FE based measures for structure borne sound radiation

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ABSTRACT

The sound emission of thin-walled radiating components is a common objective of structural optimisation. Acoustic measures are not implemented in common FE-codes. Thus, different velocitiy based measures will be compared: the kinetic energy, the equivalent radiated power (ERP) and the lumped parameter model (LPM). The most common approach - the ERP - is based on the sound intensity in normal direction and the sound pressure on the radiating surface. Assuming a unit radiation efficiency all-over the surface and neglecting local effects, this is a common approach for an upper bound of structure borne noise. Therein, the sound power finally results from the squared velocity integrated over the radiating surface and the constant fluid impedance. As ERP usually requires extra post processing to consider the velocity in normal surface direction, the kinetic energy is essential in common FEA results including all velocity components apart from the normal direction, too. Thus, it is less accurate but maybe usable for optimisation abilities. In contrast, LPM is a simplification of the Rayleigh-integral and thus gives quite accurate results but requires significant higher computational costs than ERP. Possibilities and limits of estimating the emitted sound power by these three methods will be shown.

Keywords: sound radiation, finite element method   I-INCE Classification: 23.1; 75.3

1. INTRODUCTION

Especially light and stiff thin-walled structures tend to be sensitive for structure borne sound. Thus, the sound radiation behaviour is a common optimisation criterion within the design and development. The optimisation procedure relates e.g. on the adjustable material behaviour of fibre reinforced plastics (e.g. stiffness and damping) (1). Sensitive parameters are fibre volume content, fibre orientation as well as stacking sequence resulting in non-linear dependencies (2). Either genetic or gradient based optimisations require numerous finite-element simulations. So there is a need for efficient numerical measures of structure borne sound as objective function of such processes.

Therefore, the radiated sound power is commonly used to express the behaviour of radiating surfaces, componentes and machnies and is formulated as the integral of the intensity over the surrounding and radiating surface. Exact calculations of the sound power are limited to a few academic cases. Thus, numerical approximation methods are commonly used but computationally expensive as fluid-sructure-interaction has to be solved in one or both directions.

The boundary element method (BEM) including fast-multipole techniques for large-scale problems became a very popular approach but is limited for a large frequency range or modified structures within optimisation loops (e. g. (3)).

Simplified methods are known and based on some assumptions. For hard reflecting surfaces such as stiff thin-walled structures, particle velocity and structure normal velocity are identical. Moreover, an evaluation of the sound pressure on the structure's surface is needed.

This paper compares three different approaches of sound power and the kinetic enery. Namely, there is the equivalent radiated sound power (ERP), the volume veloctiy (PVV) and the so called lumped parameter model (LPM) comparable to the Direct FEM.

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2. ENERGY AND POWER ESTIMATIONS

2.1 Kinetic Energy

Within steady state finite element simulations, the energy balance consisting of different elastic and dissipating components is estimated implicitly. Therein, the kinetic energy of the whole system or several components is given by the integral over the volume V

$$W_{kin} = \int_V \frac{1}{2}\rho_s \, \boldsymbol{v}\boldsymbol{v}^T \mathrm{d}V \tag{1}$$

with the density of the solid surface ρ_s and the surface velocity $\boldsymbol{v}$. According to common standards in acoustics the energy level

$$L_W = 10 \, \lg\left(\frac{W_{kin}}{W_0}\right) \, dB \tag{2}$$

is referred to a level of $W_0 = 10^{-12} J$. The use of global quantaties, e.g. potential and kinetic energy, to estimate the acoustic behaviour is suggested by (4).

2.2 Equivalent Radiated Power

In contrast, the radiation behaviour of vibrating components is often estimated by the radiated sound power P representing the integral of sound intensity I in normal direction over the closed surfaces Γ circumscribing the radiating object (5).

$$P = \int \vec{I}\cdot\vec{n} \, \mathrm{d}\Gamma \qquad \text{with} \qquad \vec{I} = \frac{1}{2}\Re\left(p\vec{v}\right) \tag{3}$$

Regarding to the velocity normal to the surface $v_n = \vec{v}\vec{n}$ imported from dynamic analysis, (5) compares estimates for the sound pressure analysing acoustic fields with numerical methods. Therein, the equivalent radiated power represents a simple, popular and efficient approach for the sound pressure in the local relation

$$\boldsymbol{p} \approx \rho_f \, c_f \boldsymbol{v} \tag{4}$$

with the fluid's density ρ_f as well as it's speed of sound c_f. The relation between particle velocity and sound pressure is reduced to the fluid's characteristic impedance.

$$Z_0 = \rho_f \, c_f \tag{5}$$

The approximation is typical in far fields and high frequencies and results in the sound power as an integral

$$P_{ERP} = \frac{1}{2}\rho_f \, c_f \int |v(x)|^2 \, \mathrm{d}\Gamma(x) \tag{6}$$

or discretised formulation for N_e piecewise constant elements with an area A_μ

$$P_{ERP} = \frac{1}{2}\rho_f \, c_f \sum_{\mu=1}^{N_e} A_\mu v_\mu v_\mu^* \tag{7}$$

This formulation neglects local effects such as interaction between sources assuming the same radiation efficiency $\sigma = 1$ for all elemental sources. Usually overestimating the radiation, it gives a good impression of an upper bound for convex rigid bodies and high frequencies.

2.3 Lumped Parameter Modell

For more accurate results the (5) mentioned the lumped parameter model (LPM) as well as the boundary element method (BEM) . The lumped parameter model by KOOPMANN and FAHNLINE (4, 6, 7) is based on a simplification of the RAYLEIGH-integral including a TAYLOR series for the GREEN's function as a multipole expansion. This yields to a formulation for a source at x_μ and a reciever at y_ν .

$$P_{LPM} = -\frac{1}{2}k \, \rho_f \, c_f \sum_{\mu=1}^{N_e}\sum_{\nu=1}^{N_e} S_\mu S_\nu \Im\left\{G_{\mu\nu}\right\}\Re\left\{v_\mu v_\nu^*\right\} \qquad \text{with} \qquad \Im\left\{G_{\mu\nu}\right\} = -\frac{\sin(k|x-y|)}{2\pi|x-y|} \tag{8}$$

The imaginary part of the GREEN's function weights the interacting sources. The double summation will be computationally more expensive than ERP but much more efficant than fast multipole BEM. LPM predictions are exact for dipole modes. Besides, the accuracy is dependent on the mesh refinement as well as the compliance with the assumptions of the RAYLEIGH-integral but gernerally gives appropriate results in the low and mid frequency range.

2.4 Volume Velocity

The volume velocity u is an integral of the particle velocity on the ratating surface (4).

$$u = \int v \mathrm{d}\Gamma = \sum_{\nu=1}^{N_e} v_\mu S_\mu \tag{9}$$

The derived radiatied sound power PVV

$$P_{VV} = \frac{k^2 \rho_f \, c_f}{4\pi} \, u \, u^* = \frac{k^2 \rho_f \, c_f}{4\pi} \sum_{\mu=1}^{N_e} \sum_{\nu=1}^{N_e} S_\mu S_\nu \Re \left\{ v_\mu v_\nu^* \right\} \tag{10}$$

is understood as a reduction of the LPM. It includes local antiphase vibration such as dipole effects but only requires a single sum (5). In this case all interactions have a constant freqeuency dependet radiation efficiency.Thus, PVV provides good results for the average power in the lower frequency range but is not sensitive to mesh refinement.

3. IMPLEMENTATION

Based on steady state numerical models wirh harmonic force excitation, all given sound power estimations have been determined in a post-processing script. Numerical simulations have been done in ABQUS 6.13 and thus the script is a python-algorithm.

The implementation of all previously named sound power estimates is based on piecewise constant elements and follows (5).

$$P = \sum_{\mu=1}^{N_e} \sum_{\nu=1}^{N_e} P_{\mu\nu} = \sum_{\mu=1}^{N_e} P_{\mu\mu} + 2 \sum_{\mu=1}^{N_e-1} \sum_{\nu=\mu+1}^{N_e} P_{\mu\nu} \tag{11}$$

The sound power portions $P_{\mu\nu}$ are understood as a partial sound power of all N_e constant elements acting as a monopole soure. In detail, $P_{\mu\mu}$ represents the independent source distributions whereas $P_{\mu\nu}$ ($\mu \neq \nu$) considers the interaction between these sources.

The interaction matrix is symertic and its elements can be determined by

$$P_{\mu\nu} = P_{\nu\mu} = \frac{1}{2} \rho_f \, c_f \, S_\mu \, \sigma_{\mu\nu} \, \Re \left\{ v_\mu v_\nu^* \right\} \tag{12}$$

with the dimensionless radiation efficiency $\sigma_{\mu\nu}$. For the different sound power modells $\sigma_{\mu\nu}$ acts as

$$\sigma_{\mu\nu} = \delta_{\mu\nu} \qquad \text{for ERP,} \tag{13}$$

$$\sigma_{\mu\nu} = \frac{k^2 \, S_\nu}{2\,\pi} \frac{\sin(k \, R_{\mu\nu})}{k \, R_{\mu\nu}} \qquad \text{for LPM,} \tag{14}$$

$$\sigma_{\mu\nu} = \frac{k^2 \, S_\nu}{2\,\pi} \qquad \text{for PVV.} \tag{15}$$

As mentioned, the numerical estimations are of different computational efforts. ERP and PVV sums are of order N_e whereas LPM requires N_e^2 steps. In the given implementation, reading the nodal velocity fields for all frequency points from the output database took most of the time of the whole post-processing. Array functions in Python are such efficient, that the order of summation did not effect the total time noticeably.

4. MODELLING AND RESULTS OF TEST CASES

For comparative studies of the energy and sound power estimates, academic cases such as a free-free-plate (FFFF) and a clamped plate (CCCC) as well as an oil pan have been used. The material has been assumed to be linear elastic steel with a viscous damping ratio η without frequency dependency. All parameters are given in table 1.

Table 1 – Material properties of steel

elastic parameters			damping
E/MPa	ν	$\rho/g/cm^3$	$\eta/\%$
210000	0.30	7.80	0.045

4.1 Rectangular Plates

First, the estimations have been tested on a rectangular plate of 250 x 200 2 mm with a harmonic excitation of 1 N in a frequency band from 10 Hz to 1000 Hz. The mesh consits of quadratic shell elements with 2mm mesh size. The simulations in the frequency domain are based on a modal superposition. In favour, the FFFF-plate shows 13 resonaces within that range starting at 127 Hz. Only three modes above 378 Hz are determined for the CCCC-plate.

The results in figure 1 show very similar resonances for W_{kin} and P_{ERP}. Resonance heights are generally decreasing over frequency and damping. The absolute error of W_{kin} and P_{ERP} is constantly 13 dB over the whole bandwidth. PVV and LPM differ slightly between the modes but otherwise are of very similar characteristics.

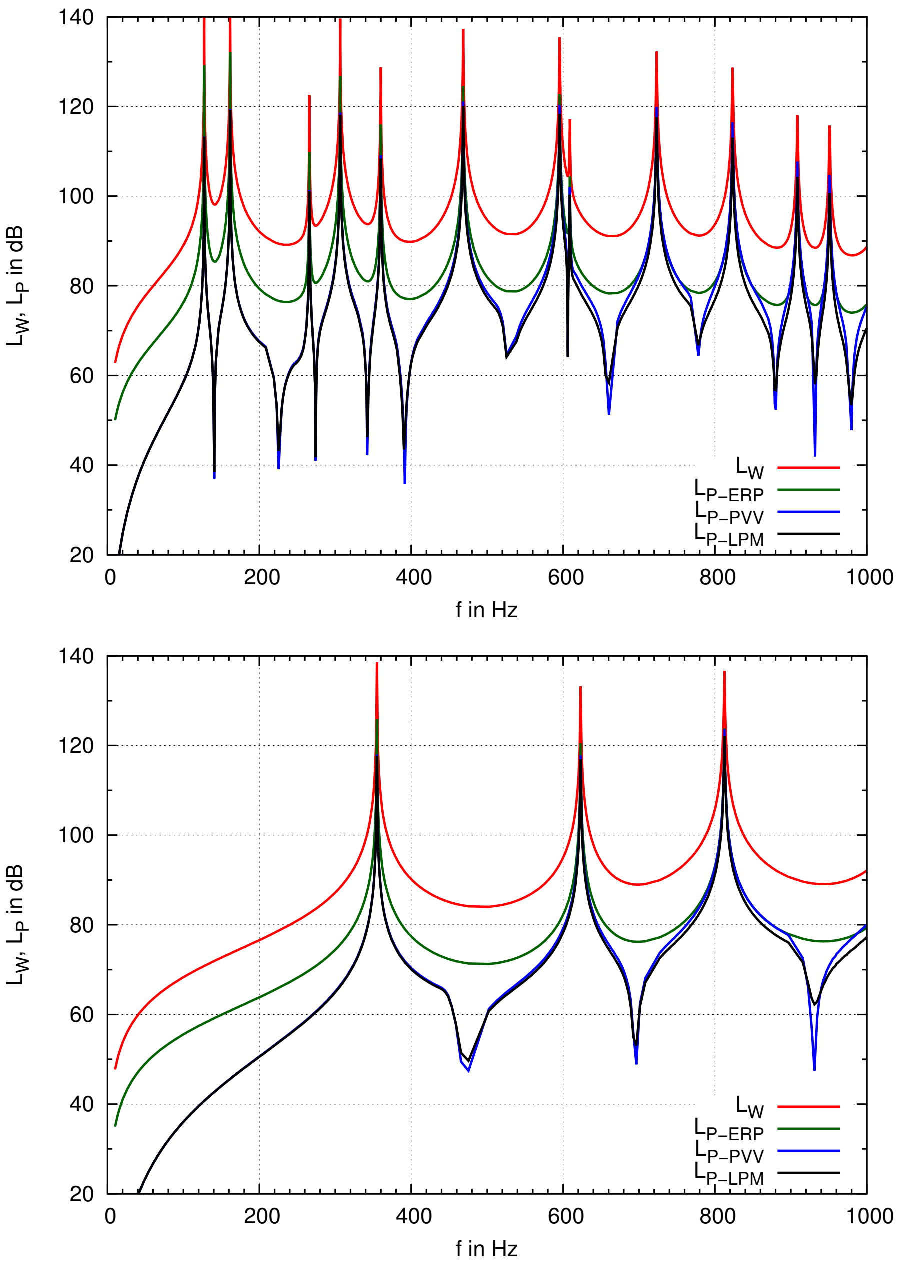

Figure 1 – plate results for FFFF and CCCC boundary conditions: kinetic energy and power estimates

Assuming the LPM beeing the most accurate estimation, the absolute errors related to the LPM are shown in figure 2. It shows the correlation of the ERP towards LPM/PVV results with increasing frequency but despite the singularities between the modes. In contrast, the slight difference of PVV and LPM increases over frequency and reaches less than 2.5 dB at 1000 Hz.

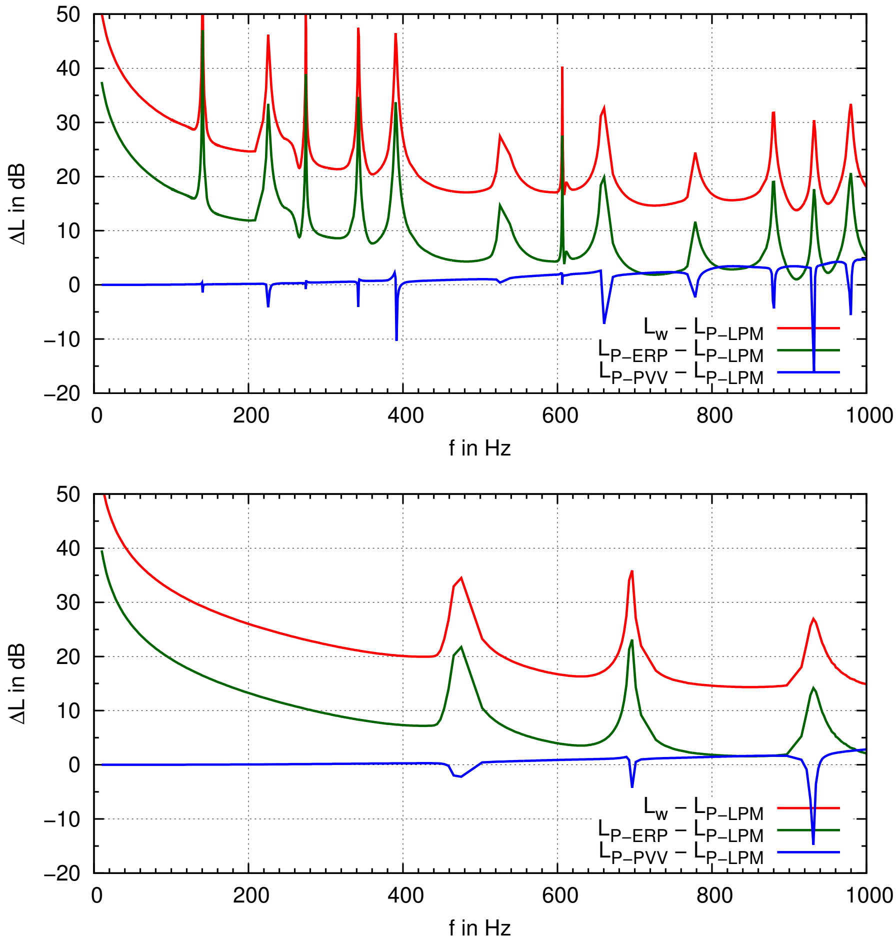

Figure 2 – Absolute error of the different estimates realted on th LPM results: plate with FFFF and CCCC boundary conditions

4.2 Oil pan

As a next step, a thin-walled formed structure of the same material has been analysed. The outer dimensions of the oil pan shape shown in figure 3 are app. 250 x 210 x 100 mm. Related to the figure, the part has been excited vertically on the top surface. The part is fixed with displacment boundary conditions along the lower edge (red). Wall thickness of the part is app. 1.7 mm and again the mesh has been developed with quadratic shell elements of 2 mm edge length. Thus, the model consits of about 8500 elements and 25000 nodes. The frequency range was increased up to 10 kHz for this model.

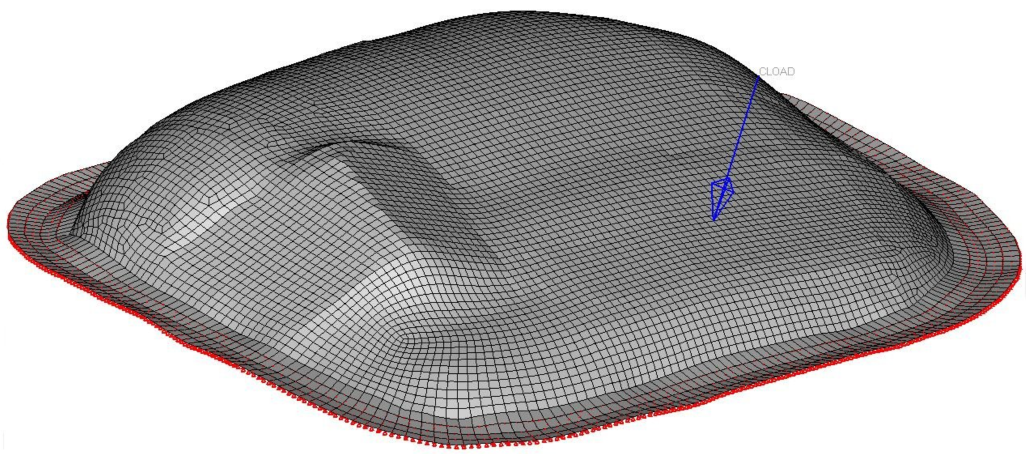

Figure 3 – Demonstrator part: oil pan

Both, kinetic energy and sound power estimates show a strong modal behaviour (figure 4). Again, the ERP and kinetic energy differ almost constantly 13 dB in the whole frequency range. Peaks are generally higher and more sharp in the energy level with a decreasing difference over the frequency. Moreover, the absolte errors for ERP and kinetic energy tend to constant values of 0 dB and 13 db above 2 kHz whereas the difference between PVV and LPM increases over frequency.

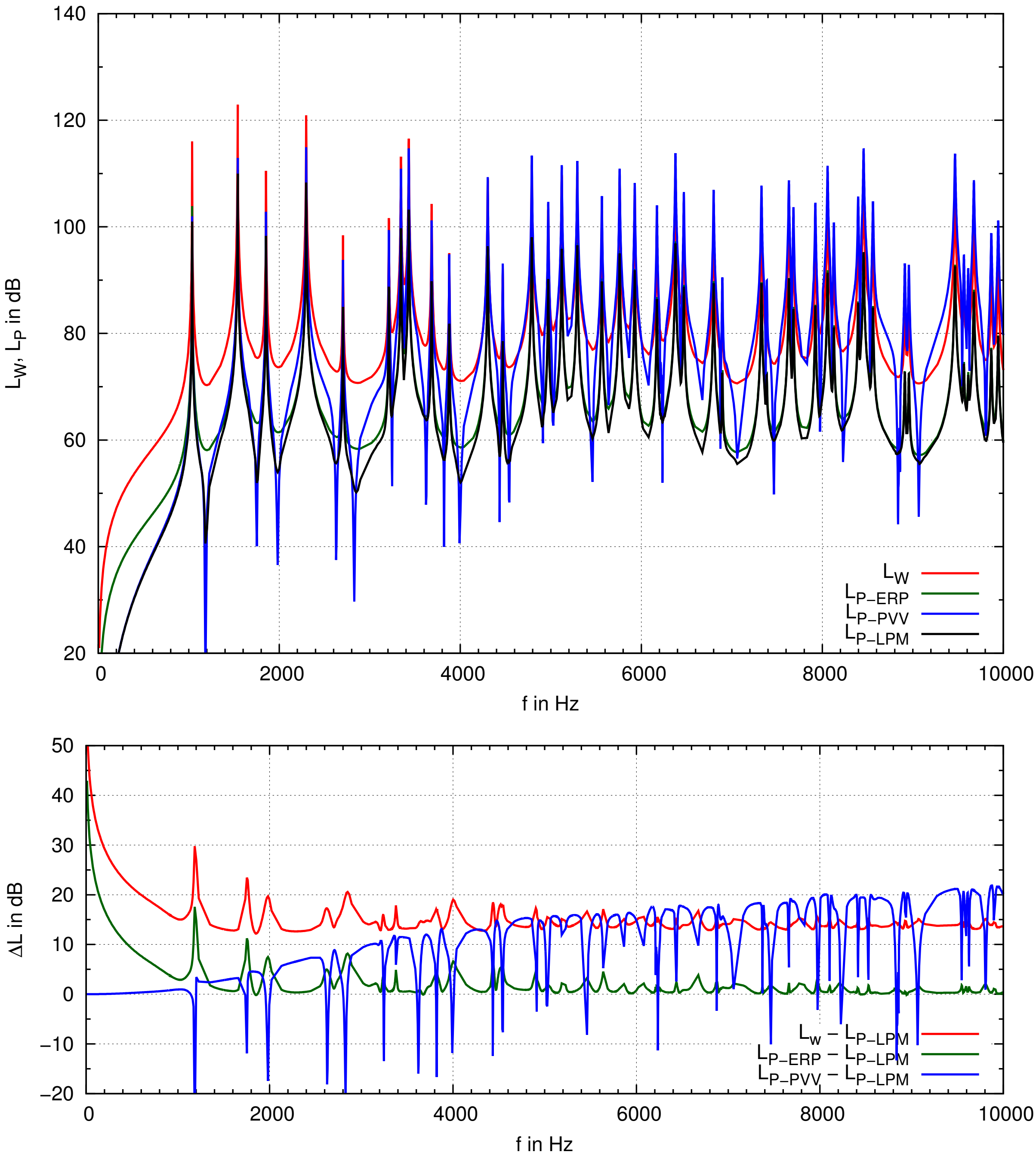

Figure 4 – Oil pan results: kinetic energy vs. sound power estimates - levels and absolute error values

5. SUMMARY AND OUTLOOK

In summary, kinetic energy and equivalent radiated sound power levels show an almost similar behaviour in the determined frequency range. The modal characteristic for lower frequencies and the decreasing dynamic of the singular peaks at higher frequencies are represented equally. The energy level strongly overestimates the sound power within the whole bandwidth but remains with an almost constant difference of 13 dB. As expected, ERP is exact in the high frequencies.

The lumped parameter model as well as the volume velocity give more accurate results with similar or higher computational costs. Especially the increasing error of PVV at high frequencies has to be taken into account for choosing the correct sound radition measure.

Though, the kinetic energy enables the possibility of an adequate qualitative acoustic characterisation showing all important dependencies but significantly deviating in the level. In addition, there are no additional post-processing efforts as it can be taken straight from the structutral dynamic simulation.

Comparing the sound power estimates, the LPM gives most accurate results and thus is the best choice for quantitative acoustic chracterisations without solving the coupled structural-acoustic system. Due to the long reading time for the velocity fields and fast matrix operations, there is no need for faster sum algorithms of ERP and PVV.

Further studies investigate the acoustic elements, the influence of the wall thickness and implement anisotropic damping models for steady state dynamic analysis. Moreover, a vibro-acoustic optimiastion for composite layups will benefit from the methodology developed here.

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