Coupled analysis of two-dimensional acoustic and membrane vibration by concentrated mass model

Satoshi ISHIKAWA[1]; Shinya KIJIMOTO[2]; Ryoma OWAKI[3]; Ataru MATSUO[4]

[1, 2, 4] Kyushu University, Japan

[3] Mitsubishi Electric Corporation, Japan

ABSTRACT

In the finite element method of a structural-acoustic coupled analysis, the mass matrix and the stiffness matrix are not symmetrical. Therefore, the modal analysis cannot be applied directly to the coupled problem. In our previous study, a concentrated mass model was proposed to analyze a two-dimensional acoustic analysis. The model consists of the masses, the connecting springs. It is very easy to couple the structure and acoustic field by the concentrate mass model. Furthermore, the mass matrix and the stiffness matrix are symmetrical. In this paper, we propose a concentrated mass model to perform a coupled analysis of a two-dimensional acoustic and a membrane vibration. And we propose a coupling method to arrange of the masses of the air near the membrane. To confirm the validity of the proposed model, the natural frequency obtained by the concentrated mass model is compared with the natural frequency by the modal coupling method. These results are in good agreement. Therefore, it is concluded that the proposed model is valid for the coupled analysis of an acoustic and a vibration analysis.

Keywords: Simulation, Modeling Method I-INCE Classification of Subjects Number(s): 76.9

1. INTRODUCTION

To reduce interior noise in transportation vehicles, structural-acoustic coupled analysis has performed by the finite element method (1)-(3). However, it is difficult to couple a structure vibration analysis and an acoustic analysis because the displacement is used at the nodal point as the variable in the structure field, but the sound pressure or velocity potential is used as the variable in the acoustic field. In the finite element model of the structural-acoustic coupled problem, the equations of motion were given by Everstine et. al. (4), and Joseph (5) solved the equations by the modal coupling using the modal coordinate in the case of rigid wall and in vacuo. However, in the equations of motion, the mass matrix and the stiffness matrix are not symmetrical. Therefore, the modal analysis cannot be applied to the coupled problem because the orthogonality is not satisfied (6). To overcome this problem, MacNeal et. al. (7) symmetrized the matrix by the coordinate transformation. However, it is complicated to handle the method, and the coordinate transformation loses physical meanings.

In our previous study, a concentrated mass model was proposed to analyze a two-dimensional acoustic analysis (8). The model consists of the masses, the connecting springs, and connecting dampers. The mass point is placed at the nodal point, and the variable of this model is the displacement of the mass point. In the case of the structural-acoustic coupled problem, the treatment of the coupling is only to place the mass point of the structure and the mass point of the acoustic field at the nodal points in each area. Therefore, it is very simple to derive the equations of motion of the mass points. Furthermore, the mass matrix and the stiffness matrix in the equations are symmetrical. Therefore, we can use the modal analysis easily by the concentrated mass model. And, the computational cost of the eigenvalue analysis becomes lower because of the symmetrical matrix. Another advantage of this model is to carry out easily a nonlinear acoustic analysis in the case of

[1] ishikawa@mech.kyushu-u.ac.jp
[2] kiji@mech.kyushu-u.ac.jp
[3] Owaki.Ryoma@dh.MitsubishiElectric.co.jp
[4] a.matsuo@sky.mech.kyushu-u.ac.jp

large amplitude sound.

In this paper, we propose a concentrated mass model to perform a coupled analysis of a two-dimensional acoustic and a membrane vibration. The sound space and the membrane are modeled as masses, and connecting springs. And we propose a coupling method between the acoustic analysis and the vibration analysis to arrange of the masses of the air near the membrane. To confirm the validity of the proposed model, the natural frequency obtained by the concentrated mass model is compared with the natural frequency by the modal coupling method.

2. CONCENTRATED MASS MODEL

We deal with a coupled problem of a two-dimensional acoustic in a rectangle-plate-shaped space and a one-dimensional membrane vibration, as shown in Fig. 1. One of the faces of the rectangle-plate-shaped space is the membrane and the other faces are rigid. The thickness of the acoustic space is *h.* The air in the acoustic space and the membrane are modeled as the concentrated mass model which consists of masses, connecting springs and connecting damper, as shown in Figs. 2 and 5.

2.1 Model of Acoustic Space (8)

The air in the acoustic space is modeled as the concentrated mass model which consists of masses, connecting springs as shown in Fig. 2. The space is divided into uniform rectangular elements such that the length is l_x, l_y, as shown in Fig. 3. The mass is concentrated on the intersection point (nodal point). The displacement of the nodal point (i, j) in x direction is $x_{i,j}$, the displacement in y direction is $y_{i,j}$, the sound pressure in element $[i, j]$ is $dp_{i,j}$. The pressure in equilibrium state is p_0, the density in equilibrium state is ρ_0, the volume in each element in equilibrium state is $V_0 = hl_x l_y$. Considering the mass of air in the shaded area in Fig. 3, the mass of each mass point, m, is written as follow

$$m = \rho_0 h l_x l_y \tag{1}$$

$dV_{i,j}$ is the variation of volume in element $[i, j]$ when the mass points move, as shown in Fig. 4. Using the displacement of each mass point, $dV_{i,j}$ is given by

$$dV_{i,j} = \frac{h}{2}\left\{l_y\left(x_{i,j} + x_{i,j-1} - x_{i-1,j} - x_{i-1,j-1}\right) + l_x\left(y_{i,j} + y_{i-1,j} - y_{i,j-1} - y_{i-1,j-1}\right)\right\} \tag{2}$$

Considering an adiabatic change, the sound pressure, $dp_{i,j}$, becomes

$$dp_{i,j} = -\frac{p_0 \gamma\, dV_{i,j}}{V_0} \tag{3}$$

Considering the force acting on the shaded area in Fig. 3, the x direction force acting on the mass point (i, j) from the sound pressure, $f_{i,j}^{kx}$, and the y direction force, $f_{i,j}^{ky}$, are written as follows

$$\left.\begin{aligned} f_{i,j}^{kx} &= \frac{hl_y}{2}\left(dp_{i,j} + dp_{i,j+1} - dp_{i+1,j} - dp_{i+1,j+1}\right) \\ f_{i,j}^{ky} &= \frac{hl_x}{2}\left(dp_{i,j} + dp_{i+1,j} - dp_{i,j+1} - dp_{i+1,j+1}\right) \end{aligned}\right\} \tag{4}$$

In addition to this spring force, the base support damper from the shear stress from the upper and lower walls and the connecting damper from the shear stress in x, y direction and from the normal stress were derived (3). However, these dampers are not considered in this paper.

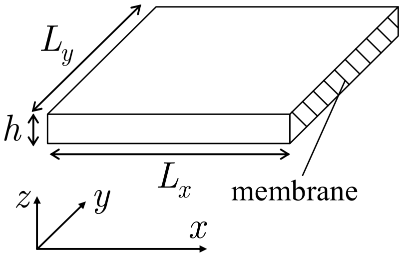


Figure 1 – Analytical space

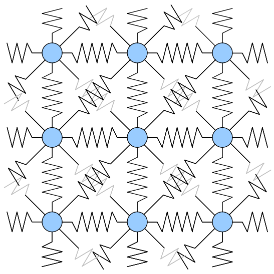


Figure 2 – Concentrated mass model

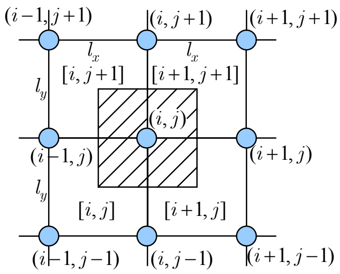


Figure 3 – Square lattice

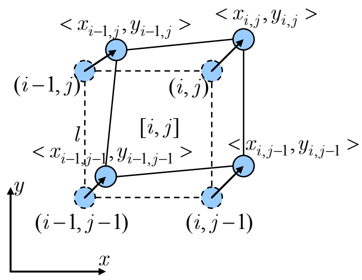


Figure 4 – Volume variation in element

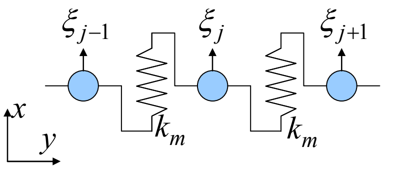


Figure 5 – Model of membrane

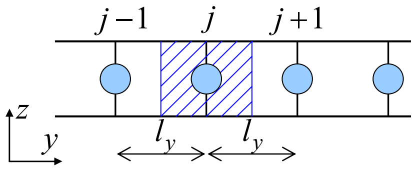


Figure 6 – Element of membrane

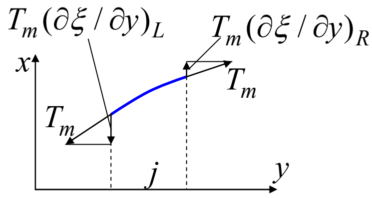


Figure 7 – Square lattice

2.2 Model of Membrane

In this section, the membrane face in Fig. 1 is model as the concentrated mass model which consists of masses, connecting springs as shown in Fig. 5. We consider a one-dimensional membrane. The space is divided into uniform elements such that the length is l_y, as shown in Fig. 6. The division number is N_y. The mass is concentrated on the nodal point. The displacement of the nodal point *j* in *x* direction is ξ_j, the tension of the membrane is T_m, the surface density is ρ_m.

Considering the mass of the membrane in the shaded area in Fig. 6, the mass of each mass point, m_m, is written as follow

$$m = \rho_m h l_y \tag{5}$$

The restoring force of the membrane is model as the spring. The forces in Fig. 7 act on the shaded area in Fig. 6. The displacement of the membrane in *x* direction is $\xi(y)$. The *x* direction component of the tension from the left side is $T_m = (\partial\xi / \partial y)_L$, so the restoring force from the left side becomes

$$f_m = T_m h \left(\frac{\partial \xi}{\partial y}\right)_L \tag{6}$$

If we relate ξ to the displacement of the mass point ξ_j, and transform Eq. (6) to the finite difference form, we get

$$f_m = T_m h \frac{\xi_j - \xi_{j-1}}{l_y} \tag{7}$$

From this equation, the constant of spring in Fig. 5 is

$$k_m = \frac{T_m h}{l_y} \tag{8}$$

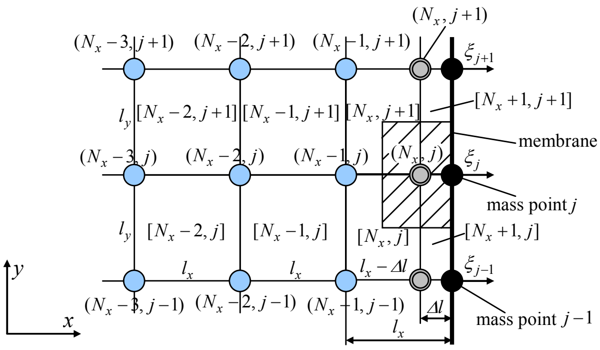


Figure 8 – Placement of mass point near membrane

2.3 Treatment of Coupled Region

In this section, we explain the treatment of the coupled region of the acoustic space and the membrane. Figure 8 shows the mass arrangement in *x*-*y* plane in the vicinity of the membrane. The heavy line at the right end indicates the membrane, and the acoustic space on the left side of the membrane is divided into uniform elements such that the length is l_x in x direction. The division number is N_x. The right end elements [N_x, *] are divided, and we put the new elements [N_x+1, *] such that the length is Δl next to the membrane. The masses indicated by the double circles are concentrated on the intersection point. Considering the mass of air in the shaded area in Fig. 8, the mass of the mass point, m', is written as follow

$$m' = \frac{1}{2}\rho_0 h l_y \left(l_x + \Delta l\right) \tag{9}$$

Using the displacement of mass point of the membrane, the variation of volume in element [N_x+1, j] is given by

$$dV_{N_x+1,j} = \frac{h}{2}\left\{l_y\left(\xi_{j-1}+\xi_j - x_{N_x,j} - x_{N_x,j-1}\right) + \Delta l\left(y_{N_x,j} - y_{N_x,j-1}\right)\right\} \tag{10}$$

The sound pressure in element [N_x+1, j], $dp_{N_x+1,j}$, is calculated by using Eqs. (3) and (10). The sound pressure in element [N_x+1, j], $dp_{N_x+1,j}$, and the sound pressure in element [N_x+1, $j+1$], $dp_{N_x+1,j+1}$, act on the mass point j of the membrane. Then, the equation of motion of the menbrane mass point becomes

$$m_m\ddot{\xi}_j = k_m\left(\xi_{j+1} - 2\xi_j + \xi_{j-1}\right) + \frac{l_y h}{2}\left(dp_{N_x+1,j} + dp_{N_x+1,j+1}\right) \tag{11}$$

2.4 Equation of Motion

Arranging the equations of motion of each mass, the matrix representation of the equations is expressed as follow

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{0} \tag{12}$$

where $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{K}$ is the stiffness matrix which consists of restoring forces of Eqs. (4) and (7), and $\boldsymbol{x}$ is the displacement vector arranging the displacement of each mass point as follow equation

$$\boldsymbol{x} = \left[x_{0,0}, x_{0,1}, \cdots, x_{n_x+1,n_y}, y_{0,0}, y_{0,1}, \cdots, y_{n_x+1,n_y}\right]^T \tag{13}$$

$\boldsymbol{M}$ is the diagonal matrix arranging the mass, and $\boldsymbol{K}$ is the symmetric matrix from the reciprocity theorem. Then, $\boldsymbol{M}$ and $\boldsymbol{K}$ are the symmetric matrix. Therefore, we can use the modal analysis easily by Eq.(12). Furthermore, the computational cost of the eigenvalue analysis becomes lower by the eigenvalue analysis method for symmetric matrix.

3. COMPARISON WITH MODAL COUPLING METHOD

To confirm the validity of the coupling model proposed in section 2, we calculate the natural frequency of the rectangle-plate-shaped space comprising five rigid walls and one membrane wall in Fig. 1, and compare the natural frequency by the proposed model with the results by the modal coupling method (9). The boundary conditions of the acoustic field are given as follows

$$x = 0 : v_x = 0 \tag{14}$$

$$x = L_x : v_x = \frac{\partial \xi}{\partial t} \tag{15}$$

$$y = 0, L_y \, ; \, v_y = 0 \tag{16}$$

where v_x is the particle velocity in x direction, v_y is the particle velocity in y direction. In Eq. (15), the particle velocity corresponds with the velocity of the membrane. The boundary conditions of the one-dimensional membrane are given as follows

$$y = 0, L_y : \xi = 0 \tag{17}$$

3.1 Modal Coupling Method (9)

The wave equation of the two-dimensional acoustic space is given by

$$\frac{\partial^2 \phi}{\partial t^2} = c^2 \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) \tag{18}$$

where ϕ is the velocity potential, c is the sound speed. If we assume the solution of this equation is $\phi = \Phi(x, y) e^{\tilde{j}\omega t}$ ($\tilde{j} = \sqrt{-1}$, ω is the angular frequency), and consider the boundary conditions (Eqs. (14), (16)), we get the eigenfunctions

$$\Phi = a_r \cos \nu_r x \cdot \cos \frac{r\pi}{L_y} y \tag{19}$$

where a_r is the arbitrary constant, r is the order, and ν_r is follow equation.

$$\nu_r^2 = \frac{\omega^2}{c^2} - \frac{r^2 \pi^2}{L_y^2} \tag{20}$$

To satisfy the boundary condition of Eq. (15), Φ is expressed as the sum of the eigenfunctions as follow equation.

$$\Phi = \sum_{r=0}^{N} a_r \cos \nu_r x \cdot \cos \frac{r\pi}{L_y} y \tag{21}$$

On the other hand, the equation of motion of the membrane is given by

$$\frac{\partial^2 \xi}{\partial t^2} - c_m^2 \frac{\partial^2 \xi}{\partial y^2} = \frac{h}{\rho_m} [p]_{x=L_x} \tag{22}$$

where $c_m = \sqrt{T_m / \rho_m}$, p is sound pressure. Differentiating this equation with respect to time, and assuming $\psi = \partial \xi / \partial t$, we can get

$$\frac{\partial^2 \psi}{\partial t^2} - c_m^2 \frac{\partial^2 \psi}{\partial y^2} = \frac{1}{\rho_m} \left[\frac{\partial p}{\partial t} \right]_{x=L_x} \tag{23}$$

If we assume $\psi = \Psi(y) e^{\tilde{j}\omega t}$, the eigenfunction satisfying the boundary conditions of Eq. (17) becomes

$$\Psi = b_r \sin \frac{r\pi}{L_y} y \tag{24}$$

where b_r is the arbitrary constant. Ψ is expressed as the sum of the eigenfunctions

$$\Psi = \sum_{r=1}^{N} b_r \sin \frac{r\pi}{L_y} y \tag{25}$$

From Eq.(21), the sound pressure, p , becomes

$$p = \rho \frac{\partial \phi}{\partial t} = \tilde{j} \rho \omega \sum_{r=0}^{N} a_r \cos \nu_r x \cdot \cos \frac{r\pi}{L_y} y \cdot e^{\tilde{j}\omega t} \tag{26}$$

Substituting this sound pressure, p , and ψ using Eq. (25) into Eq. (23), and multiply both sides by $\sin(q\pi y / L_y)$, and integrating from 0 to L_y with respect to y, we can get

$$\left(\omega^2 - \omega_q^2\right)\rho_m h \frac{L_y}{2} b_q = \omega^2 \rho h \sum_{r=0}^{N} a_r \beta_{qr} \cos\nu_r x \cdot \cos\frac{r\pi}{L_y} y \tag{27}$$

where ω_q is the q th natural frequency, ω_q and β_{qr} are follow equations

$$\omega_r = \frac{q\pi}{L_y} c_m \tag{28}$$

$$\beta_{qr} = \int_0^{L_y} \sin\frac{q\pi}{L_y} y \cdot \cos\frac{q\pi}{L_y} y dy = \begin{cases} \dfrac{2qL_y}{\left(q^2 - r^2\right)\pi} & \left(q + r \neq 2l\right) \\ 0 & \left(q + r = 2l\right) \end{cases} \tag{29}$$

From the boundary condition of Eq. (15), we get

$$\sum_{r=1}^{N} b_r \sin\frac{r\pi}{L_y} y = \sum_{r=0}^{N} a_r \nu_r \sin\nu_r L_x \cdot \cos\frac{r\pi}{L_y} y \tag{30}$$

Multiplying both sides of this equation by $\sin(q\pi y / L_y)$, and integrating from 0 to L_y with respect to y, we can get

$$\frac{L_y}{2} b_q = \sum_{r=0}^{N} a_r \beta_{qr} \nu_r \sin\nu_r L_x \tag{31}$$

Arranging Eqs. (27), (31) in order $q = 0,1,2,\cdots$, we get the system of equations about $a_1, a_2, a_3, \cdots$, $b_1, b_2, b_3, \cdots$. By finding nontrivial solutions of the system of equations, we get the natural frequencies and mode shapes of the coupled system.

3.2 Analysis by Concentrated Mass Model

To find the natural frequency and the natural mode by the concentrated mass model, the equation of motion of each mass is expressed by the matrix representation. If we assume $\boldsymbol{x} = \boldsymbol{X} e^{j\omega t}$, Eq. (13) becomes the generalized eigenvalue problem like follow equation

$$\left[\boldsymbol{K} - \omega^2 \boldsymbol{M}\right]\boldsymbol{X} = \boldsymbol{0} \tag{32}$$

This equation gives the natural frequencies and mode shapes.

The parameters of the simulation are listed in Table 1. About the division of elements near the membrane in Section 2.3, we simulate in three conditions: without division ($\Delta l = l_x$), $\Delta l = l_x / 2$, and $\Delta l = l_x / 5$.

Table 1 – Parameter values

L_x [m]	0.4	L_y [m]	0.5
N_x	40	N_y	50
h [mm]	30	p_0 [MPa]	0.1013
ρ_0 [kg/m^3]	1.27	γ	1.4
ρ_m [kg/m^2]	0.046	T_m [N]	300

3.3 Numerical Results

Table 2 shows the comparison between the natural frequency by the concentrated mass model and the natural frequency by the modal coupling theorem. The natural frequencies which has the mode number $(1,2,3,\cdots)$ are appropriate solutions. In case the mode number is slanting line, the solutions are spurious mode (10). Figure 12 shows the spurious mode at 387.80 Hz and 684.67 Hz. In the case of the spurious mode, the sound pressure is distributed in a patchy fashion. Meanwhile, the numerical results by the concentrated mass model have the zero eigenvalues because the degree of freedom of the model has a surplus. The number of the zero eigenvalues is half of the degree of freedom of the model. In the case of the zero eigenvalue, each element does not have the sound pressure though there are displacements of the mass points.

From table 2, the natural frequencies by the concentrated mass model agree well with the natural frequencies by the modal coupling theorem except in spurious mode. Comparing $\Delta l = l_x$, $\Delta l = l_x / 2$, and $\Delta l = l_x / 5$, the natural frequencies in the case of $\Delta l = l_x / 5$ most closely correspond the natural frequencies by the modal coupling method. Figure 9 shows the natural frequency to variation of $l_x / \Delta l$ in the case of 1st mode and 4th mode. The dashed line shows the natural frequency by the modal coupling method. From Fig. 9, as $l_x / \Delta l$ is large, the natural frequency approaches the results by the modal coupling method. Namely, as the mass points next to the membrane approach the membrane, the natural frequencies by the concentrated mass model are valid. The boundary condition between the acoustic space and the membrane needs to satisfy the follow equations

$$v_x = \frac{\partial \xi}{\partial t} \tag{33}$$

$$\frac{\partial p}{\partial x} = -\tilde{j} \rho \omega \frac{\partial \xi}{\partial t} \tag{34}$$

Equation (33) is the geometric boundary condition, and Eq. (34) is the mechanical boundary condition. We define B_L, B_R as the amplitudes of the left side and the right side of Eq. (33), C_L, C_R as the amplitudes of the left side and the right side of Eq. (34). The difference between the left side and the right side in Eqs. (33) and (34), err_1, err_2, are defined as follows

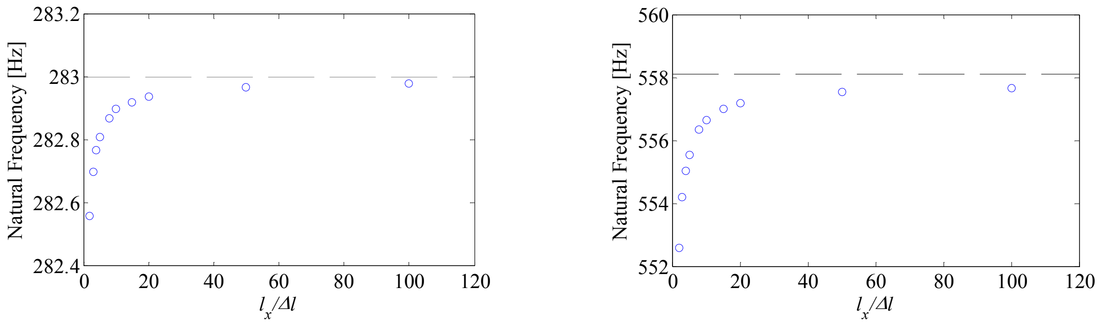
$$err_1 = \left| \frac{B_L - B_R}{B_R} \right| \tag{35}$$

$$err_2 = \left| \frac{C_L - C_R}{C_R} \right| \tag{36}$$

Figure 10 shows eer_1, eer_2 at $x = 0.5\,\text{m}$, $y = 0.15\,\text{m}$ in 1st mode by the concentrated mass model. $\partial p / \partial x$ in Eq. (33) is calculated by using finite difference form as $\partial p / \partial x = (p_{N_x+1,j} - p_{N_x,j}) / l_x$. From Fig. 10, as $l_x / \Delta l$ is large, eer_1 and eer_2 become small. Namely, as the mass points next tothe membrane approach the membrane, the model satisfies the boundary conditions of Eqs. (32), (33). The velocity of the mass point of air shown as the double circle in Fig. 8 approaches the velocity of mass point of the membrane, and the equilibrium of force comes to be satisfied as Δl

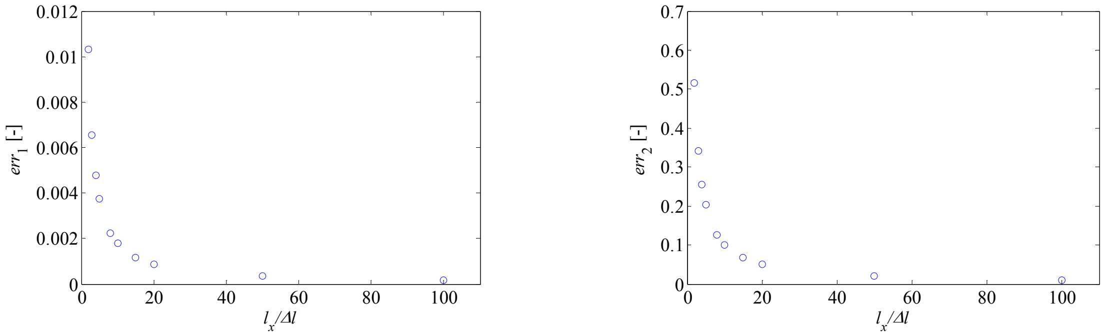
Table 2 – Natural frequency of coupled problem

Mode number	Concentrated mass model						Modal coupling
	$\Delta l = l_x$		$\Delta l = l_x / 2$		$\Delta l = l_x / 5$		
	Number	Natural frequency [Hz]	Number	Natural Frequency [Hz]	Number	Natural frequency [Hz]	Natural frequency
	1950	0	2000	0	2000	0	
1	1	282.11	1	282.56	1	282.81	283.00
2	1	305.20	1	307.79	1	309.22	310.23
	1	387.80	1	409.09	1	433.81	
3	1	468.14	1	469.95	1	470.99	471.90
4	1	547.23	1	552.58	1	555.55	558.10
5	1	667.62	1	670.00	1	671.24	672.44
	1	684.67	1	685.93	1	685.23	
	1	705.83	1	734.91	1	759.51	
6	1	741.78	1	745.75	1	748.10	750.60
7	1	745.01	1	750.53	1	753.37	756.42
8	1	902.89	1	918.68	1	927.49	935.08
	1	903.53	1	926.31	1	967.12	
9	1	949.23	1	949.36	1	949.50	951.57
10	1	1016.93	1	1023.21	1	1026.44	1031.04



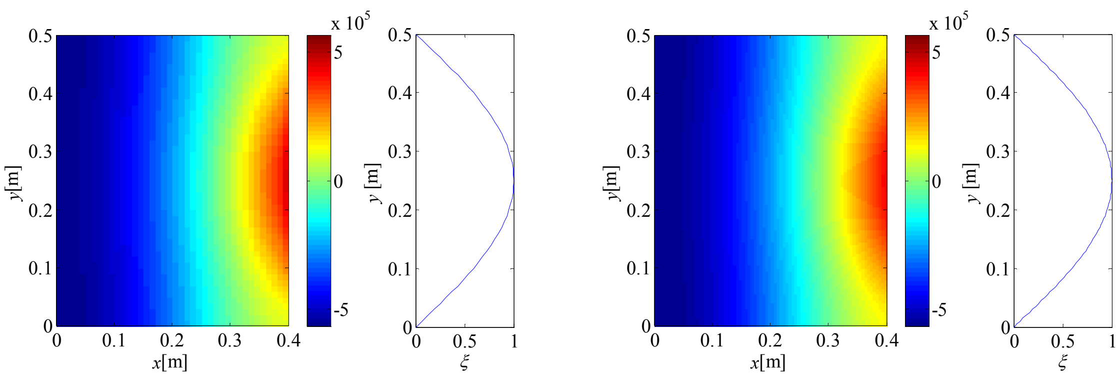
(a) 1st mode (b) 4th mode

Figure 9 – Natural frequency to variation of $l_x / \Delta l$

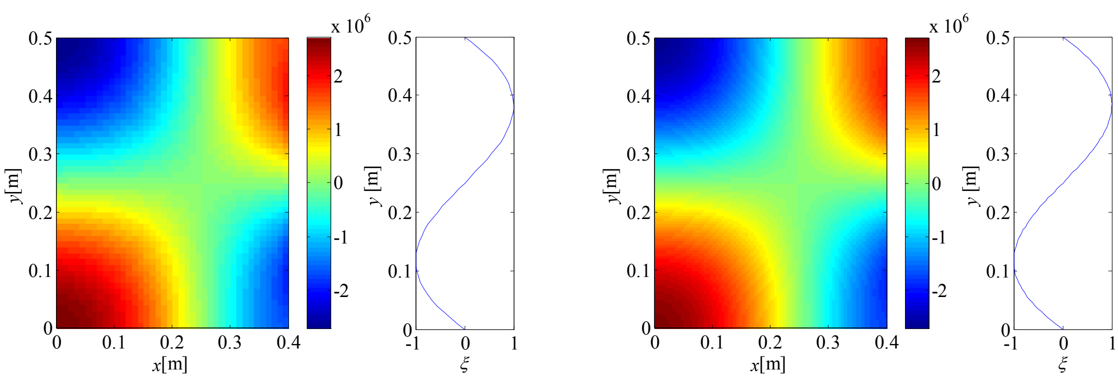


(a) Error of geometric boundary condition (b) Error of mechanical boundary condition

Figure 10 – Error of boundary condition to variation of $l_x / \Delta l$

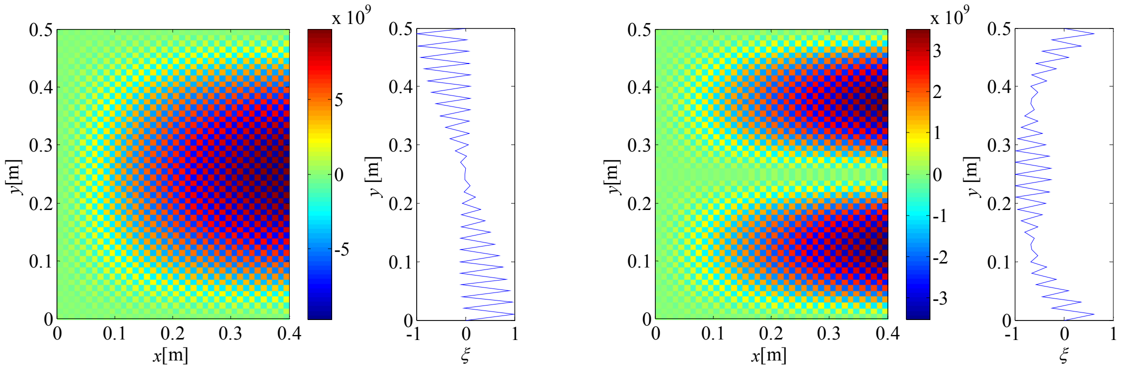


(a) Concentrated mass model (1st mode) (b) Modal coupling method (1st mode)



(c) Concentrated mass model (3rd mode) (d) Modal coupling method (3rd mode)

Figure 11 – Mode shapes of sound pressure and membrane



(a) 387.80 Hz (b) 684.67 Hz

Figure 12 – Spurious mode (sound pressure and membrane mode)

Table 3 – Natural frequency of acoustic space in case of rigid wall

Mode number	Natural frequency [Hz]
(0, 1)	334.17
(1, 0)	417.71
(1, 1)	534.93
(0, 2)	668.34
(1, 2)	788.14
(2, 0)	835.42
(2, 1)	899.78
(0, 3)	1002.51

Table 4 –Natural frequency of membrane in vacuo

Mode number	Natural frequency [Hz]
1	571.04
2	1142.08
3	1713.12
4	2284.16
5	2855.20
6	3426.24
7	3997.28
8	4568.32

becomes small.

Figure 11 is the comparison between the natural mode shape by the concentrated mass model ($\Delta l = l_x$) and the mode shape by the modal coupling method. In (a) and (b), the order of the natural mode is first, and in (c) and (d), the order is third. The left-side figure is the mode shape of the sound pressure, and the right-side figure is the mode shape of the membrane. In each mode, the mode shape by the concentrated mass model agrees well with it by the modal coupling method. The first mode is the acoustic (1,0) mode, third mode is the acoustic (1,1) mode. Table 3 shows the natural frequencies of the acoustic field in the case of the rigid wall, and table 4 shows the natural frequencies of the membrane in vacuo. So the natural frequencies of the coupled problem decrease from the natural frequency of the acoustic field in the case of the rigid wall.

As shown above, though the spurious mode and the zero eigenvalue exist in the numerical results by the concentrated mass model, the concentrated mass model is valid for the coupled analysis of acoustic and membrane vibration.

3.4 Computation time

To confirm the advantage of the proposed model to the finite element model, the computation time of eigenvalue analysis is measured in each method. The equation of FEM is given by (4),(5)

$$\begin{bmatrix} \boldsymbol{M}_{ss} & \boldsymbol{0} \\ \boldsymbol{M}_{fs} & \boldsymbol{M}_{ff} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{u}} \\ \ddot{\boldsymbol{p}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{K}_{ss} & \boldsymbol{K}_{sf} \\ \boldsymbol{0} & \boldsymbol{K}_{ff} \end{bmatrix} \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{p} \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}_s \\ \boldsymbol{0} \end{bmatrix} \tag{37}$$

The degree of freedom of the concentrated mass model is approximately twice that of FEM. The eigenvalue analysis is performed by Intel Math Kernel Library. The eigenvalue analysis method of the concentrated mass model is the divide-and-conquer method for symmetric matrix, and the method of FEM is QR method for asymmetric matrix.

Table 5 shows the comparison of computation time in each division number. The concentrated mass model is faster than FEM in spite of the twice degree of freedom.

Table 5 – Comparison of computation time

Division number	$N_x = 20,\ N_y = 25$	$N_x = 40,\ N_y = 50$	$N_x = 80,\ N_y = 100$
Concentrated mass model	0.0625 s	2.350 s	87.42 s
FEM	0.1938 s	6.116 s	463.54 s

4. CONCLUSIONS

To analyze the coupled problem of the two-dimensional acoustic in the rectangle-plate-shaped space and the membrane vibration, we propose the concentrated mass model that consists of the mass and the connecting spring. The acoustic space and the membrane are modeled as the concentrated mass model respectively. The treatment of the boundary between the acoustic space and the membrane is only the arrangement of the masses of the air near the membrane. It is very easy to couple the structure and acoustic field by the concentrate mass model. Furthermore, the mass matrix and the stiffness matrix are symmetrical. Therefore, we can use the modal analysis easily by this model. And, we confirm the validity of the concentrated mass model by comparing the natural mode computed by this model with the results by the modal coupling method. Though there are physically-meaningless modes in the solution by the proposed model like the spurious mode and the zero eigenvalues, we can obtain the natural frequency and the mode shape that agree well with the results by the modal coupling method. Furthermore, it is confirmed that the computation time of the concentrated mass model is faster than FEM in eigenvalue analysis. Therefore, the concentrated mass model we propose in this paper is valid for the coupled analysis of the two-dimensional acoustic and the membrane vibration. The future task is the elimination of the spurious mode and the zero eigenvalues.

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