

The spatial structure of an acoustic wave propagating through a layer with high sound speed gradient

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ABSTRACT

A plane acoustic wave propagating through a high sound speed gradient layer in a liquid near a pressure-release surface is considered in detail. Analogously to the solution for the refraction of the incident plane wave obtained previously, equations describing the refraction of the reflected wave are derived. For the total acoustic field in the layer, dependencies of the pressure and the vertical particle velocity on the depth and frequency are calculated for several incident grazing angle values. It is shown that these dependencies have distinct features close to the acoustic duct cut-on frequencies. Near these cut-on frequencies, both pressure and vertical velocity have strong maxima along the frequency axis, whereas along the vertical spatial axis the two variables have alternate minima and maxima, the number of which increases with frequency. It is suggested that the maxima in the vertical velocity near the surface at the cut-on frequencies are responsible for the sharp maxima in the grazing angle at the surface in the incident wave. The latter maxima were demonstrated previously. The presence of the pressure maxima and minima within the layer is confirmed by calculations using the RAMSurf parabolic equation simulation code.

Keywords: Plane wave, Refraction, Bubbles

I-INCE Classification of Subjects Number(s): 23.2

1. INTRODUCTION

The phenomenon of formation of an acoustic duct near the ocean surface under certain conditions is well-known. For example, such a duct appears if the water temperature is constant in a layer near the surface. Due to the effect of water column pressure, the sound speed increases linearly with increasing depth. The presence of such a duct may lead to substantial increase in propagation range.

Propagation of sound in such a duct is strongly influenced by the effects of wind blowing above the ocean surface. Effects of wind regarding underwater sound propagation are two-fold.

First of all, due to surface gravity waves induced by wind, the ocean surface becomes rough. When acoustic waves are reflected from the rough surface, the acoustic energy is scattered in different directions, thus significantly increasing the coherent acoustic energy loss. As a result, any accurate model of sound propagation in the duct should take the surface roughness into consideration.

Also, wind action leads to accumulation of air bubbles near the surface. These bubbles cause additional gradient in the wind speed, which may be considerable. For example, according to the model by Ainslie (1), at the wind speed of 10 m/s for the frequency of 2 kHz the sound speed changes by about 13 m/s within the top 3 m of the water column.

It is well-known that grazing angle of an acoustic wave at the ocean surface strongly affects the value of the acoustic energy loss due to the surface roughness. Therefore, to predict the surface reflection loss in the case of the rough surface, the grazing angle at the surface needs to be calculated. Due to the strong gradients of the sound speed in the bubbly layer the laws of geometrical acoustics become inapplicable there and, as a result, a wave-based propagation model should be used to calculate the grazing angle.

Jones et al. (2) suggested a model, the "JBZ" model, of the reflection loss from the rough ocean surface. The results of the calculations using the JBZ model showed good correlation with the results of Monte-Carlo simulations obtained with the use of a parabolic equation transmission code.

An integral part of the JBZ model is a wave-based theory of sound refraction in the layer filled with wind-induced bubbles. This theory is based on the formulation published by Brekhovskikh (3).

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Zinoviev et al. (4) presented the solution describing refraction of an incident plane acoustic wave within the bubbly layer with high sound speed gradient. It was shown that, in such a layer, the grazing angle at the surface of the ocean can differ considerably from the grazing angle below the layer (the incident grazing angle) thus strongly affecting the resultant surface reflection loss. The dependence of the grazing angle at the surface on the incident grazing angle was shown to have strong maxima at some frequencies for small incident grazing angles.

The main purpose of this paper is to extend the acoustic wave-based solution to the wave reflected from the ocean surface. Based on this solution and on the previously derived solution for the incident wave, spatial distributions of the total field in the duct are obtained. The maxima in the grazing angle at the surface observed by Zinoviev et al. (4) are investigated and their frequencies are compared with the cut-on frequencies of the modes of the duct formed within the layer with high sound speed gradient. The obtained spatial distribution of the total acoustic pressure in the layer is compared with the results of calculations with the use of the RAMSurf parabolic equation simulation code. Also, a spatial distribution of the acoustic field is obtained for the case where the reflection coefficient at the surface is less than unity.

Section 2 contains a description of the previous results for the incident wave obtained by Zinoviev et al. (4). Section 3 discusses the maxima of the surface grazing angle with respect to frequency for the incident wave. These frequencies are compared with the cut-on frequencies of normal modes of the duct formed within the layer with high sound speed gradient. A solution for the amplitudes of the pressure and velocity in the reflected wave is formulated in Section 4. Section 5 is devoted to an investigation of the pressure and velocity profiles obtained for the total acoustic field in the layer. The work of this paper is confined to pressure-release surface which is smooth.

2. PREVIOUS RESULTS FOR THE REFRACTION OF THE INCIDENT WAVE

2.1 Statement of the problem

Zinoviev et al. (4) considered the following problem. The acoustic medium is a fluid half-space with the sound speed $c_0 = 1500$ m/s far below the surface and with the equilibrium fluid density $\rho_0=1000$ kg/m³. The boundary of the half-space is considered to be pressure-release. In the fluid immediately below the surface the sound speed profile coincides with the sound speed profile of the transitional layer described by Brekhovskikh (3) and is determined by the following equation:

$$c(z) = c_0 \sqrt{\frac{1 - N}{1 - N \frac{e^{m(z + z_0)}}{1 + e^{m(z + z_0)}}}}, \quad z > 0,$$
(1)

where N and m are parameters of the layer. N characterises the "strength" of the layer, i.e. the difference between sound speed values on the surface and below the layer; and 1/m characterises the thickness of the layer. Zinoviev et al. (4) showed that the sound speed profile determined by Eq. (1) can be fitted with good precision to the sound speed profile in the bubbly layer as determined with the use of the model by Ainslie (1). The parameter z_0 is determined from the fitting procedure.

It is important to note that, in this paper, the emphasis is placed on the investigation of characteristics of acoustic wave refraction in the layer under consideration and the effects of the ocean surface roughness on sound refraction within the layer are not considered. The ocean surface is considered to be smooth and the layer is assumed to be horizontally uniform. Investigation of the influence of the surface roughness on the sound speed profile and on sound wave refraction is considered to be outside the scope of this work.

The incident wave is a plane acoustic wave with wavevector k_0 approaching the layer from $z \rightarrow \infty$. The incident grazing angle, i.e. the angle between the wavevector and the horizontal axis at the lower boundary of the layer, is θ_0 . The propagation of such a wave through the layer is governed by the following Helmholtz equation, which is derived from the wave equation for the acoustic pressure, p, in layered inhomogeneous media (3, 4):

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} + k^2 (z) p = 0.$$
⁽²⁾

Based on the formulation by Brekhovskikh (3), Zinoviev et al. (4) provided a solution to Eq. (2) for the acoustic pressure and velocity in the incident wave within the layer with the sound speed profile described by Eq. (1). The solution is exact within the current assumptions, and, unlike the geometrical

acoustics, takes into account wave effects in the layer.

2.2 Grazing angle at the surface

The grazing angle at the surface, θ_s , was defined as the angle between the direction of the energy flux vector, **q**, and the horizontal axis. The vector **q** is determined as follows:

$$\mathbf{q} = 0.5 \operatorname{Re}(p\mathbf{v}^*). \tag{3}$$

where \mathbf{v}^* is the complex conjugate of the fluid particle velocity vector.

Zinoviev et al. (4) used the solution for the acoustic pressure to calculate the dependence of the grazing angle θ_s on the acoustic frequency f and the incident grazing angle θ_0 . Figure 1, which was first published in Ref. (4), shows the difference $\theta_s - \theta_{sn}$, where θ_s is the grazing angle at the surface obtained using the solution of Eq. (2) and θ_{sn} is the grazing angle obtained using Snell's law of geometrical acoustics. In Figure 1, the wind speed at 19.5 m height above the ocean surface, $w_{19.5}$, is 12.5 m/s.



Figure 1 - Difference between the grazing angles at the surface calculated with the use of the solution of Eq. (2) obtained in Ref. (4) and with Snell's law for the layer with high sound speed gradient; wind speed

 $w_{19.5} = 12.5 \text{ m/s}$

It is clear from Figure 1 that, at larger incident grazing angles and higher frequency, the incident wave within the layer is governed by Snell's law. At smaller incident grazing angles and lower frequencies, the grazing angle at the surface can be either larger or smaller than that predicted by Snell's law. It is also seen that resonance-like maxima in the grazing angle at the surface can reach tens of degrees in their value whereas the incident grazing angle at the bottom of the layer is less than 1°. This difference in the grazing angles means that wave phenomena play a significant role in the layer with high sound speed gradient. Investigation of these resonances is one of the main objectives of this work.

3. MAXIMA IN THE GRAZING ANGLE AT THE SURFACE AND THE CUT-ON FREQUENCIES OF THE DUCT MODES

The layer with high sound speed gradient near the ocean surface constitutes an acoustic duct. The cut-on frequency, f_j , of a duct-associated normal mode of order j is determined by the following equations (5):

$$f_{j} = c_{0}/\lambda_{j}, \quad \lambda_{j} = \frac{2\sqrt{2}}{j - 1/4} \int_{0}^{H} \sqrt{N(z) - N(H)} dz, \quad j = 1, 2, ...,$$
(4)

where *H* is the duct depth and $N(z) = c_0/c(z)$ is the index of refraction. Table 1 contains the frequencies of all four resonances showed in Figure 1 calculated for the incident grazing angle of 0.1° as well as the cut-on frequencies of the duct modes calculated using Eqs. (4).

Mode numbers	Cut-on frequencies, Hz	Resonances in θ_s , Hz
1	1806	1772
2	4214	4196
3	6622	6605
4	9030	9011

Table 1 – Comparison of the cut-on frequencies of the duct modes with the maxima of θ_s in Figure 1

Table 1 shows that the frequencies of the resonances in θ_s observed in Figure 1 are very close to the cut-on frequencies of the duct modes. Therefore, it may be suggested that the resonances are caused by excitation of these modes. The small difference in these frequencies can be explained by the fact that Eq. (4) is derived using ray acoustics and, therefore, ignores wave effects in the layer. The wave effects, like the very large grazing angle at the surface shown in Figure 1, may increase the path length along which sound travels between consecutive reflections from the surface, thus leading to larger wavelengths and smaller frequencies for the normal modes.

4. FORMULATION OF THE SOLUTION FOR THE REFLECTED WAVE AND THE TOTAL ACOUSTIC FIELD

The following equations describe the depth-dependence of the acoustic pressure, p, and of the horizontal and vertical components of the particle velocity, v_x and v_n respectively, for the wave reflected from the surface. These are obtained analogously to the derivation of the equations for refraction of the incident wave (4):

$$\xi = -\mathrm{e}^{mz},\tag{5}$$

$$p(\xi) = (1 - \xi) F(\xi), \tag{6}$$

$$F\left(\xi\right) = \sum_{n=0}^{\infty} f_n\left(\xi\right),\tag{7}$$

$$f_0(\xi) = \xi^{i\delta - 1},\tag{8}$$

$$f_{n+1}(\xi) = f_n(\xi) \left(1 + \frac{\mu^2}{(n+1)(n-2i\delta+1)} \right) \frac{1}{\xi},$$
(9)

$$\delta = \frac{k_0}{m} \sin \theta_0, \tag{10}$$

$$\mu^2 = N k_0^2 / m^2 (1 - N), \tag{11}$$

$$v_x(\xi) = p(\xi) \cos\theta_0 / \rho_0 c_0, \qquad (12)$$

$$v_n(\xi) = \frac{\mathrm{i}}{\omega \rho_0} \left(-m\xi F(\xi) + (1-\xi)\frac{\partial F(\xi)}{\partial z} \right),\tag{13}$$

$$\frac{\partial F\left(\xi\right)}{\partial z} = \sum_{n=0}^{\infty} s_n\left(\xi\right),\tag{14}$$

$$s_0(\xi) = m(i\delta - 1)\xi^{i\delta - 1},\tag{15}$$

$$s_{n+1} = s_n \left(\xi\right) \left(1 + \frac{\mu^2}{(n+1)(n-2i\delta+1)}\right) \left(1 + \frac{1}{1-i\delta+n}\right) \frac{1}{\xi}.$$
 (16)

It may be noted that the equations describing the incident wave can be obtained from Eqs. (5) to (16) by changing the sign in front of the terms containing $i\delta$.

As the ocean surface is assumed to be pressure-release, the calculation of the total acoustic field requires the pressure in the incident and reflected waves on the ocean surface to be taken into account with the same amplitude and opposite signs.

5. DEPENDENCE OF THE TOTAL ACOUSTIC FIELD ON DEPTH AND FREQUENCY

5.1 Pressure vs. depth and frequency

Figure 3 shows the dependence of the pressure amplitude in the total acoustic field on frequency and depth for several incident grazing angle values. The pressure is normalised by the amplitude of the pressure in the incident wave. For comparison, one of the pictures is calculated for zero wind speed, i.e. for an isovelocity fluid half-space. The calculations are carried out for the wind speed, $w_{19,5}$ =12.5 m/s. Figure 2 shows the sound speed profile corresponding to this wind speed value. It is also clear from this Figure that the layer depth at this wind speed is close to 5 m.





Ainslie procedure (1); dashed line: the fitted SSP calculated using Eq. (1)

The following features can be observed in Figure 3.

- There is zero pressure at the surface due to the pressure-release boundary condition.
- At very small incident angles (Figures 3a and 3b) there are maxima of pressure within the considered sound speed profile at the cut-on frequencies of the duct modes shown in Table 1. The pressure in the layer is close to zero at non-resonance frequencies due to large vertical wavelength and the pressure release boundary conditions at the surface.
- At very small angles there are also clear maxima and minima with respect to depth, which can be explained by the spatial structure of the duct modes.
- At larger angles ($\theta_0=0.5^\circ$ and 1° , Figures 3c and 3d), the modal structure is still visible but becomes less so due to the interference between the incident and reflected waves which occurs regardless of the high sound speed gradient.
- At $\theta_0=3^\circ$, the field structures within the layer with high sound speed gradient and in the isovelocity ocean (Figures 3e and 3f respectively) are similar but still different, as there are, for example, 7 maxima above the depth of 7 metres at f=10 kHz in the presence of high sound speed gradient and only 5 maxima for the case of the isovelocity ocean.



Figure 3 - Acoustic pressure amplitude for the total field in the layer with high sound speed gradient as a function of frequency and depth for several incident grazing angle values. The pressure is normalised by the pressure amplitude in the incident wave below the layer; a) - e): the wind speed $w_{19.5} = 12.5$ m/s, f): $w_{19.5} = 0$ (isovelocity ocean). The vertical dashed lines in Figure 3d) correspond to the frequencies considered in

Section 5.4

5.2 Vertical velocity vs. depth and frequency

The dependence of the amplitude of the vertical component of particle velocity on depth and frequency is shown in Figure 4.



Figure 4 - Amplitude of the vertical component of particle velocity for the total field in the layer with high sound speed gradient as a function of frequency and depth for several incident grazing angle values. The particle velocity is normalised by the vertical component of particle velocity in the incident wave below the

layer; a) - e): the wind speed $w_{19.5} = 12.5$ m/s, f): $w_{19.5} = 0$

Below are some characteristics of the distribution of the vertical component of particle velocity shown in Figure 4.

- There is a maximum of the vertical velocity at the ocean boundary due to the pressurerelease boundary condition.
- At small incident angles (Figures 4a and 4b) there are maxima in the vertical velocity at the cut-on frequencies of the duct normal modes. The field structure corresponds to the spatial structure of each mode. The values of the velocity at these frequencies are much larger than the velocity in the incident wave (the latter value is small due to small incident grazing angle). The maxima exist only within the layer with high sound speed gradient (up to the

depth of 5 m).

- These maxima may be related to the large grazing angle at the surface in the incident wave at the cut-on frequencies shown in Figure 1.
- With increasing grazing angle the zones near the maxima become wider in frequency.
- At $\theta_0=3$ degrees (Figure 4e) all maxima corresponding to modes of order 2 and higher merge. The resulting maximum of the vertical velocity at the surface for these modes is caused by the refraction governed by Snell's law.
- The maximum corresponding to the first cut-on frequency exists for all incident grazing angles under consideration. The wave effects for this maximum still play a significant role at these grazing angles.
- As expected, at $w_{19.5} = 0$, (i.e. in the isovelocity ocean, Figure 4f) there are no maxima within the layer associated with sound refraction. The field structure is determined by interference of the incident and reflected waves.

5.3 The total field in the case of non-zero reflection loss at the surface

Figures 3 and 4 have been obtained with the assumption that there is no energy loss during reflection from the surface, which means that the amplitudes of the incident and the reflected waves are equal at the surface. Figure 5 below shows the amplitudes of the pressure and of the vertical component of particle velocity for the total field for the reflection coefficient of 0.5, which means that the corresponding surface reflection loss is 6 dB. The pressure in the incident and reflected waves at the surface remain in opposite phase. It is clear from Figure 5 that the presence of these maxima within the layer at the cut-on frequencies of the duct modes does not depend strongly on the reflection coefficient at the surface. Although not shown here, it may be noted that the maxima disappear when the reflection coefficient approaches zero, as the maxima are the result of interference of both incident and reflected waves affected by the refraction within the layer.



Figure 5 - The amplitude of the pressure (a) and of the vertical component of particle velocity (b) for the total field in the layer with high sound speed gradient as a function of frequency and depth for the reflection coefficient of 0.5. The phase difference between the pressure in the incident and reflected waves at the

surface remains 180°.

5.4 Comparison of the above results with a parabolic equation solution

The pictures of the distribution of the total pressure in the layer shown in Figure 3 have been compared with the results of calculations of the transmission loss (TL) obtained with the use of the parabolic equation transmission code RAMSurf (6) (Figure 6). The calculations have been carried out for a deep ocean with an isothermal mixed layer of 64 m depth. Superimposed on this layer is the sound speed profile corresponding to the wind speed of 12.5 m/s, which is considered in this paper. The source is located at 6m depth.

If Snell's law is applied to wave refraction in the isothermal layer, it can be shown that the grazing angle at the lower boundary of the layer with high sound speed gradient can be no larger than 2.1°, but, at the same time, it can be smaller than this value. Taking this into consideration, the spatial

distributions of TL presented in Figure 6 are compared with the spatial distribution of the acoustic pressure for the incident grazing angle of 1°, which is shown in Figure 3d.

Figure 6a displays the TL distribution for the frequency of 3.2 kHz. It can be seen from Figure 3d that this frequency lies between the cut-on frequencies of the normal modes of the duct formed in the layer with high sound speed gradient. At this frequency, an interference pattern due to sound radiated in different directions by the point source is clearly seen in Figure 6a. The pressure profile in Figure 3d at this frequency is marked by the vertical white line labelled "1". This profile has a maximum at about 1m depth and a minimum at about 2 m depth. The corresponding maximum and minimum of the acoustic pressure in Figure 6a are located at similar depths.

Figure 6b shows the TL distribution for the frequency of 4.2 kHz, which corresponds to the cut-on frequency of the second duct mode in Figure 3d (the vertical white line "2"). At this frequency, the vertical pressure profile has maxima at depths of about 0.5 m and 3 m and a minimum at about 1.5 m, which can be seen in both figures. Contrary to Figure 6a, Figure 6b does not show any significant variations of pressure along the horizontal axis, which can be explained by the prevalence of the resonant second mode within the layer.

Figures 6c and 6d are for the frequencies of 5.2 kHz and 6.6 kHz respectively. The former frequency is between the cut-on frequencies of the duct modes, while the latter one corresponds to the cut-on frequency of the third duct mode. Figure 6c shows maxima in the acoustic pressure at depths of about 0.5 m and 2 m and minima at depths of about 1 m and 3 m. In Figure 6d, two maxima are located at approximate depths of less than 0.5 m and 1.5 m and another wide maximum is at a depth of about 4 m. The corresponding vertical pressure profiles in Figure 3d are marked by white vertical lines "3" and "4" respectively. It can be clearly seen that maxima and minima at both frequencies are located at similar depths. Analogously to Figures 6a and 6b, Figure 6c shows significant variations of pressure along the horizontal axis, whereas Figure 6d does not at least for the first three kms.



Figure 6 - Transmission loss in the layer with high sound speed gradient corresponding to wind speed of 12.5 m/s superimposed on an isothermal mixed layer 64 m deep for different frequencies. The figures are obtained with the use of the parabolic equation code RAMSurf

6. SUMMARY

In this paper, an exact solution of the wave equation in a layer with large sound speed gradient similar to that caused by wind induced air bubbles is obtained for the wave reflected from the ocean surface. Together with a previously obtained solution for the incident wave, this solution describes the total acoustic field in the layer.

With the use of the obtained solution for the total field, dependencies of the amplitudes of the acoustic pressure and of the vertical component of the particle velocity on frequency and depth are calculated. It is shown that, at the frequencies close to cut-on frequencies of the normal modes of the acoustic duct formed within the layer with high sound speed gradient, the pressure and velocity amplitudes are much larger than those at other frequencies and the vertical profiles of these amplitudes have maxima and minima the number of which depends on the mode order. It is also shown that the presence of absorption at the surface cannot significantly affect the presence of these maxima and minima.

It is shown that the vertical component of the velocity in the total field can reach high values at and near the cut-on frequencies, which correlates with the previous result that the grazing angle at the surface in the incident wave can become as large as tens of degrees at these frequencies.

It is also demonstrated that the refraction within the layer with high sound speed gradient determines wave propagation if the grazing angle at the lower boundary of the layer is about 3° or smaller. For the layer studied, at larger grazing angles the influence of the refraction on the pressure and velocity profiles within the layer decreases.

The obtained vertical pressure profiles are compared with the transmission loss profiles calculated using the parabolic equation transmission code RAMSurf. It is shown that RAMSurf predicts the maxima and minima of the acoustic pressure amplitude at similar depths as the solution presented in this paper.

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