Impedance matrix of rubber-cord fluid-filled hose

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ABSTRACT

One of the main hydraulic elements to reduce noise and vibration of pipelines is a rubber-cord hose, which consists of a composite shell and attachment flanges. Within the frames of a beam model dynamic behavior of pipeline elements is described with impedance matrix 14x14, which can be determined by calculation or experiment. In the current paper a theoretical beam model of fluid-filled hose is introduced, taking into account with orthotropic and viscoelastic properties of composite shell. Expressions for impedance matrix elements are presented in analytical form. Some elements are measured and good agreement between predicted and measured characteristics is shown.

Keywords: Impedance, Hose, Pipe: 11.3.3

1. INTRODUCTION

Beam model is widely used to predict vibration of a pipeline (1, 2, 3). In this model straight element of the pipeline is described by the matrix of impedances shown in Fig. 1.

		$q'_{x(in)}$	$q'_{y(in)}$	$q'_{z(in)}$	$\varphi'_{x(in)}$	$\varphi'_{y(in)}$	$\varphi'_{z(in)}$	$V_{x(in)}$	$q'_{x(out)}$	$q'_{y(out)}$	$q'_{z(out)}$	$\varphi'_{x(out)}$	$\varphi'_{y(out)}$	$\varphi'_{z(out)}$	$V_{x(out)}$
		V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}	V_{11}	V_{12}	V_{13}	V_{14}
$F_{x(in)}$	Q_1	$Z_{1,1}$						$-Z_{7,1}$	$-Z_{8,1}$						$Z_{14,1}$
$F_{y(in)}$	Q_2		$Z_{2,2}$				$Z_{6,2}$			$-Z_{9,2}$				$Z_{13,2}$	
$F_{z(in)}$	Q_3			$Z_{2,2}$		$-Z_{6,2}$					$-Z_{9,2}$		$-Z_{13,2}$		
$M_{x(in)}$	Q_4				$Z_{4,4}$							$-Z_{11,4}$			
$M_{y(in)}$	Q_5			$-Z_{6,2}$		$Z_{5,5}$					$Z_{13,2}$		$Z_{12,5}$		
$M_{z(in)}$	Q_6		$Z_{6,2}$				$Z_{5,5}$			$-Z_{13,2}$				$Z_{12,5}$	
$P_{(in)}$	Q_7	$-Z_{7,1}$						$Z_{7,7}$	$Z_{14,1}$						$-Z_{14,7}$
$F_{x(out)}$	Q_8	$Z_{8,1}$						$-Z_{14,1}$	$-Z_{1,1}$						$Z_{7,1}$
$F_{y(out)}$	Q_9		$Z_{9,2}$				$Z_{13,2}$			$-Z_{2,2}$				$Z_{6,2}$	
$F_{z(out)}$	Q_{10}			$Z_{9,2}$		$-Z_{13,2}$					$-Z_{2,2}$		$-Z_{6,2}$		
$M_{x(out)}$	Q_{11}				$Z_{11,4}$							$-Z_{4,4}$			
$M_{y(out)}$	Q_{12}			$Z_{13,2}$		$-Z_{12,5}$					$-Z_{6,2}$		$-Z_{5,5}$		
$M_{z(out)}$	Q_{13}		$-Z_{13,2}$				$-Z_{12,5}$			$Z_{6,2}$				$-Z_{5,5}$	
$P_{(out)}$	Q_{14}	$-Z_{14,1}$						$Z_{14,7}$	$Z_{7,1}$						$-Z_{7,7}$

Figure 1 – Impedance matrix of straight pipeline section

Impedance matrix model of hose based on equations of isotropic pipeline was developed in (4, 5). This model takes into account Poisson's fluid-structure interaction and isotropic material of the hose. Orthotropic and viscoelatic properties of the hose can be considered indirectly by fitting wavenumbers from experimental data (4, 6, 7, 8, 9). Three-parameter Kelvin–Voigt model for fluid vibration in viscoelatic hose was presented in (7, 10). In (11) expressions for impedance were derived for the orthotropic shell without viscoelastic properties and only longitudinal wave propagation was considered.

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Model of the longitudinal vibration taking into account fluid-structure, as well as anisotropic properties and features of shell reinforcement design, but do not take into account the viscoelastic properties was proposed in (12).

In this paper one-dimensional model of fluid-filled hose is developed taking into account its anisotropic and viscoelastic properties. Numerical result is compared with experimental data obtained by the method (13).

2. ONE-DIMENSIONAL MODEL OF HOSE

In (14) it was shown that in a limited frequency range orthotropic fluid-filled shell can be considered a one-dimensional approach. Thus, as in the steel pipe, can be considered only four types of waves – a plane wave in the fluid, longitudinal flexural and torsional waves in the shell. Hose flanges are considered as perfectly rigid bodies.

Reinforced shell of hose is generally a multilayer composite consisting of layers of matrix and fiber.). Formulas for the elastic constants of the composite in the principal directions of elasticity are given in (15). Shell has orthotropic if the cord is laid to the meridian at an angle in a cylindrical coordinate system.

Elastic parameters of the generalized Hooke's law in cylindrical coordinates for the shell with the fiber laying angle $\pm\theta$ are given by (16)

$$\begin{aligned}
B_{11} &= B'_{11}\cos^4\theta + 2(B'_{12}+2B'_{66})\sin^2\theta\cos^2\theta + B'_{22}\sin^4\theta \\
B_{22} &= B'_{11}\sin^4\theta + 2(B'_{12}+2B'_{66})\sin^2\theta\cos^2\theta + B'_{22}\cos^4\theta \\
B_{12} &= B'_{12} + \left[B'_{11}+B'_{22}-2(B'_{12}+2B'_{66})\right]\sin^2\theta\cos^2\theta \\
B_{66} &= B'_{66} + \left[B'_{11}+B'_{22}-2(B'_{12}+2B'_{66})\right]\sin^2\theta\cos^2\theta
\end{aligned} \tag{1}$$

where

$$B'_{11} = \frac{E_1}{1-\nu_1\nu_2},\quad B'_{22} = \frac{E_2}{1-\nu_1\nu_2},\quad B'_{66} = G,\quad B'_{12} = \frac{\nu_2 E_1}{1-\nu_1\nu_2} = \frac{\nu_1 E_2}{1-\nu_1\nu_2}.$$

Here E_1, E_2, ν_1, ν_2 are the elastic parameters (principal directions).

Frequency-dependent elastic modulus and loss factor of the matrix can be derived from rheological two-element Kelvin–Voigt model:

$$E_m^*(f) = E_m^0(\tau_{E,t,1} f^2 + \tau_{E,t,2} f + 1)/(\tau_{E,l,1} f + 1)$$

$$\eta_m^*(f) = \eta_m^0(\tau_{\eta,t,1} f^2 + \tau_{\eta,t,2} f + 1)/(\tau_{\eta,l,1} f + 1)$$

where E_m^0, η_m^0 are values of elastic modulus and loss factor at 0 Hz; f is the frequency; τ is the parameter. Losses is taken in consideration as complex elastic modulus (17)

$$E_m(f) = E_m^*(f)(1 + i\eta_m^*(f))$$

2.1 Longitudinal wave propagation

Longitudinal vibration can be described by following equation (12)

$$\frac{\partial^4 P}{\partial x^4} + \alpha\frac{\partial^2 P}{\partial x^2} + \beta\cdot P = 0 \tag{2}$$

$$\alpha = \omega^2(A\rho_f + S_{11}\rho_s),\quad \beta = \omega^4\rho_f\rho_s(AS_{11} - 2BS_{12}^2)$$

$$A = \frac{c_f^2}{\rho_f} + \frac{2RS_{22}}{h} + \frac{2S_{22}(1+h/R)}{(2+h/R)},\quad B = \frac{R^2\left((R+h)^2\ln(1+h/R) + (R+h/2)h\right)}{2h^2(R+h/2)^2}.$$

Here $S_{11} = B_{22}/B_{\#}$, $S_{22} = B_{11}/B_{\#}$, $S_{12} = -B_{12}/B_{\#}$, $S_{66} = 1/B_{66}$, $B_{\#} = B_{11}B_{22} - (B_{12})^2$; h is the wall thickness; R is the internal radius; ρ_s, ρ_f are the densities of shell and fluid; c_f is the velocity of sound in fluid; ω is the angular frequency.

The solution of equation (2) is

$$P = C_1 e^{i\lambda_1 x} + C_2 e^{-i\lambda_1 x} + C_3 e^{i\lambda_3 x} + C_4 e^{-i\lambda_3 x}$$

where $\lambda_{1,3}$ are wavenumbers.

$$\lambda_1 = \sqrt{\left(-\alpha + \sqrt{\alpha^2 - 4\beta}\right)/2}$$

$$\lambda_3 = \sqrt{\left(-\alpha - \sqrt{\alpha^2 - 4\beta}\right)/2}$$

Impedance matrix elements related with longitudinal vibrations with regard for boundary conditions are given by

$$Z_{1,1} = \frac{i\omega\rho_s S}{(\lambda_1\mu_3 - \lambda_3\mu_1)}\left(-\frac{\lambda_3\lambda_1^2 \cos(\lambda_1 l)}{\sin(\lambda_1 l/2)} + \frac{\lambda_1\lambda_3^2 \cos(\lambda_3 l)}{\sin(\lambda_3 l/2)} + A\omega^2\rho_f\left(\frac{\lambda_3 \cos(\lambda_1 l)}{\sin(\lambda_1 l/2)} - \frac{\lambda_1 \cos(\lambda_3 l)}{\sin(\lambda_3 l/2)}\right)\right) + i\omega m_x$$

$$Z_{8,1} = \frac{i\omega\rho_s S}{(\lambda_1\mu_3 - \lambda_3\mu_1)}\left(-\frac{\lambda_3\lambda_1^2}{\sin(\lambda_1 l)} + \frac{\lambda_1\lambda_3^2}{\sin(\lambda_3 l)} + A\omega^2\rho_f\left(\frac{\lambda_2}{\sin(\lambda_1 l)} - \frac{\lambda_1}{\sin(\lambda_3 l)}\right)\right)$$

$$Z_{7,1} = \frac{2iS_{22}\omega^3\rho_f\rho_s}{(\lambda_1\mu_3 - \lambda_3\mu_1)}\left(\frac{\lambda_3 \cos(\lambda_1 l)}{\sin(\lambda_1 l/2)} - \frac{\lambda_1 \cos(\lambda_3 l)}{\sin(\lambda_3 l/2)}\right)$$

$$Z_{14,1} = \frac{2iS_{11}\omega^3\rho_f\rho_s}{(\lambda_1\mu_3 - \lambda_3\mu_1)}\left(\frac{\lambda_3}{\sin(\lambda_1 l/2)} - \frac{\lambda_1}{\sin(\lambda_3 l/2)}\right)$$

$$Z_{7,7} = \frac{-i\omega\rho_f}{\pi R^2(\lambda_1\mu_3 - \lambda_3\mu_1)}\left(\frac{\mu_1 \cos(\lambda_3 l)}{\sin(\lambda_3 l/2)} - \frac{\mu_3 \cos(\lambda_1 l)}{\sin(\lambda_1 l/2)}\right)$$

$$Z_{14,7} = \frac{-i\omega\rho_f}{\pi R^2(\lambda_1\mu_3 - \lambda_3\mu_1)}\left(\frac{\mu_1}{\sin(\lambda_3 l)} - \frac{\mu_3}{\sin(\lambda_1 l)}\right)$$

where $\mu_1 = \lambda_1^3 - A\omega^2\rho_f\lambda_1$; $\mu_3 = \lambda_3^3 - A\omega^2\rho_f\lambda_3$; m_x is the mass of the flange without fluid; l is the length of elastic part of the hose.

2.2 Transverse wave propagation

Transverse vibration can be given by following equation:

$$B_{11}I_y\frac{\partial^4 w}{\partial x^4} + \omega^2\rho_e I_y\left(1 + \frac{B_{11}}{\chi B_{66}}\right)\frac{\partial^2 w}{\partial x^2} - \left(\rho S\omega^2 - \frac{\rho_e^2 I_y\omega^4}{\chi B_{66}}\right)w = 0 \tag{3}$$

where ρ_e is the effective density; I_y is the moment of inertia of shell cross-section; S is the area of shell cross-section; χ is the shear coefficient.

Effective density is given by

$$\rho_e = \rho_s + \frac{\rho_f}{(1 + h/R)^2 - 1}$$

According to (18) shear coefficient of orthotropic beam with ring cross-section is given by

$$\chi = \frac{3B_{11}(1-q^4)(1+q^2)/2}{B_{66}B_{12}(2q^6 + 18q^4 - 18q^2 - 2)/B_{22} - B_{11}(7q^6 - 27q^4 - 27q^2 - 7)/4}$$

where $q = R/(R+h)$.

The solution of equation (3) is

$$w(x) = C_1 \cos k_b x + C_2 \sin k_b x + C_3\ chk_b x + C_4\ shk_b x$$

Two waves propagate in the beam described by equation (3), but the second wave is inhomogeneous in low-frequency region. Wavenumber of the first wave is given by

$$k_b = \sqrt{\frac{2\omega^2 \rho_e}{B_{11}} \cdot \left[\sqrt{\left(1 - \frac{B_{11}}{4\chi \cdot B_{66}}\right)^2 + \frac{B_{11} S}{\omega^2 \rho_e \cdot I_y}} + \left(1 + \frac{B_{11}}{4\chi \cdot B_{66}}\right) \right]}$$

Impedance matrix elements related with transverse vibrations are given by

$$Z_{2,2} = \frac{iB_{11} I_y k_b^3}{4\omega} \left(C_1 \sin(k_b l) - C_2 \cos(k_b l) + C_3 \sinh(k_b l) + C_4 \cosh(k_b l)\right) + i\omega m_y$$

$$Z_{92} = \frac{iB_{11} I_y k_b^3}{4\omega} \left(-2C_2\right)$$

$$Z_{6,2} = \frac{iB_{11} I_y k_b^2}{4\omega} \left(-C_1 \cos(k_b l) - C_2 \sin(k_b l) + C_3 \cosh(k_b l) + C_4 \sinh(k_b l)\right) +$$
$$+ l_1 \frac{iB_{11} I_y k_b^3}{4\omega} \left(C_1 \sin(k_b l) - C_2 \cos(k_b l) + C_3 \sinh(k_b l) + C_4 \cosh(k_b l)\right)$$

$$Z_{13,2} = \frac{iB_{11} I_y k_b^2}{4\omega} \left(-2C_1 + 2 l_1 k_b C_2\right)$$

here $C_1 = -C_3 = (\cos(k_b l) - \cosh(k_b l)) / (2 - 2\cosh(k_b l)\cos(k_b l))$;

$C_2 = -C_4 = (\sin(k_b l) + \sinh(k_b l)) / (2 - 2\cosh(k_b l)\cos(k_b l))$;

l_1 is the length of the flange; m_y is the mass of the flange.

$$Z_{5,5} = -\frac{iB_{11} I_y k_b^2}{4\omega} \left(-C_1 \cos(k_b l) - C_2 \sin(k_b l) + C_3 \cosh(k_b l) + C_4 \sinh(k_b l)\right) -$$
$$- l_1 \frac{iB_{11} I_y k_b^3}{4\omega} \left(-C_1 \sin(k_b l) - C_2 \cos(k_b l) + C_3 \sinh(k_b l) + C_4 \cosh(k_b l)\right) + i\omega J_y$$

$$Z_{12,5} = \frac{2iB_{11} I_y k_b^2}{4\omega} \left(C_1 + l_1 k_b C_2\right)$$

here $C_1 = -C_3 = -\frac{l_1 k_b \left(\cos(k_b l) - \cosh(k_b l)\right) + \sin(k_b l) - \sinh(k_b l)}{2k_b \left(1 - \cosh(k_b l)\cos(k_b l)\right)}$;

$C_2 = -C_4 = \frac{-l_1 k_b \left(\sin(k_b l) + \sinh(k_b l)\right) + \cos(k_b l) - \cosh(k_b l)}{2k_b \left(1 - \cosh(k_b l)\cos(k_b l)\right)}$;

J_y is the moment of inertia of the flange.

2.3 Torsional wave propagation

Coloumb, Saint-Venant and Timoshenko beam theories give the same results for the ring cross-section (19). Therefore technical theory of rod is used to describe torsional vibration.

$$\frac{\partial^2 \phi}{\partial x^2} - k_t^2 \phi = 0$$

where

$$k_t = \sqrt{\frac{\omega^2 \rho_s}{B_{66}}}$$

here ρ_s is the density of rod.

$$Z_{4,4} = \frac{iI_x B_{66} k_t \cos k_t l}{\omega \sin k_t l} + i\omega J_x$$

$$Z_{11,4} = \frac{iI_x B_{66} k_t}{\omega \sin k_t l}$$

where I_x is the moment of inertia; J_x is the moment of inertia of flange without fluid.

3. EXPERIMENTS

To verify the analytical model, numerical results were compared with experimental data. The parameters of the test hose are shown in Table 1.

Table 1 – Parameters of hose

Parameter	Value	Parameter	Value
B_{11}, Pa	$1.64 \cdot 10^8$	m_x, kg	20.6
B_{22}, Pa	$4.2 \cdot 10^8$	m_y, kg	21.8
B_{12}, Pa	$2.54 \cdot 10^8$	l, m	0.7
B_{66}, Pa	$2.52 \cdot 10^8$	l_1, m	0.15
ρ_s, kg/m^3	1166	J_x, kg·m^2	0.109
ρ_f, kg/m^3	1000	J_y, kg·m^2	0.085
c_f, m/s	1400	E_m^0, Pa	$1 \cdot 10^7$
h, m	0.009	η_m^0	0.19
R, m	0.05	$\tau_{E,t,1}$; $\tau_{E,t,1}$; $\tau_{E,l,1}$	$1.64 \cdot 10^{-6}$; 0.012; 0.0082
		$\tau_{\eta,t,1}$; $\tau_{\eta,t,2}$; $\tau_{\eta,l,1}$	$3.5 \cdot 10^{-5}$; 0.1; 0.11

Measurements were performed using the method described in (13). Comparison of the measured data and numerical results is shown in Fig. 2.

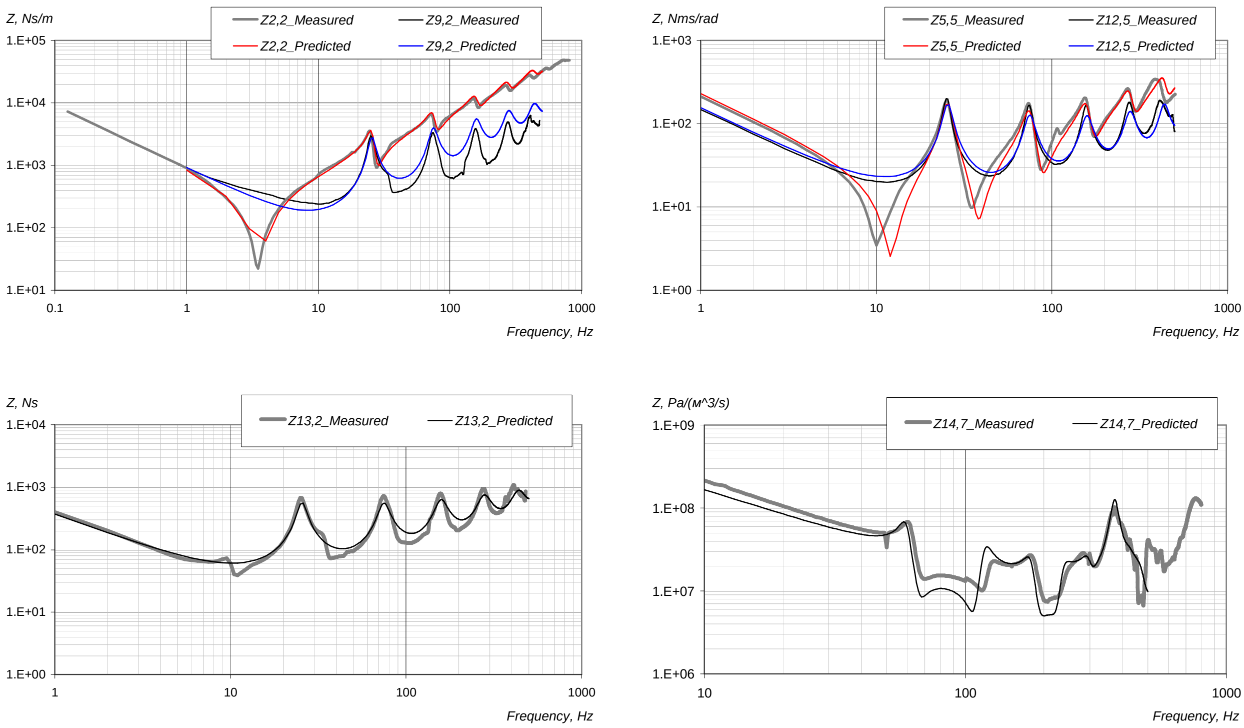

Figure 2 – Impedance matrix of straight pipeline section

4. CONCLUSIONS

One-dimensional model of the fluid-filled hose taking into account fluid-structure interaction, viscoelastic and orthotropic properties is introduced. Analytical expressions for elements of the impedance matrix based on this model are obtained. Comparison shows good agreement between experimental data and numerical predictions.

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