S2

f, Hz	$A_w$ , V	$A_1$ , V	$\varphi_1$ , °	$A_2$ , V	$\varphi_2$ , °	$L_{redS1}$ , dB	$L_{redS2}$ , dB
140	25	55	165	0	0	24,3	
140	25	70	182	0	0		31,4
140	25	0	0	50	160	25,1	
140	25	0	0	35	181		31,7
230	25	180	109	0	0	35,6	
230	25	110	231	0	0		33,5
230	25	0	0	115	121	34,8	
230	25	0	0	50	225		32,2
370	25	210	252	0	0	33,0	
370	25	155	232	0	0		35,2
370	25	0	0	170	253	33,9	
370	25	0	0	70	230		35,9
540	25	350	272	0	0	41,1	
540	25	305	193	0	0		41,3
540	25	0	0	240	286	41,1	
540	25	0	0	120	186		41,0

#### 4. CONCLUSIONS

The paper presents numerical analysis of the effects that different material composition of chosen shapes of piezoactuators have on their efficiency in plates vibration reduction and structural noise reduction.

Performed simulations showed that a step change in composition of piezoelements does not influence its efficiency in significant way. Vibration reduction levels obtained were roughly the same, the differences between different compositions can be attributed to optimization procedure.

It can also be theorized that with electrical separation between the inner and the outer parts these elements could be used as a sensor-actuator combo. With the inner layer being the sensor and the outer layer being actuator. The efficiency of reduction should not be significantly impaired.

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