Beat period control of bell sound using an operational modal analysis

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ABSTRACT

Beats in the sound of a ringing bell are periodic variations in volume that are generated by a slight asymmetry of the bell structure. However, beat periods are often too long or short due to uncontrollable elements in the bell-casting process. To optimize the beat period, beat tuning is performed after bell casting using the mode data of the bell. In the case of a large oriental bell, which is sounded by striking it with a heavy wooden hammer, mode data should be extracted only using the response data such as acceleration or sound pressure. In this study, we introduce a beat-tuning method using the operational deflection shape (ODS) for a large Oriental bell. Using the ODS, we extract the frequency and mode pair, which cause the beating sound, and we tune the beat period of the sound through a structural modification technology.

Keywords: Beating Sound, Operational Deflection Shape, Beat Period Control

1. INTRODUCTION

A large Korean bell is rung by striking it with a heavy wooden hammer. A strong beat with a proper period makes listeners feel as if the bell were alive and crying. The beat phenomenon in a bell results from the asymmetry caused by inevitable variation in the bell diameter and thickness[1, 2]. However, the beat period is often too long or short due to uncontrollable elements in the bell-casting process. To optimize the beat period, the inside of the bell is ground after casting. Grinding position and the amount are determined considering the original beat period, striking position and mode pair position[3, 4]. Therefore, frequency pair and mode pair data are necessary to perform beat tuning. Usually, mode data can be obtained by using frequency response functions when both the excitation and response signals are available. In a large bell, however, the impact force of a heavy wooden hammer cannot be measured, so only the response signal is available. To obtain the mode pair data from the response signals, this study employs the ODS (Operational Deflection Shape) method for a large bell[5]. Transmissibility functions for the ODS method are measured, and the mode data are extracted. Using the mode pair data, beat tuning is performed on a large Koreanbell (Po-cheon Citizen Bell), which was cast in 2006 by a Korean bell founder. Table 1 lists the specifications of the bell, and Figure 1 shows an image of the bell.

Table 1 – Specifications of the bell

Height	3300 mm
Maximum diameter	2040 mm
Maximum thickness	205 mm
Height of striking point	800 mm
Weight	15 tons

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Figure 1 Po-cheon Citizen Bell

2. MODE DATA EXTRACTION

In the striking of the bell, the impact does not contain any dominant frequency component, and the natural frequency components of the bell are relatively uniformly excited. Therefore, the natural frequencies can be identified from the peaks of the power spectrum. Further, mode shapes can be obtained by using the transmissibility function between a reference point and roving points as shown in Figure 2. Assuming a single force on k-th point, the transmissibility function is determined as follows[6]:

$$T_{ij}^{k} = \frac{X_i(s)}{X_j(s)} = \frac{H_{ji}(s)F_k(s)}{H_{jk}(s)F_k(s)} = \frac{H_{ik}(s)}{H_{jk}(s)} \tag{1}$$

The frequency response function is represented by modal parameters as follows:

$$H_{ik}(s) = \sum_{m=1}^{N_m}\left(\frac{\phi_{ir}\,L_{kr}}{s-\lambda_r} + \frac{\phi_{ir}^{*}\,L_{kr}^{*}}{s-\lambda_r^{*}}\right) \tag{2}$$

where N_m is the number of modes, and λ_r, ϕ_{ir}, and L_{ir} are the *r-th* pole, mode vector, and modal participation factor, respectively. Applying Eq. (2) to Eq. (1).

$$\lim_{s-\lambda_r} T_{ij}^{k}(s) = \lim_{s-\lambda_r} \frac{(s-\lambda_r)\,H_{ik}(s)}{(s-\lambda_r)\,H_{jk}(s)} = \frac{\phi_{ir}}{\phi_{jr}} \tag{3}$$

Eq. (3) indicates that mode vector can be obtained from the transmissibility function between a roving point *i* and a fixed reference point *j*.

Twenty-four transmissibility functions between 24 roving points and the reference point (striking point 1) were measured in sequence. The measured acceleration signal was digitally processed for DFT using a signal analyzer (Harmony & dBFa, 01dB). Figure 2 shows the experimental set up.

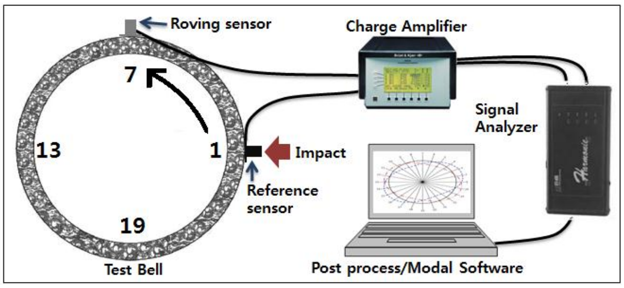


Figure 2 Schematic diagram of the experiment.

In this study, we focus on the beat control of the 1st mode as its beating sound lasts relatively long, compared to beats by other modes, which disappear rapidly. Table 2 shows the natural frequency pair of the 1st mode. Slight asymmetry of the bell structure makes the frequency pair (L,H). The inverse of the frequency difference becomes the beat period. Figure 3 (a) shows the anti-nodal lines of the 1st mode pair, which makes beating sound depicted in Figure 3 (b). The striking position is close to the anti-node of the 1st-L mode (solid line); therefore, the 1st-L mode is strongly excited. As the result, the beat is not clear. Furthermore, a beat period of 16 s is too long, and listeners can barely hear the beating sound. This long beat period indicates that the asymmetry of the bell is very small. Consequently, beat tuning is necessary to obtain a strong beat with shorter beat period.

Table 2 Natural frequency pair and beat data

1st- L	79.156 Hz
1st- H	79.219 Hz
Δ f	0.063 Hz
Beat period	15.9 sec

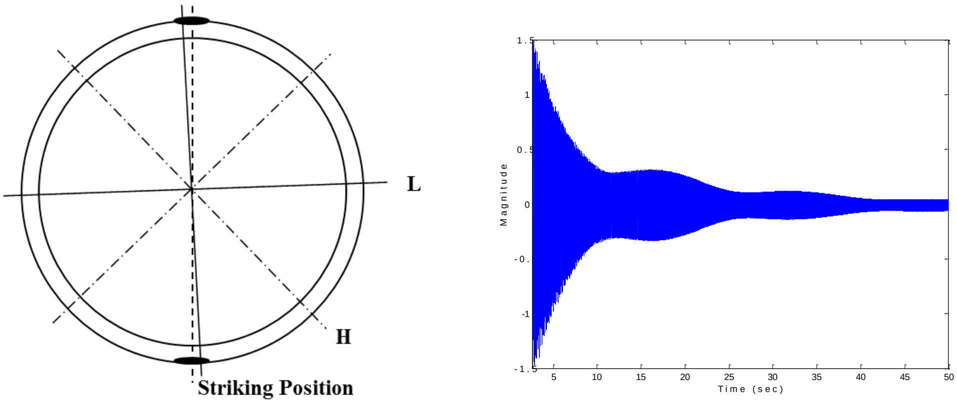


Figure 3 Anti-nodal lines (top view) and beat of the 1st mode.

3. BEAT TUNING

Based upon the beat theory of cylindrical shells[7], for a strong beat, the bell should be struck at the center of the nodes of the L and H modes, as shown in Figure 4. In this case, the L and H modes are equally excited and a clear beat occurs. However, this condition is not often satisfied by uncontrollable asymmetric elements in the casting process. This condition can be achieved by modifying the effective mass and stiffness of the 1st mode.

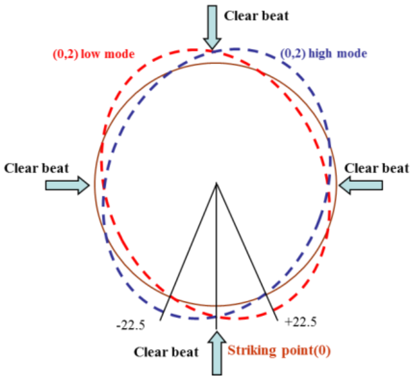


Figure 4 Striking positions for a clear beat

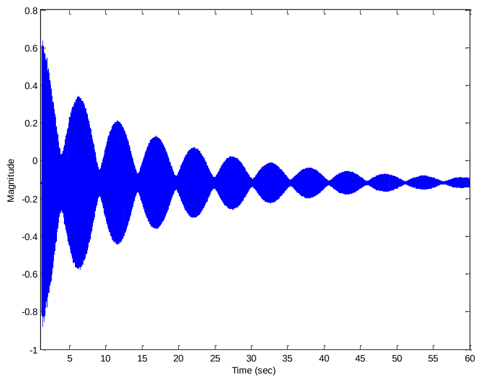
According to the numerical simulation[3], it was found that decreasing the thickness at 22.5° from the striking point creates the mode configuration shown in Figure 4. Thickness decrement at 22.5° decreases the bending stiffness at the position and 22.5° becomes the antinode of the 1st-L mode. In this bell, 22.5° is closer to the anti-node of the 1st-L mode, so grinding that position reduces the bending stiffness of the 1st-L mode more. This increases the frequency difference between the L and H modes and shortens the beat period. We ground away the thick band of the bell bottom in a stepwise manner over a width of 10° and depth of 4 mm. Table 3 shows the natural frequencies, position of L mode antinode, and beat periods during tuning. At the final step, the beat period was shortened to 5.3 s, from an original period of 15.9 s. In addition, the antinode of the L mode moved towards 22.5°, which means that striking the bell can create a stronger beat.

Table 3 Tuning effect of the 1st mode pair

	Initial state	1st step	2nd step	Final step
1st- L	79.156 Hz	79.070 Hz	79.047 Hz	79.000 Hz
1st- H	79.219 Hz	79.195 Hz	79.195 Hz	79.188 Hz
L-antinode	3°	16°	20°	22°
Beat period	15.9 s	8.0 s	6.8 s	5.3 s

Figure 5 shows the grinding work for beat tuning and the effect on the beat. We can see a strong beat with a proper period of 5.33 s.

a) Grinding in of the bell inside



b) Beat after grinding

Figure 5 Beat tuning and its result

4. Conclusion

Beat tuning work was performed on a large Korean bell in order to make a strong beat with a proper period in the sound. To extract the mode data for the beat tuning, transmissibility functions between a reference point and 24 roving points were measured. In the original bell, (L, H) frequency pair of the 1st mode showed a very small difference and this made a very long beat period. Also, beat in the sound was not strong as the striking point was close to the anti-node of L mode. To make a strong and a proper period beat in the sound, effective bending stiffness of the mode pair was modified by locally decreasing the thickness of the bell bottom. By grinding the thick bottom band of the bell around 22.5° from the striking point, L mode frequency decreased more than H mode frequency and the beat period was shortened to 5.3 s. Furthermore, through the tuning work, the striking point shifted in between the anti-nodes of the L, H modes and consequently, this made a strong beat in the sound.

REFERENCES

1. J. S. Hong and J. M. Lee, “Vibration of circular ring with local deviation”, Transactions of the ASME, Journal of Applied Mechanics, 61, pp. 317-322, (1994).
2. H. G. Park., Y. J. Kang. and S. H. Kim., Dual Mode Tuning Strategy of a Slightly Asymmetric Ring, J. Acoust. Soc. Am., 123(3), 1383~1391, (2008).
3. J. M. Lee, S. H. Kim, S. J. Lee, J. D. Jeong and H. G. Choi, “A Study on the vibration characteristics of a large size Korean bell”, Journal of Sound and Vibration, 257(4), 779-790, (2002).
4. S. H. Kim., J. H. Lee., "Beat period tuning method using an equivalent bell model", Journal of the Acoustical Society of Korea, 31(8), 561~568, (2012).
5. C. Devriendt, G. Steenackers, G. De Sitter, P. Guillaume, “From operating deflection shapes towards mode shapes using transmissibility measurements”, Mech. Syst. Sig. Proc., 24(3), 665-677, (2010).
6. Devriendt, C., Guillaume, P., "Identification of Modal Parameters from Transmissibility Measurements", Journal of Sound and Vibration, 314(1), 343~356, (2008).
7. S. H. Kim, W. Soedel, and J. M. Lee, “Analysis of the Beating Response of Bell Type Structures”, Journal of Sound and Vibration, 173(4), 517-536, (1994).