Free vibrations of a box-type structure by plates with arbitrary boundary conditions

Kaipeng ZHANG[1]; Tao ZHANG [2]; Han WU[3] Dongyan SHI[4]

[1] College of Mechanical and Electrical Engineering, Harbin Engineering University

No. 145 Nantong Street, Harbin, Heilongjiang, 150001, P. R. China

[2] College of Mechanical and Electrical Engineering, Harbin Engineering University

No. 145 Nantong Street, Harbin, Heilongjiang, 150001, P. R. China

[3] AVIC International Trade and Economic Development Ltd, Xi'an Branch, Xi'an, 710075, PR China

[3] College of Mechanical and Electrical Engineering, Harbin Engineering University

No. 145 Nantong Street, Harbin, Heilongjiang, 150001, P. R. China

ABSTRACT

In this paper, a modified Fourier solution is developed for the free vibrations of a box-type structure as four elastically coupled rectangular plates with arbitrary boundary conditions. The modified Fourier solution for the problem is obtained using improved Fourier series method, in which both three displacements of the rectangular plates are represented by a new form of trigonometric series functions with a drastically improved convergence as compared with the conventional Fourier series. It is shown that the general coupling and boundary conditions are accounted for using the artificial spring technique and can be easily obtained by assigning the springs with corresponding values. All the unknown series expansion coefficients are treated as the generalized coordinates and solved using the Rayleigh-Ritz technique. The efficiency, accuracy and reliability of the proposed approach are demonstrated by comparison with the Finite Element Method (FEM) results. In addition, the current approach offers an easy analysis operation for the entire model parameters and the change of any parameter from one case to another is as easy as changing structure parameters without need of making any change to the solution procedure, thus this will make a parametric study and further mechanism analysis easier compared to most existing procedures.

Keywords: box-type structure, vibration, improved Fourier series method, arbitrary boundary conditions

I-INCE Classification of Subjects Number(s): 42

1. INTRODUCTION

Cylindrical shells are usually used as the simplified models to investigate the structure-borne sound of some practical engineering structures, such as the fuselages of aircraft, the hull of submarine, and so on. The reduction and control of the structure-borne sound is achieved by adopting the passive methods in the mid and high frequency range and the active structural acoustic control technique in the low frequency range in the practical applications. Quite a few of experimental and analytical studies have been conducted recently on the basis of cylindrical shell model, but few reports are concerned with another familiar engineering structure, the built-up plate structure, which is also widely used as the ship hull, piping systems and the cabin of vehicles, etc., with the advantages of having the light weight while retaining enough curved and wrest resistant strength. In practice, the FEM is a convenient and effective choice to investigate the free and forced vibration characteristics for some complex configurations including the box-type structure. A great effort was made by Lin [2]

1 zhangkaipeng@hrbeu.edu.cn
2 zhangtao@hrbeu.edu.cn
3 hannahwu613@gmail.com
4 shidongyan@hrbeu.edu.cn

to illustrate the vibrational features of a box-type structure using the finite element method. The infinite number of modes was classified into six groups due to their symmetry and each group of modes was investigated in detail. The sound characteristics, such as the sound radiation directivity and patterns of the box structure were also discussed [3]. Although the FEM is an effective means to make the dynamic analysis of a complex structure in the low frequency range, it is not more favorable and convenient than the analytical method to perform a further investigation of parameterization and mechanisms. For example, when the built-up box structure is appropriately modeled as a four side-panels configuration with adjacent plates coupled at right angle, the vibration analysis of this structure can be classified as the problem of coupled plates. This sort of problem can be analytically investigated for some classical boundary conditions. A typical work was presented by Pan and Farag [1] decade ago. They developed a rigidly coupled model consisting of two rectangular plates coupled at arbitrary angle to illustrate the dynamic response and power flow characteristics assuming that the coupled model had all other edges simply supported despite the coupling edge. In their calculation, the in-plane vibration was involved at the rigidly connected edge to satisfy the force and displacement equilibriums, the coupling angle and the contribution of components of flexural force which have an effect on power flow were also considered. They found that the moment played an important role in coupling below the cut-off frequency of the first in-plane mode.

In this paper, a built-up box structure is modeled as four rectangular plates elastically connected at right angles under general boundary conditions. A great amount of research work has been reported in the literature to calculate the eigenpairs of rectangular plate structure under typical boundary conditions [4-7] and then the dynamic response and energy flow characteristics of beam/plate structure has been widely investigated employing the power flow technique [9-16]. Although the widely accepted fact is that the analytical solution of a rectangular plate only exists for some classical boundary conditions, the problem of generally restrained plate attracts many researchers' attention [8,17-21]. An improved double Fourier series method was proposed by Li et al. [21] to analytically investigate the flexural vibration of a rectangular plate. By adding several auxiliary functions into the displacement expressions, the potential discontinuity through the whole plate area is overcome effectively for the arbitrary boundary restraint problem of a rectangular plate. Subsequently, this technique was extended into the in-plane eigenpairs calculation of a rectangular plate under artificially restrained edges [22,23] and a combination of these two displacement expressions was employed to elucidate the free vibration characteristics of two elastically coupled rectangular plates by Du et al. [24].

In this work, an analytical built-up box-type structure model is presented by treating and modeling the structure as four rectangular plates elastically connected at right angles under general boundary conditions. The exact double Fourier series solutions considering both the flexural and in-plane vibrations are derived by using the Rayleigh-Ritz approach. The change of the boundary conditions can be easily achieved by only varying the stiffness of the three sets of the boundary springs at the all boundaries of the box-type structure without the need of making any change to the solutions. The excellent accuracy of the current result is validated by comparison with those obtained from other analytical approach as well as the Finite Element Method (FEM). Numerical results are presented to illustrate the current method is not only applied to the classical homogeneous boundary conditions but also other interesting and practically important boundary restraints on free vibrations of the box-type structure with varying stiffness of boundary springs.

2. THEORETICAL FORMULATIONS

A generally coupled plate model is schematically presented in Figure. 1, together with the global and local coordinates employed in this paper. This model indicates ample information including general boundary condition, right angles and elastically coupled condition. Each edge of the four component panels is restrained by three groups of translational springs distributed uniformly along the *x*, *y*, *z* axes separately and one group of rotational springs around the *y*-axis to simulate the given or typical boundary conditions expressed in the form of transverse shear force, in-plane shear force, in-plane longitudinal force, and the flexural moment, respectively. These linear springs are assigned very high stiffness values (in calculations artificial stiffness values of 10^{14}) for a clamped edge and zero for a free boundary condition. Four additional groups of springs resembling the boundary springs are uniformly arranged at the coupling edge to artificially connect the two adjacent panels by regulating the values of coupling stiffness.

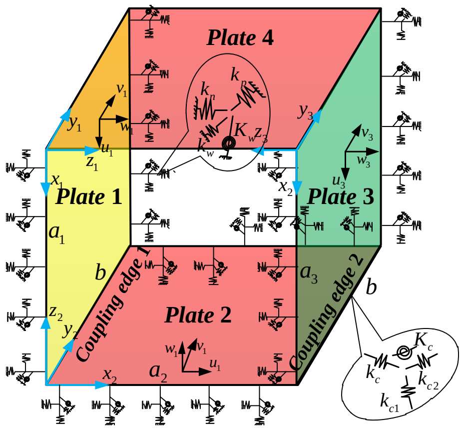


Figure 1 –A box-type built-up structure.

In this paper, the double Fourier method is adopted and served to illustrate the free vibration features of a box-type structure. The result can be considered as the exact solution by solving the Rayleigh-Ritz equation which can be deduced from the combination of governing equations, boundary conditions for the whole system and continuity conditions at the coupling edges.

The displacement presentation for the out-of-plane transverse vibration of plate i can be expressed as the following form according to the local coordinates:

$$w_i(x,y)=\sum_{m=-4}^{\infty}\sum_{n=-4}^{\infty}A_{i,mn}\varphi_{a_i m}(x)\varphi_{b_i n}(y) \tag{1}$$

The in-plane displacements in x- and y- directions are separately described as

$$u_i(x,y)=\sum_{m=-2}^{\infty}\sum_{n=-2}^{\infty}B_{i,mn}\varphi_{a_i m}(x)\varphi_{b_i n}(y) \tag{2}$$

$$v_i(x,y)=\sum_{m=-2}^{\infty}\sum_{n=-2}^{\infty}C_{i,mn}\varphi_{a_i m}(x)\varphi_{b_i n}(y) \tag{3}$$

where $A_{i,mn}$, $B_{i,mn}$, $C_{i,mn}$ denotes the series expansion coefficients, and

$$\varphi_{a_i m}(x)=\begin{cases}\cos\lambda_{a_i m}x & m\ge 0\\ \sin\lambda_{a_i m}x & m<0\end{cases}\qquad \lambda_{a_i m}=m\pi/a_i \tag{4}$$

$$\varphi_{b_i n}(x)=\begin{cases}\cos\lambda_{b_i n}x & n\ge 0\\ \sin\lambda_{b_i n}x & n<0\end{cases}\qquad \lambda_{b_i n}=n\pi/b_i \tag{5}$$

Mathematically, the series in the form of equations (1)-(3) are able to expand and uniformly converge to any function. So, the admissible functions expressed in a new form of trigonometric series expansions are introduced to remove the potential discontinuities with the functions and their derivatives.

Four groups of springs which are associated with the transverse moment, out-of-plane shear force, in-plane longitudinal force, and in-plane shear force are introduced to illustrate the boundary restraints and the coupling interactions. The coupling effect of the plates is described by these internal forces which can be artificially regulated by changing the stiffness of corresponding springs. For a rigid connection, the stiffness of all the springs can be assigned an infinite value (in calculation the value 10^{14} was used). Thus, attached potential energy stored in the boundary and coupling springs should be considered. With the assumption of thin plate theory, the total potential energy and kinetic energy of the model, marked as V and T separately, can be concisely written as

$$V=\sum_{i=1}^{4}(V_{dt}^i+V_{dj}^i+V_{bt}^i+V_{bj}^i)+V_c \tag{5}$$

$$T=\sum_{i=1}^{4}(T_{dw}^i+T_{dj}^i) \tag{6}$$

In detail, V_{dt}^i and V_{dj}^i mean the deformational energy related to the transverse and in-plane vibrations of plate i, respectively. V_{bt}^i and V_{bj}^i are the additional potential energy associated with the transverse and in-plane boundary conditions of plate i, too.

T_{dw}^i and T_{dj}^i separately indicate the kinetic energy for the transverse and in-plane vibration of plate j. The specific expressions of terms in Eqs. (5) and (6) are as follows:

$$V_{dt}^{i}=\frac{D_i}{2}\int_0^{a_i}\int_0^{b_i}\left[\frac{\partial^2 w_i}{\partial x_i^2}+\frac{\partial^2 w_i}{\partial y_i^2}+2\mu_i\frac{\partial^2 w_i}{\partial x_i^2}\frac{\partial^2 w_i}{\partial y_i^2}+2\left(1-\mu_i\right)\left(\frac{\partial^2 w_i}{\partial_i^2\partial_i^2}\right)^2\right]dx_i dy_i \tag{7}$$

$$\begin{aligned}V_{bt}^{i}==&\frac{1}{2}\int_0^{b_i}\left[k_{wi0}w_i^2+K_{wi0}\left(\frac{\partial w_i}{\partial x_i}\right)^2\right]_{x_i=0}dy_i+\frac{1}{2}\int_0^{b_i}\left[k_{wi1}w_i^2+K_{wi1}\left(\frac{\partial w_i}{\partial x_i}\right)^2\right]_{x_i=a_i}dy_i\\&+\frac{1}{2}\int_0^{a_i}\left[k_{wi0}w_i^2+K_{wi0}\left(\frac{\partial w_i}{\partial x_i}\right)^2\right]_{y_i=0}dx_i+\frac{1}{2}\int_0^{a_i}\left[k_{wi1}w_i^2+K_{wi1}\left(\frac{\partial w_i}{\partial x_i}\right)^2\right]_{y_i=b_i}dx_i\end{aligned} \tag{8}$$

$$V_{dj}^{i}=\frac{G_i}{2}\int_0^{a_i}\int_0^{b_i}\left[\left(\frac{\partial u_i}{\partial x_i^2}+\frac{\partial v_i}{\partial y_i^2}\right)^2-2\left(1-\mu_i\right)\frac{\partial u_i}{\partial x_i^2}\frac{\partial v_i}{\partial y_i^2}+\frac{\left(1-\mu_i\right)}{2}\left(\frac{\partial u_i}{\partial y_i^2}+\frac{\partial v_i}{\partial x_i^2}\right)^2\right]dx_i dy_i \tag{9}$$

$$\begin{aligned}V_{bj}^{i}=&\frac{1}{2}\int_0^{b_i}\left[k_{nxi0}u_i^2+k_{pxi0}v_i^2\right]_{x_i=0}dy_i+\frac{1}{2}\int_0^{b_i}\left[k_{nxi1}u_i^2+k_{pxi1}v_i^2\right]_{x_i=a_i}dy_i\\&+\frac{1}{2}\int_0^{a_i}\left[k_{nxi0}v_i^2+k_{pxi0}u_i^2\right]_{y_i=0}dx_i+\frac{1}{2}\int_0^{a_i}\left[k_{nxi1}v_i^2+k_{pxi1}u_i^2\right]_{y_i=b_i}dx_i\end{aligned} \tag{10}$$

$$T_{dw}^{i}=\frac{1}{2}\int_0^{a_i}\int_0^{b_i}\rho_i h_i\left(\frac{\partial w_i}{\partial t}\right)^2 dx_i dy_i=\frac{1}{2}\int_0^{a_i}\int_0^{b_i}\rho_i h_i\omega^2 w_i^2 dx_i dy_i \tag{11}$$

$$T_{dj}^{i}=\frac{1}{2}\int_0^{a_i}\int_0^{b_i}\rho_i h_i\left(\frac{\partial u_i}{\partial t}+\frac{\partial v_i}{\partial t}\right)^2 dx_i dy_i=\frac{1}{2}\int_0^{a_i}\int_0^{b_i}\rho_i h_i\omega^2(u_i^2+v_i^2)dx_i dy_i \tag{12}$$

Here, ω is the angular frequency, D, G, m, ρ and h denote, respectively, the flexural rigidity, extensional rigidity, Poisson's ratio, mass density and the thickness of plates. The symbols (k_{wi0}, K_{wi0}) separately mean the linear and the torsional boundary springs for transverse vibration of plate i at the edge x_i=0 while symbols (k_{nxi0}, k_{pxi0}) separately denote the normal and the parallel linear springs for the in-plane vibration at the edge x_i=0. The other boundary springs given above are similarly defined. In this paper, the uniform material parameters for all the plates are considered. The surface integral expressions in Eqs. (7) and (9) are the deformation energy stored in the plate and the contour integral terms in Eqs. (8) and (10) are the elastic energy stored in the boundary springs. The energy of the coupling springs between each two adjacent plates consists of four parts and can be written as Eq. (13). Taking plates 1 and 2 as examples, the coupling energy is expressed in Eq. (14), and the other terms in Eq. (14) will have the similar representations as well as Eq. (14)

$$V_c=V_{c\_12}+V_{c\_23}+V_{c\_34}+V_{c\_14} \tag{13}$$

$$\begin{aligned}V_{c\_12}=&\frac{1}{2}\int_0^{b_1}\left[K_{c\_12}\left(\partial w_1/\partial x_1\big|_{x_1=0}-\partial w_2/\partial x_2\big|_{x_2=a_2}\right)^2+k_{cw\_12}\left(w_1\big|_{x_1=0}+u_2\big|_{x_2=a_2}\right)^2\right.\\&\left.+k_{cu\_12}\left(u_1\big|_{x_1=0}-w_2\big|_{x_2=a_2}\right)^2+k_{cv\_12}\left(v_1\big|_{x_1=0}-v_2\big|_{x_2=a_2}\right)^2\right]dy_1\end{aligned} \tag{14}$$

where K_{c_12}, k_{cw_12}, k_{cu_12} and k_{cv_12} are the stiffness values of those four coupling springs connecting plates 1 and 2. The other coupling springs are similarly defined. With ϑ as the unknown coefficients and L as the Lagrangian, the Lagrange equation for the whole system can be obtained as

$$\frac{\partial L}{\partial \vartheta}=0 \tag{15}$$

Make a differential calculation in Eq. (15), a group of linear equations can be derived in the matrix form:

$$\left(\mathbf{K}-\omega^2\mathbf{M}\right)\mathbf{E}=\mathbf{0} \tag{16}$$

The unknown coefficients in the displacement expressions can be expressed in the vector form as **E**.

$$\mathbf{E}=\begin{Bmatrix} A^1_{-2,-2} & A^1_{-2,-1} & \cdots & A^1_{m',0} & A^1_{m',1} & \cdots & A^1_{m',n'} & \cdots & A^1_{M,N} \\ B^1_{-2,-2} & B^1_{-2,-1} & \cdots & B^1_{m',0} & B^1_{m',1} & \cdots & B^1_{m',n'} & \cdots & B^1_{M,N} \\ C^1_{-2,-2} & C^1_{-2,-1} & \cdots & C^1_{m',0} & C^1_{m',1} & \cdots & C^1_{m',n'} & \cdots & C^1_{M,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ A^4_{-2,-2} & A^4_{-2,-1} & \cdots & A^4_{m',0} & A^4_{m',1} & \cdots & A^4_{m',n'} & \cdots & A^4_{M,N} \\ B^4_{-2,-2} & B^4_{-2,-1} & \cdots & B^4_{m',0} & B^4_{m',1} & \cdots & B^4_{m',n'} & \cdots & B^4_{M,N} \\ C^4_{-2,-2} & C^4_{-2,-1} & \cdots & C^4_{m',0} & C^4_{m',1} & \cdots & C^4_{m',n'} & \cdots & C^4_{M,N} \end{Bmatrix}^{\mathrm{T}} \quad (17)$$

In Eq.(16), the **K** is the stiffness matrix for the plate, and the **M** is the mass matrix. They can be expressed separately as:

$$\mathbf{K}=\begin{bmatrix} \mathbf{K}_{1-1} & \mathbf{K}_{1-2} & \mathbf{K}_{1-3} & \mathbf{K}_{1-4} & \cdots & \mathbf{K}_{1-12} \\ \mathbf{K}_{2-1} & \mathbf{K}_{2-2} & \mathbf{K}_{2-3} & \mathbf{K}_{2-4} & \cdots & \mathbf{K}_{2-12} \\ \mathbf{K}_{3-1} & \mathbf{K}_{3-2} & \mathbf{K}_{3-3} & \mathbf{K}_{3-4} & \cdots & \mathbf{K}_{3-12} \\ \mathbf{K}_{4-1} & \mathbf{K}_{4-2} & \mathbf{K}_{4-3} & \mathbf{K}_{4-4} & \cdots & \mathbf{K}_{4-12} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{K}_{12-1} & \mathbf{K}_{12-2} & \mathbf{K}_{12-3} & \mathbf{K}_{12-4} & \cdots & \mathbf{K}_{12-12} \end{bmatrix} \quad \mathbf{M}=\begin{bmatrix} \mathbf{M}_{1-1} & \mathbf{M}_{1-2} & \mathbf{M}_{1-3} & \mathbf{M}_{1-4} & \cdots & \mathbf{M}_{1-12} \\ \mathbf{M}_{2-1} & \mathbf{M}_{2-2} & \mathbf{M}_{2-3} & \mathbf{M}_{2-4} & \cdots & \mathbf{M}_{2-12} \\ \mathbf{M}_{3-1} & \mathbf{M}_{3-2} & \mathbf{M}_{3-3} & \mathbf{M}_{3-4} & \cdots & \mathbf{M}_{3-12} \\ \mathbf{M}_{4-1} & \mathbf{M}_{4-2} & \mathbf{M}_{4-3} & \mathbf{M}_{4-4} & \cdots & \mathbf{M}_{4-12} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{M}_{12-1} & \mathbf{M}_{12-2} & \mathbf{M}_{12-3} & \mathbf{M}_{12-4} & \cdots & \mathbf{M}_{12-12} \end{bmatrix} \quad (18)$$

Obviously, the natural frequencies and eigenvectors can now be readily obtained by solving a standard matrix eigenproblem. Since the components of each eigenvector are actually the expansion coefficients of the modified Fourier series, the corresponding mode shape can be directly determined from Eq. (1).

3. NUMERICAL EXAMPLES AND DISCUSSION

In this section, a systematic comparison between the current solutions and theoretical results published by other researchers or finite element method (FEM) results is carried out to validate the excellent accuracy, reliability and feasibility of the modified Fourier method. This model is composed of four rectangular plates rigidly coupled along the *y*-direction with the same thickness h=2.5 mm, occupying the space (0.76 ×0.6×0.6 m^3). These four panels are made of aluminum and own identical material properties: mass density ρ=7800 kg/m^3, Young's modulus E=200 GPa and Poisson ratio μ=0.3. The comparison is conducted in the frequency range up to 500Hz. Each edge of the FEM model is divided into 100 linear elements, thus the FEM results are accurate enough to be as a reference. As shown in Figure.2, we will give four boundary conditions to demonstrate the current method, S, C, F and E represent the simply-support, clamped edge, free and elastic boundary condition, the boundary set spring values shown in Table 1. By truncating the series of both transverse and in-plane displacements to 12, the difference between the current calculations and the FEM results throughout the whole range is acceptable, with the maximum value of about 5 Hz.

The Figures 3-6 shows the results of natural frequencies comparison the present method and the FEM with different boundary conditions (Figure 3 analysis of boundary conditions (a), Figure 4 analysis of boundary conditions (b), Figure 5 analysis of boundary conditions (c), and Figure 6 analysis of boundary conditions (a)). In addition, in order to validate the method, the first six mode shape calculated by the present method and FEM-based ABAQUS are shown in the Figure 7 and Figure 8. Through the Figs. 3-6 and Figs. 7-8, we could know the present method not only solved the classical boundary, but also employed the method to study the elastic boundary condition.

Table 1 – Values of spring for four type boundary conditions.

Boundary condition	Spring stiffness			
	k_w	K_w	k_n	k_p
Free edge (F)	0	0	0	0
Clamped edge (C)	1×10^{14}	1×10^{14}	1×10^{14}	1×10^{14}
Simply-supported (S)	1×10^{14}	0	1×10^{14}	1×10^{14}
Elastic-supported (E)	5×10^{8}	5×10^{8}	5×10^{8}	5×10^{8}

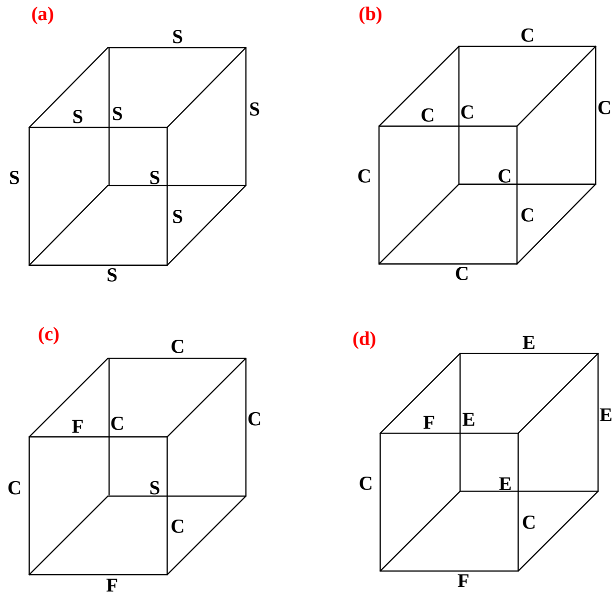


Figure 2 –The four type boundary conditions in this section.

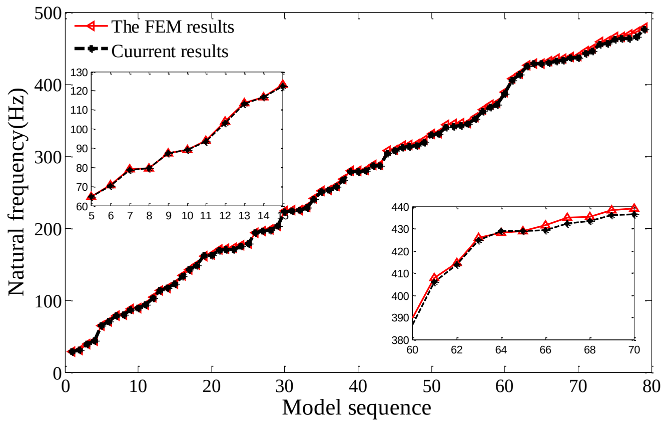


Figure 3 –Comparison of natural frequencies between the present method and the FEM with the boundary condition (a)

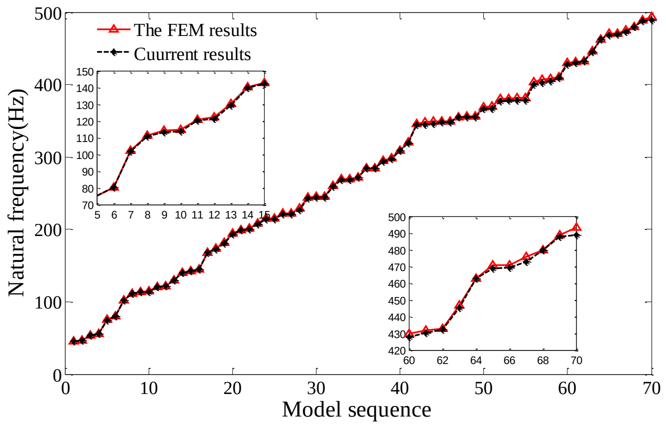


Figure 4 – Comparison of natural frequencies between the present method and the FEM with the boundary condition (b)

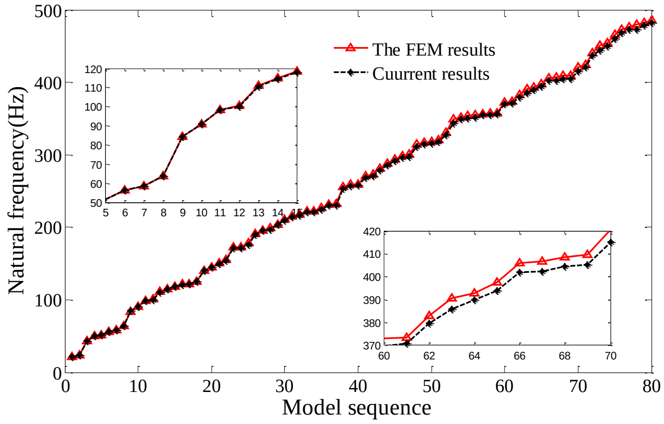


Figure 5 – Comparison of natural frequencies between the present method and the FEM with the boundary condition (c)

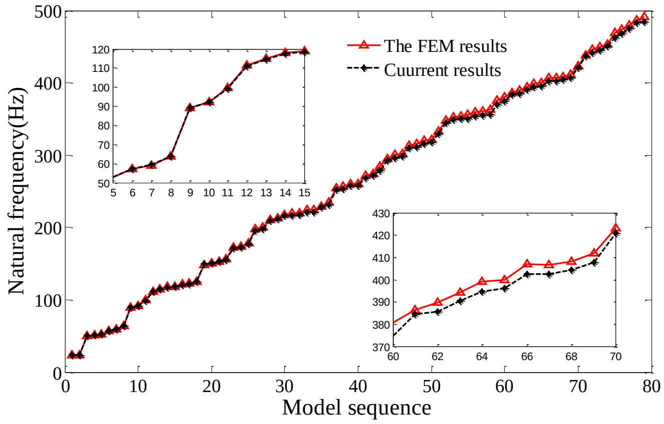
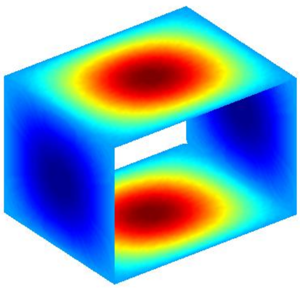
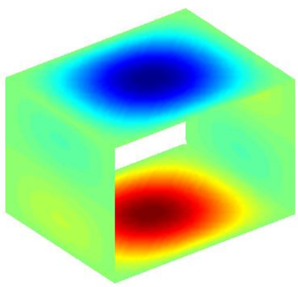


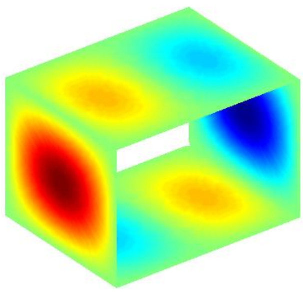
Figure 6 – Comparison of natural frequencies between the present method and the FEM with the boundary condition (d)



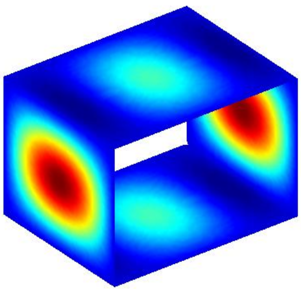
(a) 1st mode shape



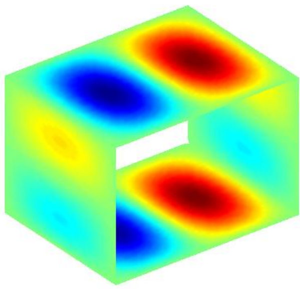
(b) 2nd mode shape



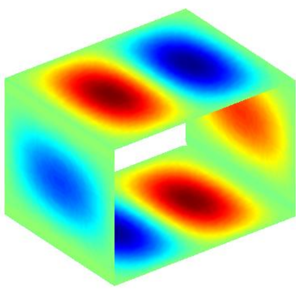
(c) 3rd mode shape



(d) 4th mode shape

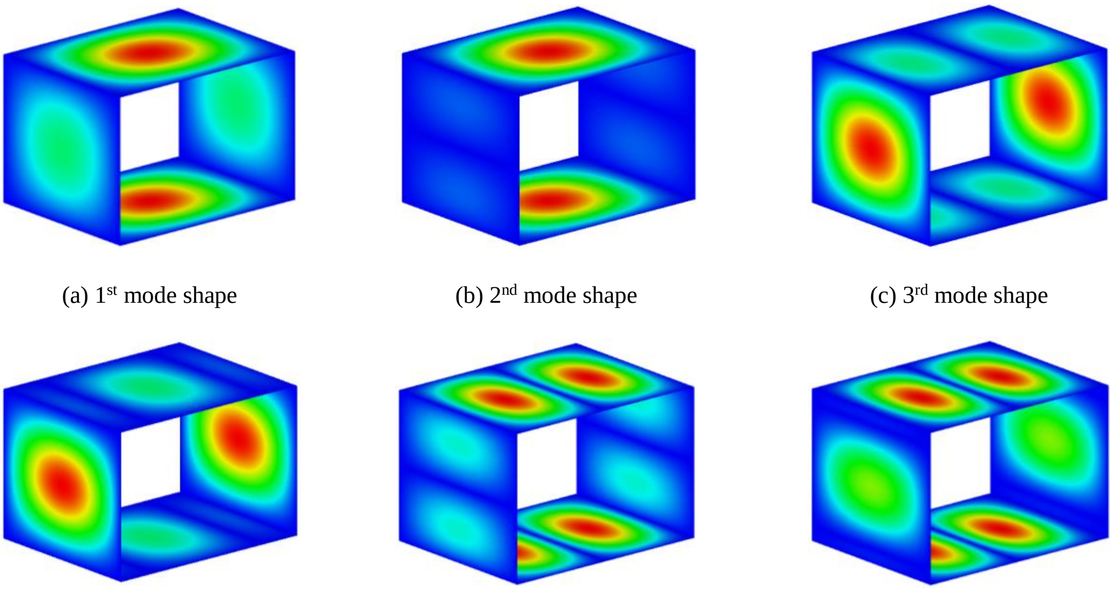


(e) 5th mode shape



(f) 6th mode shape

Figure 7 – The first six mode shapes calculated by present approach with the boundary condition (b)



(a) 1st mode shape (b) 2nd mode shape (c) 3rd mode shape

(d) 4th mode shape (e) 5th mode shape (f) 6th mode shape

Figure 8 – The first six mode shapes calculated by ABAQUS with the boundary condition (b)

4. CONCLUSIONS

In this paper, a modified Fourier solution is developed for the free vibrations of a box-type structure as four elastically coupled rectangular plates with arbitrary boundary conditions. The out and in-plane displacements components of the plate, regardless of boundary conditions, are invariantly expressed as a new form of trigonometric series expansions with a drastically improved convergence as compared with the conventional Fourier series to ensure and accelerate the convergence of the solution. It is shown that the general coupling and boundary conditions are accounted for using the artificial spring technique and can be easily obtained by assigning the springs with corresponding values. Instead of seeking a solution in strong forms in the previous studies, all the Fourier coefficients will be treated equally and independently as the generalized coordinates and solved directly from the Rayleigh–Ritz technique. The change of the boundary conditions can be easily achieved by only varying the stiffness of the four sets of boundary springs along all edges of the box-type structure without involving any change to the solution procedure. The efficiency, accuracy and reliability of the proposed approach are demonstrated by comparison with the Finite Element Method (FEM) results. In addition, the current approach offers an easy analysis operation for the entire model parameters and the change of any parameter from one case to another is as easy as changing structure parameters without need of making any change to the solution procedure, thus this will make a parametric study and further mechanism analysis easier compared to most existing procedures.

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