Individual Error Signal Design in Narrowband Active Noise Control Systems

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ABSTRACT

A narrowband active noise control (NANC) system is often applied to reduce undesired noise with multiple tones whose frequencies are close. Generally, an NANC system uses a single error signal to update a bank of adaptive filters configured in direct/parallel or parallel form. This work proposes a delayless filterbank to participate the frequency components of the error signal into individual error signals with the same frequencies components of the input signals to update the corresponding adaptive filters. A new adaptive algorithm based on the optimized performance index for the enhanced NANC system is also developed. Theoretical analysis is performed to demonstrate the increased convergence speed. Computer simulations are conducted to verify the analysis results and demonstrate the improved performance of the proposed NANC system.

Keywords: narrowband active noise control, adaptive filter, delayless filterbank, performance index
I-INCE Classification of Subjects Number(s): 37.7

1. INTRODUCTION

Rotating mechanisms periodically generate primary noise containing multiple harmonics in low frequency range. However, passive noise control uses sound-absorbing materials, which is only effective in canceling high-frequency noise. The ANC system generates a secondary noise with equal amplitude and opposite phase of the primary noise to cancel the undesired noise, which is especially suitable for reducing low-frequency noise [1] and is widely used for industrial applications. A NANC application often uses a mechanical devices such as a tachometer to synchronize internally generated reference noise for the adaptive filter [2]. Therefore, an NANC system does not require a reference microphone, thus avoids acoustic feedback. A second-order adaptive filter with the weights updated by the filtered-X least mean square (FXLMS) algorithm has been proven effective for an ANC system to cancel interference with only one harmonic [3]. Thus, most parallel NANC structures use several second-order adaptive filters connected in parallel to cancel the narrowband noise with numerous tones at the fundamental frequency and some dominant harmonic frequencies.

NANC systems include direct, direct/parallel and parallel forms. A direct form NANC structure uses one adaptive filter with the conventional FXLMS algorithm to cancel the undesired narrowband noise, but its performance degraded as the number of harmonics of undesired interference increased [1]. The NANC proposed by Glover used a sum of sinusoids as an input signal to an adaptive filter with a length much larger than two [6]. In this controller, convergence is low when the frequency separation between two neighboring harmonics is small. The direct/parallel form NANC system uses multiple reference signal generators to increase frequency separation at each channel [4]. Therefore, for each channel, frequency separation between two neighboring sinusoids is increased by separating various sinusoids into mutually exclusive sets. The convergence rate can be increased by increasing frequency separation [5]. However, since this technique uses the same error signal to update all adaptive filters, the performance of the NANC degraded by error signals from other channels. Several signal blocks with input signals along with sinusoidal waves in parallel were presented for NANC in [7]. An additional bandpass filter was used to decompose the output of each filter. However, it still used the same error signal to update all adaptive filters in the NANC system.

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In this paper, multiple second-order adaptive filters with modified cost function are derived for the proposed parallel structure NANC system. Besides, multiple delayless bandpass filters are also connected in parallel to split the error signal into corresponding channels according to the frequency components of input signals. Therefore, each second-order adaptive filter is designed to reduce only one harmonic and is updated by the corresponding individual error signal containing the same frequency components as the input signal. Therefore, the order of the adaptive filter and the complexity of the parallel NANC system are greatly reduced. The convergence speed of each adaptive filter is also examined by the modified FXLMS algorithm based on the upper bound of step size.

2. MODIFIED NANC SYSTEM DESIGN

2.1 Parallel forms NANC systems

A general parallel form NANC system separates the frequency components in the primary noise into several channels. Each channel contains only a single frequency, which can be canceled by a simple two-weight adaptive filter. Figure 1 shows the details of the m^{th} channel of a multiple-frequency NANC in parallel configuration. Suppose the undesired interference contains M sinusoidals at frequencies f_m, $m=1,\cdots,M$ and M second-order adaptive FIR filters $W_m(z)$ have two weights connected in parallel to attenuate these narrowband noise components. Let $d(n)$ denote the primary noise, $P(z)$ and $S(z)$ denote the primary and secondary paths of the NANC system, respectively, and $\hat{S}(z)$ denote the estimated secondary path. Each independent input signal $x_m(n)$ contains a single cosine or sine wave is generated from the information provided by a synchronization signal generator. That is, the m^{th} signal generator produces

$$x_m(n)=\cos(2\pi f_m n)\,, \qquad m=1,\cdots,M\,, \tag{1}$$

which is used as the reference input of the adaptive filter $W_m(z)$ with two weights. Each adaptive filter is connected in parallel, and the error signal is used to update the FXLMS algorithm. Figure 1 thus shows that canceling signal $u(n)$ is a sum of the output of the M adaptive filters

$$u(n)=\sum_{m=1}^{M}u_m(n)\,, \tag{2}$$

$$u_m(n)=w_{m,0}(n)x_m(n)+w_{m,1}(n)x_m(n-1)\,, \tag{3}$$

where $w_{m,0}(n)$ and $w_{m,1}(n)$ are the two weights of the m^{th} channel second-order adaptive filter $W_m(z)$ for $m=1,\cdots,M$. Thus, the NANC output signal is

$$u'(n)=\sum_{i=0}^{a-1}s_i\cdot u(n-i), \tag{4}$$

and the filtered signal used for the FXLMS algorithm is

$$x'_m(n)=\sum_{i=0}^{a-1}\hat{s}_i\cdot x_m(n-i), \tag{5}$$

where s_i and $\hat{s}_i$ are the coefficients of the $S(z)$ and $\hat{S}(z)$, respectively, and a is the respective filter lengths. The residual noise is $e(n)=d(n)-u'(n)$, and the mean square error (MSE) is

$$\psi(n)=E[e^2(n)]=E[(d(n)-u'(n))^2]\,, \tag{6}$$

where $E[\cdot]$ denotes the expectation operation. Generally, the conventional NANC system uses the FXLMS algorithm to update the two-weight adaptive FIR filters as:

$$w_{m,0}(n+1)=w_{m,0}(n)+2\mu e(n)x'_m(n) \quad \text{and} \quad w_{m,1}(n+1)=w_{m,1}(n)+2\mu e(n)y'_m(n)\,, \tag{7}$$

where μ is the step size and $y'_m(n)$ is in phase quadrature (90°) with $x'_m(n)$.

The reference signal $x(n)$, the primary noise $d(n)$ and the error signal noise $e(n)$ can then be expressed as multiple frequency components:

$$x(n)=\sum_{m=1}^{M}x_m(n)\,,\quad d(n)=\sum_{m=1}^{M}d_m(n)\,,\ \text{and}\quad e(n)=\sum_{m=1}^{M}e_m(n)\,, \tag{8}$$

where $x_m(n)$ has only one frequency component and $e_m(n)$ and $d_m(n)$ have the same frequency components associated with $x_m(n)$. Therefore, adaptive filter $W_m(z)$ determines the frequency components in its own input signal. However, the conventional correction term of adaptive filter $W_m(z)$ is the product of its input signal $x'_m(n)$ and error signal $e(n)$ (Eq. (7)). This conflicts with the main purpose of the parallel form NANC system, which is to split frequency components. Further, when only an $e_m(n)$ approaches zero and other components $e(n)$ do not, the other components cause interference in $e_m(n)$, which degrades the performance of the adaptive filter $W_m(z)$ and causes misalignments in the adaptive weights. Therefore, instead of Eq. (6), a new cost function is therefore proposed as

$$\psi'(n) = \sum_{m=1}^{M} E[e_m^2(n)] . \tag{9}$$

By minimizing the summation of M independent squared error signals, this cost function focuses on the square error summation of each set in different frequency components instead of on the overall squared error signal. Where the processing frequency in the adaptive filter $W_m(z)$ is f_m, the difference between $\psi(n)$ and $\psi'(n)$ is

$$\psi(n) - \psi'(n) = 2\sum_{\substack{i,j=1 \\ i \neq j}}^{M} E[e_i(n)e_j(n)] , \tag{10}$$

which contains only the crossover term with frequency components unrelated to f_m. Therefore, the new cost function is a reasonable performance index for adjusting the adaptive filter. When using the new performance index and gradient estimator, the adaptive FXLMS algorithm for the two-weight FIR filters is changed to

$$w_{m,0}(n+1) = w_{m,0}(n) + 2\mu e_m(n)x'_m(n) \quad \text{and} \quad w_{m,1}(n+1) = w_{m,1}(n) + 2\mu e_m(n)y'_m(n) . \tag{11}$$

In order to realize Eq. (11), a complete design of parallel form NANC system with a bank of delayless bandpass filters are shown in Fig.1. In Fig. 1, the filterbank $R_m(z)$ ($m = 1,\cdots,M$) is used to split the frequency component for each band, where

$$R_m(z) = \frac{\left(\dfrac{1-r_m^2}{1+r_m^2}\right)s_m z^{-1} - (1-r_m^2)z^{-2}}{1 - s_m z^{-1} + r_m^2 z^{-2}} , \tag{12}$$

$0 << r_m < 1$ determines the bandwidth of a bandpass filter and is typically chosen close to 1 to give a narrow bandwidth, and s_m ($-2r_m < s_m < 2r_m$) determines the center frequency of the passband. By properly selecting r_m and s_m, the filter $R_m(z)$ forms a passband at a given frequency $f_m = \dfrac{1}{2\pi}\cos^{-1}\left(\dfrac{s_m}{1+r_m^2}\right)$. Also, the filter $R_m(z)$ has unity gain and zero phase shift at the center of the passband [8]. Thus, the bandpass filter does not incur additional delay in the secondary path.

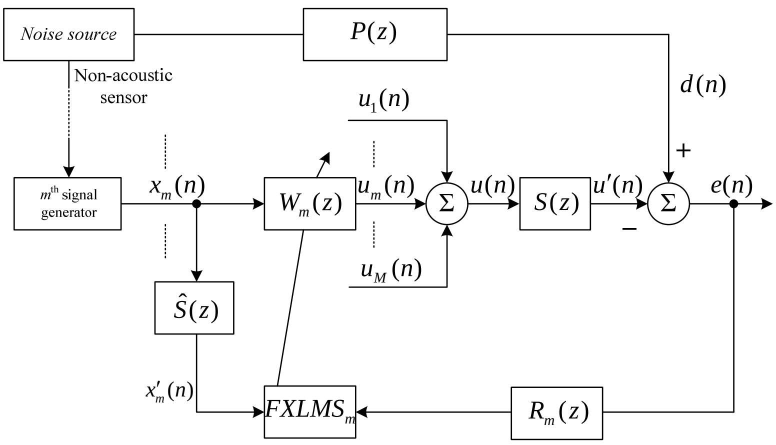

Fig. 1. Parallel form NANC system at m^{th} channel with delayless bandpass filter $R_m(z)$.

2.2 Adaptive Algorithm to Tune the Passband

In practical narrowband ANC applications, the tonal frequencies of primary noise are changing because of the varying speed of rotating machines. The center frequencies of the bandpass filters must be changed accordingly so the fixed parameter s_m defined in (12) becoming a time function $s_m(n)$. In this paper, we use the background adaptive algorithm for adapting $s_m(n)$ as shown in Fig. 3, where the reference signal $x_m(n)$ is used as both the desired signal and the input signal to the background $R_m(z)$. It is clear that the center frequency of adaptive filter $R_m(z)$ will converge to f_m to minimize the error signal $\varepsilon_m(n)$. The converged filter coefficient $s_m(n)$ is then copied to the foreground for filtering $e(n)$ to generate $e_m(n)$.

By setting the performance index as $\varepsilon_m^2(n) = \left(x_m(n) - y_m(n)\right)^2$, the adaptation of $s_m(n)$ using the steepest descent method can be expressed as

$$\begin{aligned} s_m(n+1) &= s_m(n) - \mu_m^{bp} \frac{\partial \varepsilon_m^2(n)}{\partial s_m(n)} \\ &= s_m(n) + 2\mu_m^{bp} \varepsilon_m(n) \alpha_m(n), \end{aligned} \tag{13}$$

where μ_m^{bp} is the step size for adapting $s_m(n)$, $\alpha_m(n) = \dfrac{\partial y_m(n)}{\partial s_m(n)}$, and

$$y_m(n) = \left(\frac{1-r_m^2}{1+r_m^2}\right) s_m(n) x_m(n-1) - \left(1-r_m^2\right) x_m(n-2) + s_m(n) y_m(n-1) - r_m^2 y_m(n-2). \tag{14}$$

Thus, the recursive algorithm for updating $\alpha_m(n)$ can be derived as

$$\alpha_m(n) = \left(\frac{1-r_m^2}{1+r_m^2}\right) x_m(n-1) + y_m(n-1) + s_m(n)\alpha_m(n-1) - r_m^2 \alpha_m(n-2). \tag{15}$$

The adaptation of $s_m(n)$ by the algorithm defined by (13)-(15) can tune the peak frequency of the background filter $R_m(z)$. As shown in Fig. 2, the converged $s_m(n)$ is copied to the foreground filter $R_m(z)$ for separating frequency components of the error signal $e(n)$ into corresponding channels, so each ANC filter $W_m(z)$ can be updated by the independent error signal $e_m(n)$ that contains only the narrowband residual noise with the same frequency as the reference signal.

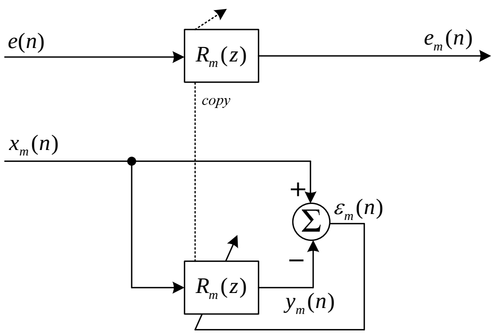

Fig. 2. Adaptive algorithm for tuning the passband of the bandpass filter.

2.3 Convergence Analysis

The conventional parallel narrowband ANC system uses the same error signal $e(n)$ that contains all the residual noise components to update all adaptive filters but the proposed algorithm separates the frequency components into corresponding independent channels. At each channel, the adaptive filter $W_m(z)$ handles only one frequency component f_m in the reference input signal $x_m(n)$ and the error signal $e_m(n)$. Therefore, the adaptation of m^{th}-channel adaptive filter is expressed as

$$\bar{\mathbf{w}}_m(n+1) = [\mathbf{I} - 2\mu_m^* \mathbf{R}_m] \bar{\mathbf{w}}_m(n) + 2\mu_m^* \mathbf{P}_m \tag{16}$$

and the misalignment vector becomes

$$\mathbf{V}'_m(n+1) = [\mathbf{I} - 2\mu_m^* \mathbf{\Lambda}_m] \mathbf{V}'_m(n) . \tag{17}$$

Because the MSE is a quadratic function of $\bar{\mathbf{w}}(n)$, the cost function $\xi_m(n)$ can be expressed as [3]

$$\xi_m(n) = \xi_{\min} + \mathbf{V}_m'^T \Lambda_m \mathbf{V}'_m \approx \xi_{\min} + \left(\lambda_0 \cdot (1-2\mu_m^* \lambda_{m,0})^{2n} v_0'^2(0) + \lambda_1 \cdot (1-2\mu_m^* \lambda_{m,1})^{2n} v_1'^2(0)\right). \tag{18}$$

Equation (18) converges to $\xi_{\min}$ as $n \to \infty$ according to the gradient estimation theorem. Thus, the step size has range of $0 < \mu_m^* < \frac{1}{\lambda_{m,l}}$ for $l = 0,1$. To guarantee a stable adaptive algorithm, the upper bound of step size is set as

$$0 < \mu_m^* < \frac{1}{\max(\lambda_{m,0}, \lambda_{m,1})}. \quad (19)$$

However, since $\max(\lambda_{m,0}, \lambda_{m,1}) \le \sum_{l=0}^{1} \lambda_{m,l} = 2\sigma_{x_m}^2$, the step size bound can be estimated as $0 < \mu_m^* < \frac{1}{2\sigma_{x_m}^2}$ Comparing the step sizes of conventional μ_m^c and proposed narrowband ANC systems, we have [9]

$$\frac{\mu_m^c}{\mu_m^*} \propto \frac{1}{(M-1)^2}. \quad (20)$$

Therefore, the proposed bandpass filters for the complete parallel ANC algorithm supports a much larger upper bound of step size as compared to the conventional algorithm, especially when M is large.

3. Simulation Results

Several performance tests were performed to evaluate the proposed parallel structure NANC system and to confirm its fast convergence results. The primary path $P(z)$ and the secondary path $S(z)$ in Fig. 1 are obtained from [9].

The first simulation tests the primary noise with three harmonics (M = 3). Three second-order bandpass IIR filters connected in parallel are applied to split the frequency components of the error signal. For the primary noise containing three sinusoids at 200 Hz, 250 Hz, and 300 Hz with the amplitudes equal to 1, the passbands of the corresponding bandpass filters, $R_m(z)$, m = 1, 2, 3, are centered at these frequencies to split the residual noise $e(n)$. The sampling rate is 1.5 kHz. The parameters used for these three bandpass filters $R_m(z)$ are $r_m = 0.95$ (for m = 1, 2, 3), $s_1 = 1.2730$, $s_2 = 0.9513$, $s_3 = 0.5879$. Figure 3 compares the learning curves obtained by three different ANC algorithms: the direct form with filter length of 10, the conventional parallel form, and the proposed parallel form. The respective step sizes for direct, conventional and proposed parallel forms are 0.125, 0.125 and 1. The simulation results confirm that all three algorithms can reduce narrowband noise, the residual noise amplitudes of the direct form, the conventional parallel structure, and the complete parallel from are about 0.1, 0.05 and almost 0 after 1000 iterations, respectively.

We consider the case that the primary noise containing eight harmonics ($M = 8$) with changing frequencies. Suppose the initial frequencies of the primary noise are 170 Hz, 180 Hz, 190 Hz, 200 Hz, 300 Hz, 310 Hz, 320 Hz and 330 Hz with amplitudes all equal to 1 and the initial bandpass filter parameters are $s_1(0) = 1.5772$, $s_2(0) = 1.5392$, $s_3(0) = 1.4992$, $s_4(0) = 1.4574$, $s_5(0) = 0.9513$, $s_6(0) = 0.8932$, $s_7(0) = 0.8340$ and $s_8(0) = 0.7738$ with $r_m = 0.95$ $(m = 1 \sim 8)$. Therefore, eight-channel narrowband ANC system is designed for the proposed parallel form. Consider engines as example, when the speeds of engines changed, the frequency of each harmonic will change accordingly. So, we increased the frequency of each harmonic by 10 Hz and increased 1.3 times of magnitudes after 3 seconds (5400^{th} iterations) to verify the performance of proposed parallel ANC system. The step sizes used for the conventional and proposed parallel forms ANC systems are 0.1 and 1 and the sampling rate is 1.8 kHz.

Figure 4 examines convergence speed in terms of parameters $s_m(n)$ that determines the peak frequency of the adaptive IIR bandpass filter. Since the primary noise frequencies changed, the passbands of the bandpass filters show the corresponding changes. The weights quickly converge to $s_1 = 1.5392$, $s_2 = 1.4992$, $s_3 = 1.4574$, $s_4 = 1.4138$, $s_5 = 0.8932$, $s_6 = 0.8340$ $s_7 = 0.7738$ and $s_8 = 0.7127$ for the frequencies equal 180 Hz, 190 Hz, 200 Hz, 210 Hz, 310 Hz, 320 Hz, 330 Hz and 340 Hz within hundreds iterations. Figure 5 depicts the learning curves by the conventional and proposed parallel ANC systems, which clearly show the much faster convergence speed achieved by the proposed parallel structure with bandpass filterbank. Comparison shows that convergence speed obtained by the proposed algorithm is faster than that of the conventional algorithm.

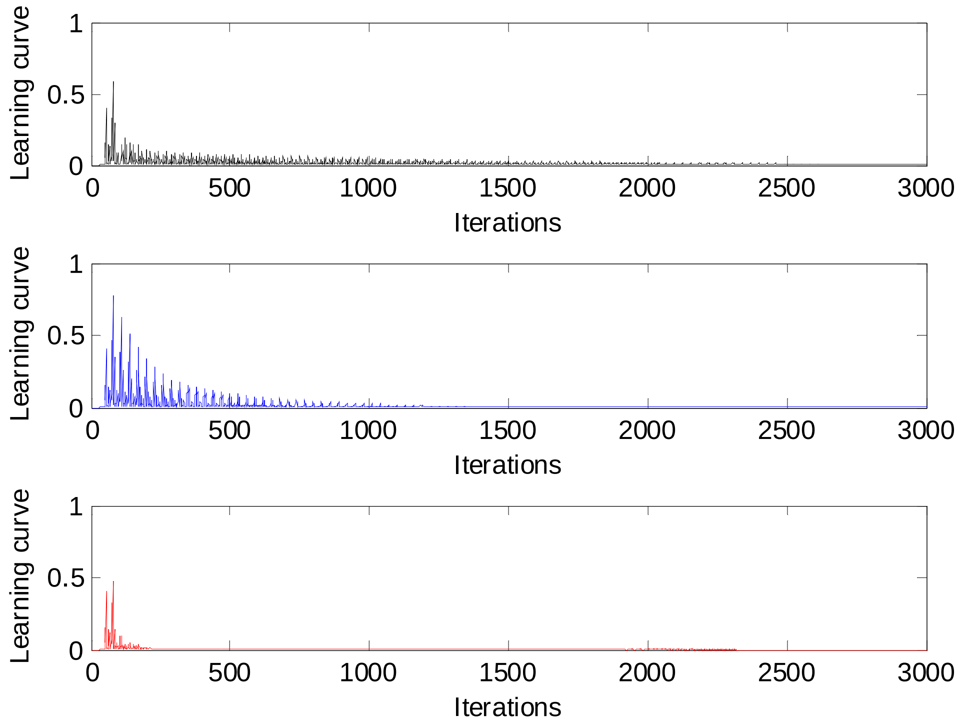

Fig. 3. Learning curves of three narrowband ANC algorithms for *M*=3: direct form ANC (upper), conventional parallel ANC (middle), and proposed complete parallel ANC (lower).

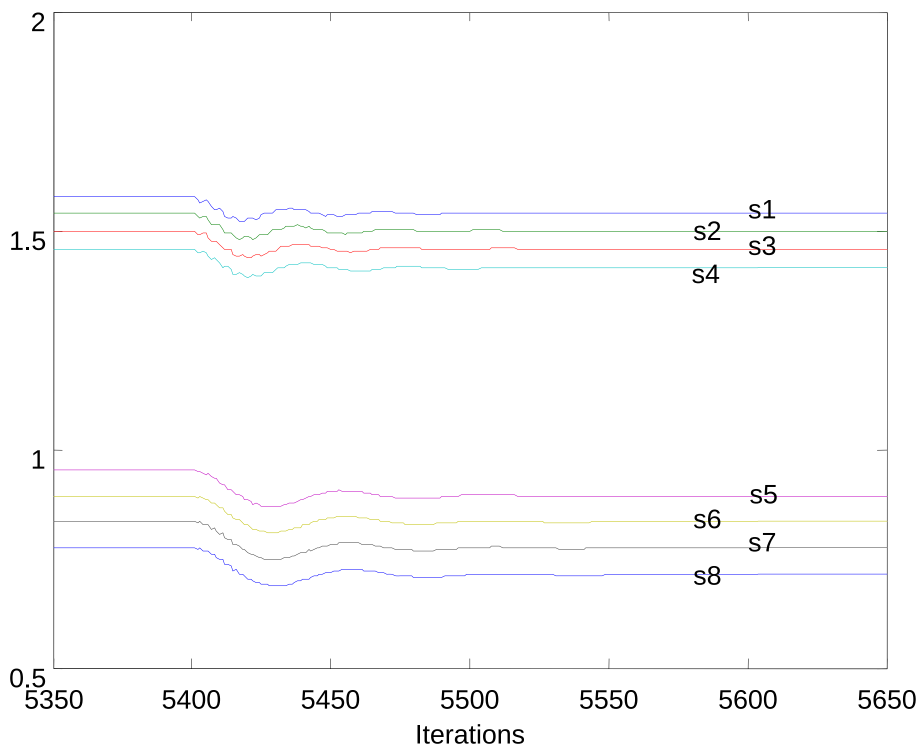

Fig. 4. Convergence of bandpass filter parameters $s_1 \sim s_8$.

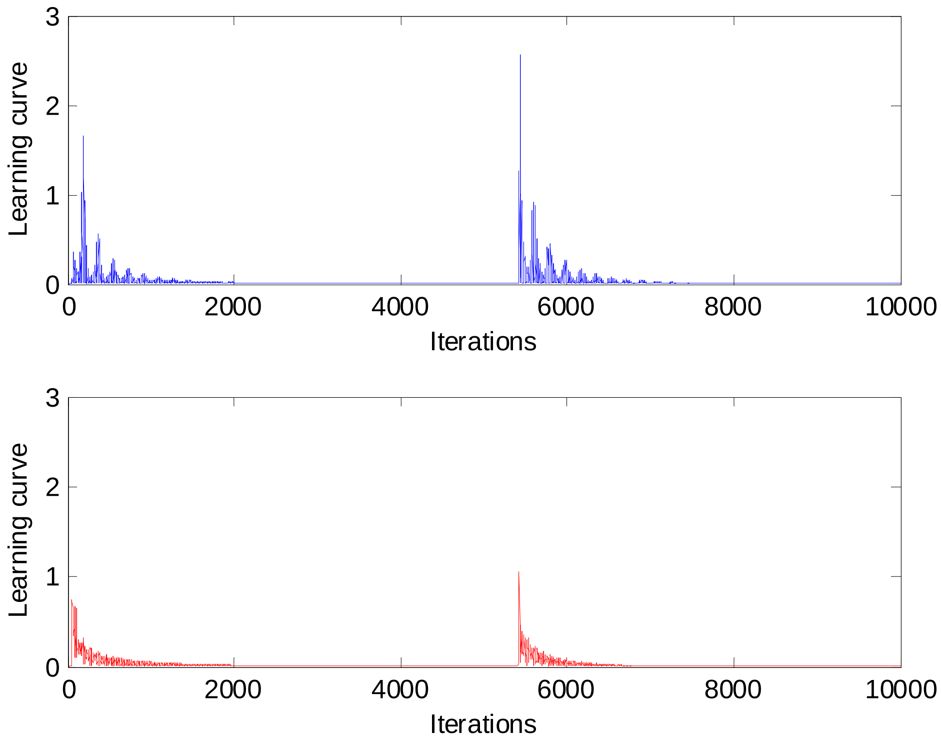

Fig. 5. Performance index (M=8), upper: conventional method, lower: proposed method.

4. CONCLUSIONS

A parallel-form NANC system was developed with a bandpass filter bank to split the frequency components of the error signal without adding delay. The new adaptive algorithm was derived based on the proposed cost function such that each adaptive filter can be tuned according to its frequency component to efficiently reduces noise. Comparison of the upper bounds of step sizes using theoretical analysis of convergence speed also confirmed that the proposed algorithm outperforms the conventional one.

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