Multivariable control of tonal disturbances using minimization of the maximum error signal through adaptive error signal weighting

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ABSTRACT

In many multichannel active noise and vibration control systems the controller is adapted to minimize the 2-norm of the error signals. This may, however, lead to a large spatial variance in the residual error. A method of achieving a more uniformly controlled error field using a weighted squared error strategy has previously been proposed, although the presented method of defining the error weighting parameters results in a very slow convergence rate. This convergence rate limitation has been overcome by the minimax algorithm which minimizes, in a least-squares sense, the maximum error signal at each iteration. However, due to the inherent switching in this algorithm, for fast convergence speeds it suffers from significant misadjustment and in a tonal control problem this introduces additional unwanted spectral components. In this paper an alternative method of minimizing the maximum error signal is proposed which uses an adaptive error-weighting matrix that is bounded and so avoids the slow convergence speeds previously reported. It is also shown that the proposed algorithm does not suffer from the same misadjustement problems shown by the minimax algorithm. The details of the proposed method are first outlined and then its performance is compared to the previously proposed methods through a series of time-domain simulations employing measurements of a physical system.

Keywords: Active noise and vibration control, Adaptive control algorithms
I-INCE Classification of Subjects Number(s): 38.2, 46.4

1. INTRODUCTION

Active noise and vibration control has been employed in a wide variety of noise control problems where weight restrictions and performance requirements mean that passive control solutions are not practical or sufficient. For example, active noise control has typically been applied in road vehicles for engine (1) and road noise control (2), in aircrafts to control propeller-induced cabin noise (3), in headphones to control exterior noise (4) and to control vibration in a wide variety of structures and for vibration isolation (5, 6). However, active control also benefits from the ability to manipulate the sound field, the radiated sound or the structural response with a greater degree of flexibility than passive control techniques. For example, more recent attention has been focused on the active control of sound quality (7, 8), which attempts to achieve a specific level after control rather than necessarily achieving the maximum attenuation of the error signals. In a similar vein, attention has also been given to the spatial properties of active control systems and this is the focus of this paper.

In multivariable active control applications the sum of the squared error signals, or the 2-norm of the error signal vector is generally minimised (9). However, this can result in a sound field or a vibration distribution that has large differences between its maximum and minimum values. To overcome this problem it is necessary to minimise the infinity-norm of the error signal vector. To achieve a more spatially uniform error, Elliott *et al* (9) propose the use of a weighted squared error strategy. However, when the proposed weighting strategy is used to minimise the infinity-norm it is susceptible to very slow convergence speeds. This problem is overcome by Gonzalez *et al* (10) through the derivation of the minimax algorithm, which minimises the maximum error signal at each iteration of the algorithm. However, the minimax algorithm suffers from misadjustment, or chattering at high convergence gains and this may limit the use of the minimax algorithm when fast convergence is critical.

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In this paper, the multivariable tonal control problem is first outlined and the general least-squares steepest descent algorithm is defined. The minimax control problem is introduced and the minimax algorithm proposed by Gonzalez *et al* (10) is summarised. It is then shown that the minimax algorithm can also be formulated in terms of the generalised cost function and the corresponding generalised update algorithm with a switching error signal weighting matrix. An alternative method of adapting the error signal weighting matrix to achieve the minimax criterion is then described, which avoids the switching properties of the minimax algorithm. The performance of the proposed algorithm is then compared with the conventional least squares algorithm and the minimax algorithm proposed in (10) through simulations using measured transfer responses. Finally, conclusions regarding the proposed algorithm are presented.

2. MULTICHANNEL TONAL CONTROL PROBLEM

The response of the multichannel feedforward control system shown in Figure 1 at a single frequency is given by

$$\mathbf{e}(e^{j\omega_0 T_s}) = \mathbf{d}(e^{j\omega_0 T_s}) + \mathbf{G}(e^{j\omega_0 T_s})\mathbf{u}(e^{j\omega_0 T_s}), \tag{1}$$

where $\mathbf{e}$ is the $(L \times 1)$ vector of error signals, $\mathbf{d}$ is the $(L \times 1)$ vector of disturbance signals, $\mathbf{G}$ is the $(L \times M)$ matrix of plant responses and $\mathbf{u}$ is the $(M \times 1)$ vector of control signals. For the remainder of this paper the dependence on $\omega_0 T_s$ will be dropped for conciseness. The optimal vector of control signals for this multichannel problem is dependent on the cost function to be minimised. In many applications the cost function is defined as the sum of the squared error signals, which is given by

$$J = \mathbf{e}^H \mathbf{e}. \tag{2}$$

When the number of error sensors is greater than the number of control actuators, i.e. $L > M$, then the problem is overdetermined and the optimal set of control signals is given by the closed-form solution

$$\mathbf{u}_{opt} = -\left(\mathbf{G}^H \mathbf{G}\right)^{-1} \mathbf{G}^H \mathbf{d}. \tag{3}$$

A more general cost function is given by the weighted sum of the squared error and control signals

$$J = \mathbf{e}^H \mathbf{Q} \mathbf{e} + \mathbf{u}^H \mathbf{R} \mathbf{u} \tag{4}$$

where $\mathbf{Q}$ and $\mathbf{R}$ are $(L \times L)$ and $(M \times M)$ weighting matrices for the error and control signals respectively. In this more general case the optimal set of control signals is given by

$$\mathbf{u}_{opt} = -\left(\mathbf{G}^H \mathbf{Q} \mathbf{G} + \mathbf{R}\right)^{-1} \mathbf{G}^H \mathbf{Q} \mathbf{d}. \tag{5}$$

The error and control weighting matrices can be selected in this case to achieve different requirements. For example, the error sensor weighting matrix could be selected in order to weight the relative importance of minimising the error signal at different error sensors and the control signal weighting matrix could be defined in order to avoid over driving the control actuators.

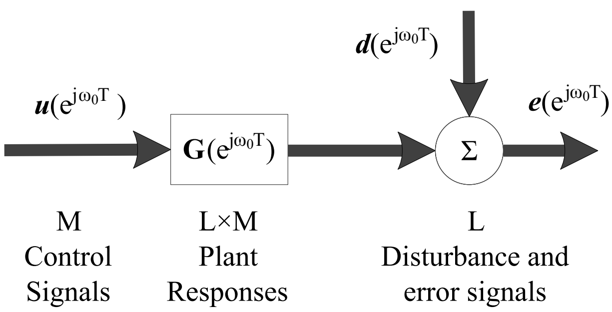


Figure 1 – Block diagram of a multichannel tonal control system operating at a frequency of ω_0.

In many practical active control systems, a fixed control filter is not suitable due to changes in the disturbance signal and the plant response over time. The generalised cost function given by equation 4 can be minimised in practice by iteratively adjusting the control signal vector. Using a gradient-descent approach the control signal update equation is given by

$$\mathbf{u}(n+1) = (\mathbf{I} - \alpha \mathbf{R})\mathbf{u}(n) - \alpha \mathbf{C} \mathbf{Q} \mathbf{e}(n) \tag{6}$$

where n is the iteration index, α is the convergence gain, $\mathbf{I}$ is the identity matrix and $\mathbf{C}$ is a complex matrix update operator, which for the steepest-descent algorithm is equal to $\mathbf{G}^H$. The update algorithm given by equation 6 is derived by assuming that the error signals have reached their steady state values before the subsequent iteration. In practice this means that the algorithm will be very slow to converge and, therefore, it is common practice to compute the next iteration prior to the steady state condition (11). The algorithms considered in this paper are thus all updated at the sampling frequency, $1/T_s$ Hz, and may therefore be referred to as Instantaneous Harmonic Control (IHC) algorithms.

3. MINIMISATION OF THE MAXIMUM ERROR SIGNAL

As highlighted in the introduction, one particular criterion that is of interest in active noise and vibration control is the minimisation of the maximum error signal. This infinity norm criterion provides a more spatially uniform residual error and this is of particular interest in a number of applications. Although there is no closed-form solution to this infinity norm problem, an iterative solution has been proposed by Gonzalez *et al* (10). The infinity norm cost function is given by

$$J_\infty = ||\mathbf{e}||_\infty = \max_{1\leq l\leq L} |e_l| = |e_{max}|, \tag{7}$$

where e_{max} is the error signal with the largest magnitude. It is shown by Gonzalez *et al* (10) that this cost function is convex and, therefore, a steepest-descent type algorithm can be used successfully to iteratively update the control signals. The gradient of the cost function given by equation 7 is (10)

$$\boldsymbol{\Delta}_\infty = (|e_{max}|)^{-1}\mathbf{g}_{max}^H e_{max} \tag{8}$$

where $\mathbf{g}_{max}$ is the row of the plant response matrix, $\mathbf{G}$, corresponding to the error signal with the maximum magnitude. Following the method of steepest-descent the iterative algorithm that minimises the maximum error signal is given by

$$\mathbf{u}(n+1) = \mathbf{u}(n) - \mu\boldsymbol{\Delta}_\infty = \mathbf{u}(n) - \alpha\mathbf{g}_{max}^H e_{max}(n) \tag{9}$$

where $e_{max}(n)$ is the error signal with the largest magnitude at each iteration. This iterative algorithm is called the minimax algorithm in (10) and this naming convention will be employed here.

3.1 Switched Error Signal Weighting Matrix

The minimisation of the maximum error signal has been achieved iteratively by Gonzalez *et al* (10) as outlined by equation 9. At each iteration this algorithm updates the control signals using only the maximum error signal at each iteration according to the steepest-descent method. This operation can also be achieved using the generalised gradient-descent algorithm given by equation 6 by setting $\mathbf{R} = 0$, $\mathbf{C} = \mathbf{G}^H$ and employing a time-varying sensor weighting matrix which selects the maximum error signal and the corresponding row of the plant response matrix at each iteration. The time varying sensor weighting matrix at each iteration is defined by setting the diagonal element of $\mathbf{Q}(n)$ corresponding to the maximum error signal to unity and all other elements to zero. That is

$$\begin{aligned} Q_l(n) = 1 \quad & \text{if} \quad |e_l(n)| = \max(|\mathbf{e}(n)|), \\ Q_l(n) = 0 \quad & \text{if} \quad |e_l(n)| \neq \max(|\mathbf{e}(n)|), \end{aligned} \tag{10}$$

where Q_l is the l-th diagonal element of the $\mathbf{Q}$ matrix. It is important to highlight that this sensor weighting matrix is equivalent to a logic-based switching strategy that switches between L adaptive controllers, each of which adapt to minimise one of the L individual error signals. The control signal update equation which describes the full controller can thus be expressed as

$$\mathbf{u}(n+1) = \mathbf{u}(n) - \alpha\mathbf{G}^H\mathbf{Q}(n)\mathbf{e}(n) \tag{11}$$

and this behaves identically to the minimax algorithm given by equation 9, however, it does not benefit from the reduced computational demands discussed by Gonzalez *et al* (10).

The stability of the minimax controller has been analysed in (10) and the condition for convergence has been given as

$$0 < \alpha \leq \frac{1}{\max(\mathbf{g}_l\mathbf{g}_l^H)}, \tag{12}$$

where $\mathbf{g}_l$ is the l-th row of the full plant response matrix, $\mathbf{G}$, and $\max(\mathbf{g}_l\mathbf{g}_l^H)$ is the maximum of the inner product of each of the L rows of the plant matrix. The derivation of equation 12 is based on the steady-state assumption that any transients in the error signals have decayed away before they are measured, however, when an iterative algorithm is updated at the sample rate the time between each iteration will typically be small compared to the transient response of the system under control and, therefore, equation 12 will give an excessively high maximum convergence gain. Additionally, the stability analysis in (10) also assumes that the error signal with the largest power does not change during the convergence, which will in general not be true.

In the context of the filtered-x LMS algorithm the stability of the controller has typically been determined following similar steady-state assumptions to the analysis of the minimax algorithm in (10), however, a more thorough analysis which does not require these assumptions has been performed by representing the filtered-x LMS algorithm as an equivalent feedback compensator (9, 11, 12). This allows the performance and stability of the controller to be assessed using well known control theory. However, in the context of the minimax algorithm it is not straightforward to perform such analysis due to the switching nature of the controller and instead it is necessary to analyse the stability using Lyapunov stability theory, for example (13). This analysis is beyond the scope of the present paper and, although it will form the basis of future work, it is shown in the results presented in Section 4 that the minimax controller is generally limited by it performance rather than its stability.

3.2 Adaptive Error Signal Weighting Matrix

An alternative method of attempting to minimise the maximum error signal based on minimising a weighted squared error is suggested by Elliott *et al* (9). The proposed method sets the error weighting terms, corresponding to the diagonal elements of the $\mathbf{Q}(n)$ matrix in equation 11, to the associated averaged squared error term. The averaged squared error for the l-th error signal is approximated using a moving average such that the l-th diagonal element of the error weighting matrix is calculated as

$$Q_l(n) = \frac{\left[e_l^2(n) + e_l^2(n-1)\right]}{2}. \tag{13}$$

Using this method the $\mathbf{Q}(n)$ matrix acts as a time varying error weighting function, opposed to its switching function operation in the minimax algorithm. It turns out, however, that this weighted error algorithm does not minimise the maximum error signal, but instead minimises the 4-norm of the error. Although this produces less variation in the residual error signals than the least squares solution, it does not give the same results as the minimax algorithm. It is highlighted in (9) that the p-norm can be minimised by setting Q_l to the average value of $e_l^{(p-2)}$ and, therefore, the minimax solution can be approximated by setting p to a large number. However, in this case it can be appreciated that Q_l can take on a very large range of values and, therefore, in practice the algorithm can be very slow to converge.

The slow convergence problems associated with the weighted error algorithm proposed in (9) can be avoided by defining the diagonal elements of the sensor weighting matrix as

$$Q_l(n) = \left(\frac{|e_l(n)|}{|e_{max}(n)|}\right)^p \tag{14}$$

where $|e_l(n)|$ is the magnitude of the l-th error signal at the n-th iteration and p must be large to approximate the infinity-norm optimisation. This method of defining the sensor weightings means that $Q_l(n)$ is bounded as

$$0 < Q_l(n) \leq 1, \quad \forall\ l \tag{15}$$

and, therefore, it is expected that the convergence of the algorithm will not be as significantly affected by the adaptation of the error signal weighting matrix as when the diagonal elements of the sensor weighting matrix, Q_l, are positive but unbounded as reported in (9).

To give some insight into the effect that the adaptive $\mathbf{Q}$ matrix has on the convergence of the algorithm, the convergence of the control signals will initially be considered following the approach applied in (14) for example. Using equations 1 and 5, with $\mathbf{R} = \mathbf{0}$, the difference between the control signal vector at each iteration, give by equation 11, and the optimal control signal can be expressed as

$$\mathbf{u}(n+1) - \mathbf{u}_{opt} = \left[\mathbf{I} - \alpha\mathbf{G}^H\mathbf{Q}\mathbf{G}\right]\left(\mathbf{u}(n) - \mathbf{u}_{opt}\right). \tag{16}$$

If it is assumed that $\mathbf{u}(0) = \mathbf{0}$ and equation 16 is applied recursively then

$$\mathbf{u}(n) - \mathbf{u}_{opt} = -\left[\mathbf{I} - \alpha\mathbf{G}^H\mathbf{Q}\mathbf{G}\right]^n \mathbf{u}_{opt}. \tag{17}$$

Using singular value decomposition of the plant response matrix, the Hessian matrix can be expressed as

$$\mathbf{G}^H\mathbf{Q}\mathbf{G} = \mathbf{V}\boldsymbol{\Sigma}\mathbf{V}^H \tag{18}$$

where $\mathbf{V}$ is a unitary matrix of complex normalised eigenvectors and $\boldsymbol{\Sigma}$ is a square diagonal matrix of eigenvalues. Using the normalised eigenvectors, a normalised control vector can be defined as

$$\mathbf{v}(n) = \mathbf{V}^H(\mathbf{u}(n) - \mathbf{u}_{opt}) \tag{19}$$

and using equations 17 and 18 this can then be written as

$$\mathbf{v}(n) = [\mathbf{I} - \alpha\boldsymbol{\Sigma}]^n\, \mathbf{v}_{opt}. \tag{20}$$

Since $\boldsymbol{\Sigma}$ is a diagonal matrix, each component of the normalised control vector is independent and can be written as

$$v_m(n) = [1 - \alpha\lambda_m]^n\, v_{m:opt}. \tag{21}$$

This leads to the standard convergence condition given by

$$0 < \alpha < \frac{2}{\lambda_m} \quad \forall\ m \tag{22}$$

and the convergence coefficient is thus limited by the maximum eigenvalue of the Hessian matrix according to

$$0 < \alpha < \frac{2}{\lambda_{max}}. \tag{23}$$

For $\alpha\lambda_m \ll 1$ equation 21 can be written as

$$v_m(n) = \mathrm{e}^{-\alpha\lambda_m n} v_{m:opt}, \tag{24}$$

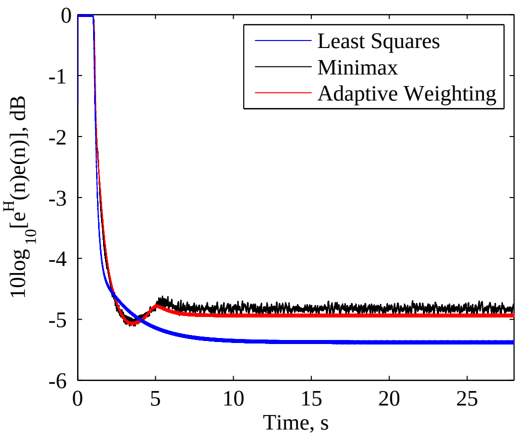
which shows that the algorithm converges in a series of modes with time constants determined by $1/(\alpha\lambda_m)$. The slowest mode thus corresponds to the smallest eigenvalue of $\mathbf{G}^H\mathbf{Q}\mathbf{G}$ and since rapid convergence is given by setting $\alpha = 1/\lambda_{max}$ the minimum time constant for the slowest mode is $\lambda_{max}/\lambda_{min}$, or the eigenvalue spread. Although this derivation makes the same steady-state assumptions outlined in the previous section with regard to equation 12 and, therefore, may over predict the maximum convergence gain when the algorithm is updated at the sample rate, it does provide some insight into the expected behaviour of the algorithm.

For the case of a time varying error weighting matrix, $\mathbf{Q}(n)$, it can be appreciated from equations 23 and 24 that both the maximum convergence gain and the modes of convergence will be affected by the change in the $\mathbf{Q}$ matrix with time, since this directly affects the eigenvalues of the Hessian matrix, $\mathbf{G}^H\mathbf{Q}(n)\mathbf{G}$. For example, if the adaptation of $\mathbf{Q}$ means that λ_{max} increases, then the maximum convergence gain will be reduced. If the adaptation of $\mathbf{Q}$ also means that the eigenvalue value spread is increased, then the speed of convergence will be reduced. This would therefore suggest that it may be necessary to adapt the convergence gain whilst also adapting the error weighting matrix in order to achieve maximum performance. However, since $\mathbf{Q}$ is diagonal and its elements are bounded between 0 and 1, this effect may be small and this will be considered in the simulations presented in the following section.

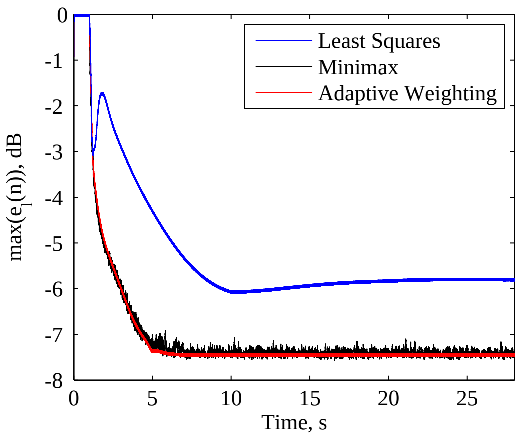
4. SIMULATIONS

To assess the performance of the minimax and adaptive error signal weighting algorithms presented in the previous section, a series of time-domain simulations have been conducted using a set of transfer responses measured between four control loudspeakers, a single primary loudspeaker and eight microphones in a rectangular enclosure with dimensions of 2.4 × 1.2 × 1.1 m. The physical plant responses and the primary path responses have been modelled using finite impulse response filters and the primary disturbance pressures have been produced by a single tone at 250 Hz. The nominal plant response model used in the update algorithms has thus been calculated at the control frequency of 250 Hz.

The performance of the steepest-descent least squares, minimax and adaptive error signal weighting algorithms are shown in Figure 2. The convergence gain, α, for each algorithm has been set to achieve a similar initial convergence speed. It is not possible to achieve identical convergence speeds for the three algorithms due to the differences in their convergence properties, as discussed in the previous sections. Figure 2a shows the convergence of the sum of the squared error signals for the three algorithms. From this plot it is clear that, as expected, the steepest-descent least squares algorithm achieves the highest level of attenuation in the sum of the squared error signals, while the minimax and adaptive error signal weighting algorithms achieve a slightly lower, but similar level of control after convergence. That said, it can be seen that the converged minimax algorithm shows significant variations about the converged level and this has been reported in (10). These variations are produced as the algorithm switches between the minimisation of the different error signals depending on which has the largest magnitude at each iteration. The magnitude of these variations can be reduced by reducing the convergence gain, α, however, this will inherently reduce the convergence rate of the algorithm.



(a) Sum of the squared errors signals.



(b) Maximum squared error signal at each iteration.

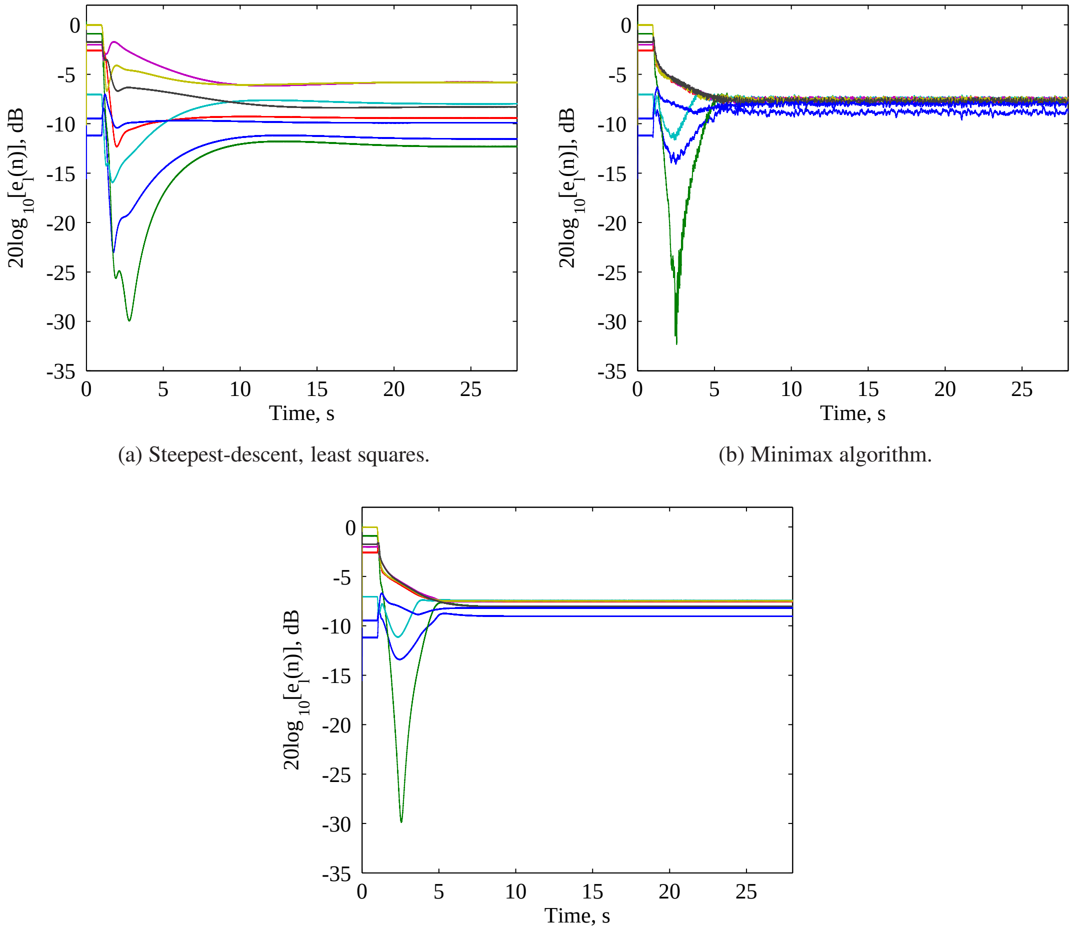
Figure 2 – The convergence of (a) the sum of the squared error signals and (b) the maximum squared error signal for the steepest-descent least squares algorithm (blue), the minimax algorithm (10) (black), and the proposed adaptive error weighting matrix steepest-descent algorithm (red).

Figure 2b shows the convergence of the maximum error signal for each of the three algorithms. From this plot it can be seen that, as expected, the minimax algorithm and the adaptive error signal weighting algorithm achieve a higher level of attenuation in the maximum error signal than the steepest-descent least squares algorithm. However, once again it is clear that the minimax algorithm suffers from the problems due to switching shown in the sum of the squared error convergence results.

An important property of the minimax algorithm is its ability to reduce the variation between the residual error signals. It has previously been highlighted by Elliott *et al* (9) that the adaptive error signal weighting method will only approximately minimise the infinity-norm of the error signals provided that p in equation 13, or in the case of the proposed method equation 14, is set to a large value. In the presented simulation results this parameter has been set to $p = 20$, however, it is important to verify that the algorithm has achieved the same performance as the minimax algorithm. Therefore, Figure 3 shows the convergence of the individual error signals for the three algorithms considered herein. From these results it can be seen that for the steepest-descent least squares algorithm shown in Figure 3a there is a 6.5 dB difference between the maximum and minimum error signal levels after convergence, while both the minimax algorithm and the adaptive error signal weighting algorithm, shown in Figures 3b and 3c respectively show only 1.5 dB variation in the levels between the maximum and minimum error signals.

To provide further insight into the behaviour of the minimax and adaptive error weighting algorithms, Figure 4 shows the variation of the diagonal elements of the $\mathbf{Q}$ matrix over the first 10 seconds for the two methods. From these results it can be seen that the elements of the $\mathbf{Q}$ matrix in the minimax algorithm are characterised by rapid switching, whereas for the adaptive error signal weighting algorithm the elements of the $\mathbf{Q}$ matrix can be seen to converge to a relatively constant value. For the minimax algorithm it can be seen that Q_1 is mainly zero and, correspondingly, the adaptive error weight is low. Conversely, for Q_5 the minimax algorithm shows a high density of switching between 0 and 1 and thus the adaptive error weight is relatively close to unity. This difference in behaviour helps to indicate why the minimax algorithm suffers from variations around the converged level.

The convergence conditions of the minimax and adaptive error weighting algorithms have been discussed in the previous section, although no formal proof of stability has been provided due to the complexity of studying such switched and time-varying controllers, although this intended for future work. Nevertheless, it is interesting to highlight how the convergence behaviour of the minimax and proposed adaptively weighted error signal algorithms differ for higher convergence gains. In general a higher convergence gain will provide



(a) Steepest-descent, least squares.

(b) Minimax algorithm.

(c) Adaptive error signal weighting.

Figure 3 – The convergence of the individual squared error signals for (a) the steepest-descent least squares algorithm, (b) the minimax algorithm (10), and (c) the proposed adaptive error signal weighting steepest-descent algorithm.

a more rapid convergence provided that the controller remains stable, but this is likely to be traded-off for a higher level of misconvergence. Figure 5 shows the convergence of the three algorithms when the convergence gain for each has been increased to achieve a similarly rapid initial convergence. From these results it can be seen that the steepest-descent least squares algorithm converges, in terms of both the sum of the squared errors and the maximum error, to almost identical levels to those presented in Figure 2 for the slower algorithm. The minimax and the adaptive error weighting algorithms, however, are more significantly affected by the increased convergence speed. The adaptive error signal weighting algorithm now has a similar level of variation to the slower minimax algorithm presented in Figure 2. Whereas the minimax algorithm now has such a large level of variation that the algorithm is unlikely to be of any practical use. For example, in an acoustic active noise control application the additional spectral components introduced by the algorithm would be clearly audible. This highlights the potential limitations of the minimax algorithm proposed in (10), which is acknowledged by the authors, but it also highlights the potential increase in convergence speed that can be achieved using the proposed adaptive error signal weighting method.

5. CONCLUSIONS

Active noise and vibration control not only provides the potential to increase the levels of practical noise or vibration reduction achievable compared to passive control techniques, but can also facilitate enhanced manipulation of the noise or vibration. In this area, previous work has investigated control algorithms that are able to provide a more spatially uniform response after control. In this paper it has been shown that the previously proposed minimax algorithm suffers from variations, or chattering about the convergence point due to the switching behaviour of the algorithm. To overcome this limitation, which becomes more signifi-

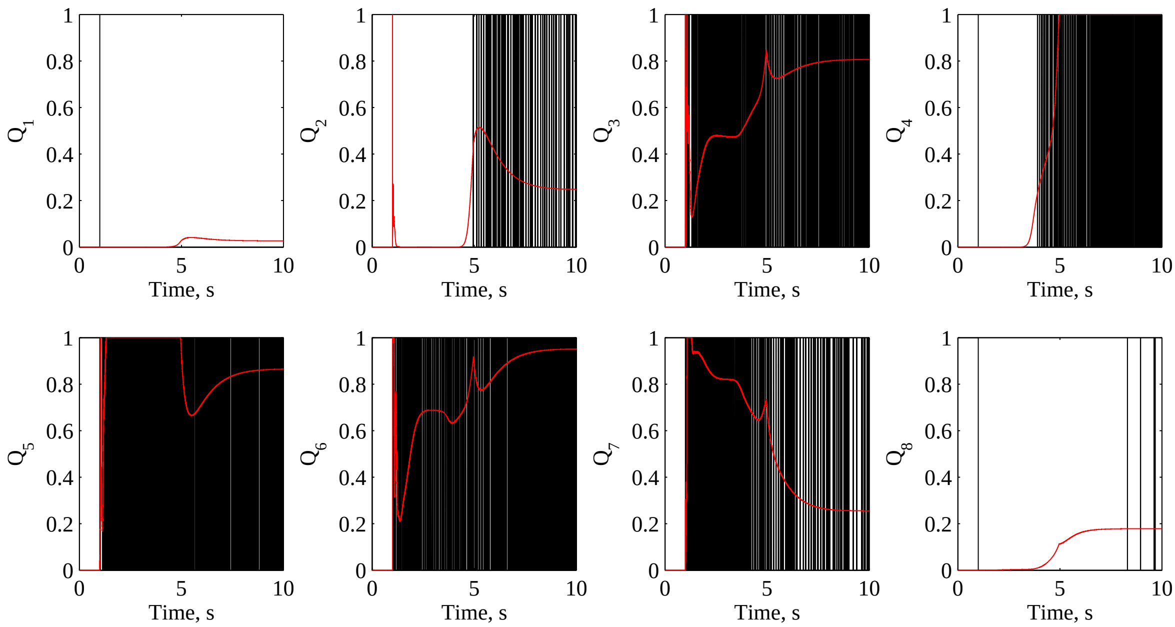
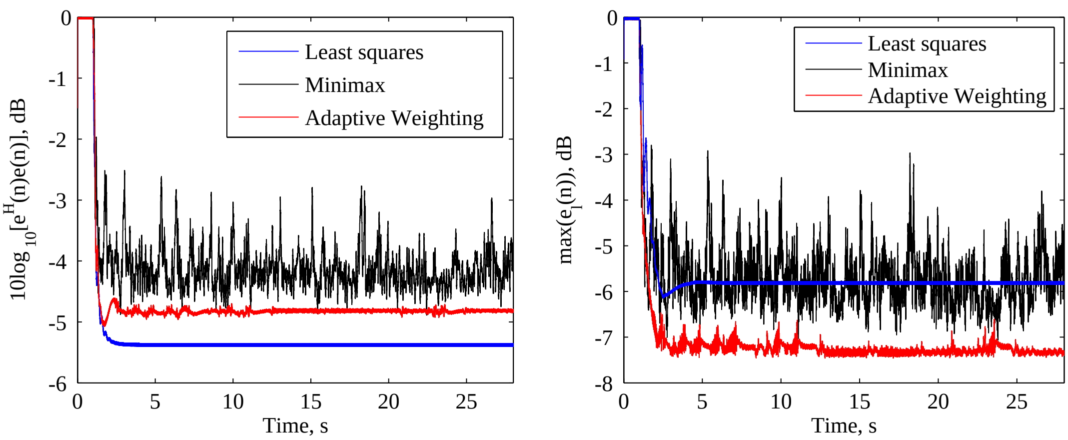


Figure 4 – The error sensor weighting matrix **Q** plotted over time for the minimax (black) and adaptive error signal weighting (red) algorithms.



(a) Sum of the squared errors signals.

(b) Maximum squared error signal at each iteration.

Figure 5 – The convergence of (a) the sum of the squared error signals and (b) the maximum squared error signal for the steepest-descent least squares algorithm (blue), the minimax algorithm (10) (black), and the proposed adaptive error weighting matrix steepest-descent algorithm (red).

cant under rapid convergence, a new method of adapting the error signal weighting matrix has been proposed. The proposed method adapts the diagonal elements of the error signal weighting matrix according to the p-th power of the magnitude of the error signals normalised by the maximum error signal magnitude; thus, the error weights are bounded between 0 and unity. It has been shown through a series of simulations employing measured transfer responses that the proposed error weighting strategy overcomes the slow convergence limitations of previously proposed error weighting strategies, which is a result of the weightings being bounded. It has also been shown that the proposed control strategy does not suffer the same levels of variation about the convergence point as the minimax algorithm. This means that the proposed strategy can achieve a faster convergence rate for the same level of error as the minimax algorithm. The stability and convergence properties of the minimax and the proposed algorithm have been discussed, however, further work is planned to provide a rigorous stability analysis.

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