



Research on vibration and sound radiation characteristics of ship stiffened composite plate structure

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ABSTRACT

Based on Statistical Energy Analysis method, the vibro-acoustic characteristics of stiffened composite plates are investigated in this paper. First, the vibration theoretical model of the stiffened composite plate is developed by using the Hoff sandwich plate theory. The method to calculate the modal loss factor of stiffened composite plate structure is proposed. Then the influence of damping layer thickness, elastic modulus and constraint layer elastic modulus on the vibro-acoustic characteristics of the stiffened composite plate has been studied. The results show that the above three parameters have a great impact on the structural vibro-acoustic characteristics. On this base, the vibro-acoustic characteristics of the cabin segment in composite are investigated. The influence of different constraint damping structure parameters and laying positions on the ship vibration radiation are analyzed. Results show that the laying location of constrained layer damping (CLD) has a great impact on the acoustic radiation reduction. The constrained layer damping laid in the excitation source place can achieve better acoustic radiation reduction.

Keywords: Acoustic Radiation, Modal Loss Factor, Statistical Energy Analysis I-INCE Classification of Subjects Number(s): 47.2

1. INTRODUCTION

Stiffened composite plate is one of the main methods adopted by reducing the ship vibration noise, which now is widely used in ship design and manufacture process. Noise can be spread by solid structure. By adopting vibration isolation, sound insulation and other measures, sound source vibration can be controlled and the spread of vibration in the solid structure can be prevented to reduce noise. In order to reduce strength of ship vibration noise, hull structure surface is usually attached a damping layer and constraining layer to form composite plate frame structure, which is named as constrained layer damping (CLD) structure. It is great effective to reduce the hull vibration and noise. About constrained damping structure, Ker win[1] applied the middle add damping layer of three layer board structure by using sine function to represent the transverse displacement of beam. At the same time, the form of complex stiffness is adopted to express beam bending stiffness. The effect on reducing vibration of damping layer of multilayer composite structure was studied. For highly nonlinear and complexity of damping structural vibration, the approximate response surface method was adopted for vibration frame acoustic radiation parameters by Xinhua Liang, who mainly considered the mean square of the velocity, acoustic radiation power and system parameters. The acoustic radiation along with the change trend of the design parameters was got[2]. Zhengxing Wang studied the characteristic of viscoelastic damping structure materials and optimized calculation[3]. Active CLD vibration control was studied by Tianxiong Liu, via using Hamilton principle to establish the kinetics equations of constrained damping layer board modeling and the seven degrees of freedom to deal with any node. Standard second order constant linear model was got[4]. Hoff sandwich plate theory was used by Zhongze Guo to study fast and efficient calculating the natural frequency and loss factor of CLD structure. It is presented that structure natural frequency and modal loss factor, which

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is calculated by Hoff sandwich plate theory, have high precision. The theory can be used to calculate natural frequency and loss factor of CLD structure [5].

Because today there is a lack of the research of vibro-acoustic characteristics of the stiffened composite plate, which are based on the Statistical Energy Analysis(SEA) method. In this paper, the method to calculate the modal loss factor of stiffened composite plate structure is proposed, using the Hoff sandwich plate theory, and the vibration theoretical model of the stiffened composite plate is developed. Then the influence of damping layer thickness, elastic modulus and constraint layer elastic modulus on the vibro-acoustic characteristics of stiffened composite plate has been studied. On this base, the vibro-acoustic characteristics of the cabin model laid CLD are investigated. The influence of different constraint damping structure parameters and laying CLD positions on the ship acoustic radiation are analyzed. It is reported that the laying location of CLD has a great impact on the vibration noise reduction.

2. BASIC THEORY

Loss factor refers to the ratio of the losing energy and the average store energy of the system in unit of frequency and time, which changes with damping layer thickness, elastic modulus and constraint layer elastic modulus, etc. By getting the corresponding modal loss factor of structure under different parameters, a calculation method of high efficiency and accuracy is proposed in this research.

2.1 The Method to Calculate the Modal Loss Factor of Composite Plate Structure

CLD plays an important role in improving the structure vibration resistance, stability, and reducing noise. CLD structure generally includes base layer, damping layer and constraint layer. Its structural form is shown in figure 1.

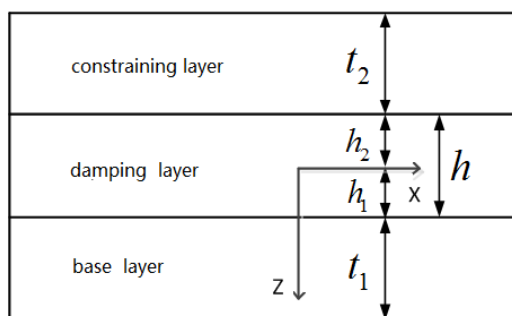


Figure 1 – The constrained damping structure profile sketch

where t_1 denotes base layer thickness, while t_2 is constraint layer thickness. The material of base and constraint layer is identical. E , μ and ρ_f are Elastic modulus, Poisson's ratio and density, respectively. h and ρ_c are damping layer thickness and damping layer density. In practical analysis, complex modulus is usually adopted. According to the linear viscoelastic theory, shear modulus of damping material is $G = G'(1 + j\eta)$, where G' is storage modulus of damping material while η is loss factor of damping material ($j = \sqrt{-1}$).

Bending rigidity and sandwich panel shear strain are considered by Hoff sandwich plate theory. Its basic assumptions are as follow: (1) the base layer and constraint layer are regarded as classical thin plates. (2) damping layer stress which parallels to the x-y plane is ignored assuming that $\sigma_x = \sigma_y = \tau_{xy} = 0$. (3) stress and strain of z direction in the damping layer is also ignored. (4) with ideal combination between each layer, there is no relative slide[5].

Only transverse vibration considered, natural vibration equation of constrained damping structure is as follow.

$$\begin{aligned}
D \left[\frac{\partial^2 \varphi_x}{\partial x^2} + \frac{1-\mu}{2} \frac{\partial^2 \varphi_x}{\partial y^2} + \frac{1+\mu}{2} \frac{\partial^2 \varphi_y}{\partial x \partial y} \right] + C \left[\frac{\partial \omega}{\partial x} - \varphi_x \right] &= 0 \\
D \left[\frac{\partial^2 \varphi_y}{\partial y^2} + \frac{1-\mu}{2} \frac{\partial^2 \varphi_y}{\partial x^2} + \frac{1+\mu}{2} \frac{\partial^2 \varphi_x}{\partial x \partial y} \right] + C \left[\frac{\partial \omega}{\partial y} - \varphi_y \right] &= 0 \\
C \left[\nabla^2 w - \frac{\partial \varphi_x}{\partial x} - \frac{\partial \varphi_y}{\partial y} \right] - D_f \nabla^2 \nabla^2 w - \rho \omega^2 w &= 0
\end{aligned} \tag{1}$$

Where φ_x and φ_y are the angles in x-z and y-z plane of the corresponding points of attachment after deformation around, which are on the surface of base and constraining layer.

$$D = D_1 + D_2 \quad D_i = \frac{E \left[h_i + \frac{t_i}{2} \right] t_i}{1 - \mu^2} \quad (i = 1, 2) \tag{2}$$

$$D_f = D_{f1} + D_{f2} \quad D_{fi} = \frac{E t_i^3}{12(1 - \mu^2)} \tag{3}$$

$$C = G \frac{\left[h + \frac{t_1 + t_2}{2} \right]^2}{h} \tag{4}$$

By substituting ω and f into Eq. (1) angle and deflection, one obtains:

$$\varphi_x = \frac{\partial \omega}{\partial x} + \frac{\partial f}{\partial y}, \varphi_y = \frac{\partial \omega}{\partial y} - \frac{\partial f}{\partial x}, w = \omega - \frac{D}{C} \nabla^2 \omega \tag{5}$$

By substituting Eq. (5) into Eq. (1), one obtains:

$$\begin{aligned}
\frac{D}{2} (1 - \mu) \nabla^2 f + C f &= 0 \\
D \nabla^4 w + \left[1 - \frac{D}{C} \nabla^2 \right] (D_f \nabla^2 \nabla^2 w - \rho \omega^2 w) &= 0
\end{aligned} \tag{6}$$

Where

$$\rho = \rho_f (t_1 + t_2) + \rho_c h$$

According to the corresponding boundary conditions, the plural form inherent circular frequency ω of CLD structure, structural modal loss factor and natural frequency can be obtained by the analysis expression and are expressed as follow.

$$\eta = \text{Im}(\omega^2) / \text{Re}(\omega^2) \quad f = \sqrt{\text{Re}(\omega^2)} / 2\pi \tag{7}$$

where:

$$h_1 = \frac{2ht_2 - t_1^2 + t_2^2}{2(t_1 + t_2)} \quad h_2 = \frac{2ht_1 + t_1^2 - t_2^2}{2(t_1 + t_2)} \tag{8}$$

2.2 The Calculation Results

Using the conclusion of Mead and Markus'[6] boundary conditions affect, the transformation of the boundary conditions can make the corresponding frequency of the maximum loss factor change. But the maximum loss factor η_{\max} of structure would not change. The boundary condition which is simply supported at two opposite edges can be considered according to the conclusion. Substituting $f=0$, $w_1 = A_{\min} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$ into analysis expression:

$$\omega^2 = \frac{DK^2 + \left| 1 + \frac{D}{C} K \right| DK^2}{\rho \left| 1 + \frac{D}{C} K \right|} \tag{9}$$

where: $K = \left| \frac{m\pi}{a} \right|^2 + \left| \frac{n\pi}{b} \right|^2$ ($m, n=1, 2, \dots$) while a, b are the length and width of constrained damping structure[5].

Substituting ω^2 into Eq. (1) and the modal loss factor η and natural frequency f of the constrained damping structure under the condition of simply supported can be got.

2.3 The Verification of Theory

The correctness of the above method is verified with the theoretical solution comparing with the calculation results of finite element method. A numerical example of Johnson is given: laminated plate structure is isotropic and four edges simply supported with a cross section of $a \times b = 0.3048 \text{ m} \times 0.3480 \text{ m}$. The thickness of the top and bottom surface is 0.762mm. The Young's modulus is taken as $E = 6.89 \times 10^4 \text{ MPa}$. The fit parameters for this material are given as $\mu = 0.30$ and $\rho_f = 2740 \text{ kg/m}^3$. The thickness of intermediate sandwich is 0.254mm. The fit parameters for sandwich plate are given as $\mu = 0.49$ and $\rho = 999 \text{ kg/m}^3$. The shear modulus is $E = 0.896 \text{ MPa}$ and the loss factor is $\eta = 0.50$.

The above method and the calculations presented by literature [7] are given in Table 1. Alignment and accuracy of the above method and the theoretical solution are very high. So it can be used in the design calculation of CLD structure.

Table 1 – Comparing with other author’s results

Modal m, n	Analytical solution of ref.[7]		FEM solution of ref.[7]		Solution of the above	
	Frequency f/Hz	Modal loss factor η	Frequency f/Hz	Modal loss factor η	Frequency f/Hz	Modal loss factor η
1,1	60.3	0.190	57.4	0.176	60.2	0.190
1,2	115.4	0.203	113.2	0.188	115.2	0.203
2,1	130.6	0.199	129.3	0.188	130.4	0.199
2,2	178.7	0.181	179.3	0.153	178.5	0.181
1,3	195.7	0.174	196.0	0.153	195.4	0.174

3. RESEARCH ON ACOUSTIC VIBRATION OF STIFFENED COMPOSITE PLATE

3.1 The Model of Vibro-acoustic Characteristics

Vibro-acoustic characteristics of stiffened composite plate are researched. The characteristics of CLD structure parameters effect on plate vibro-acoustic are got. First, SEA model of stiffened composite plate is established. The model size is $1.2\text{m} \times 0.9\text{m}$ and the thickness of base plate is $h=0.005\text{m}$. The fit parameters for material of stiffened plate are given as $E = 2 \times 10^{11} \text{ Pa}$, $\mu = 0.3$ and $\rho = 7800 \text{ Kg/m}^3$. Concentrated force (1N) is exerted in the panel midpoint and semi-infinite flow field from the panel 5 m is established, whose medium is water ($\rho = 1000 \text{ Kg/m}^3$). The range of frequency is 200 ~ 20 kHz 1/3 octave. It is shown in Figure 2.

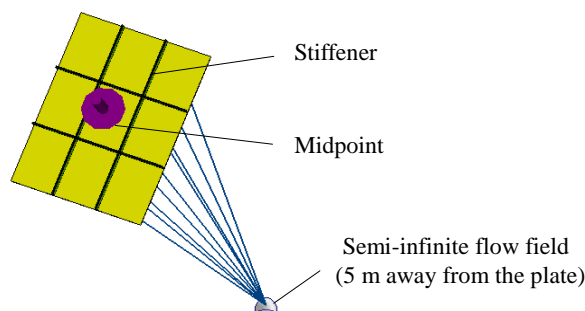


Figure 2 –The SEA model of ship stiffened composite frame

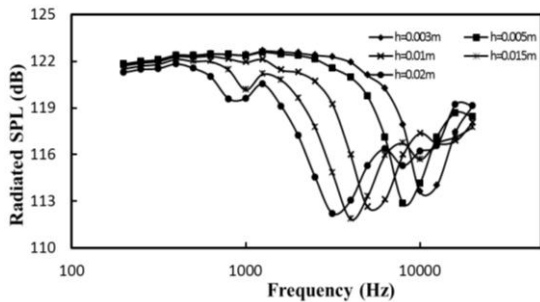
CLD is used on the base layer. The thicknesses of base and constraint layer plate are $h = 0.005\text{m}$.

The fit parameters for their materials are given as $E=2 \times 10^{11}$ Pa, $\mu=0.3$ and $\rho=7800$ Kg / m³, respectively. Damping layer is viscoelastic material and its thickness is $h=0.005$ m. The fit parameters for the material of damping layer are given as $E=3 \times 10^6$ Pa, $\mu=0.5$, $\rho=1000$ Kg / m³ and $\eta=0.5$, respectively.

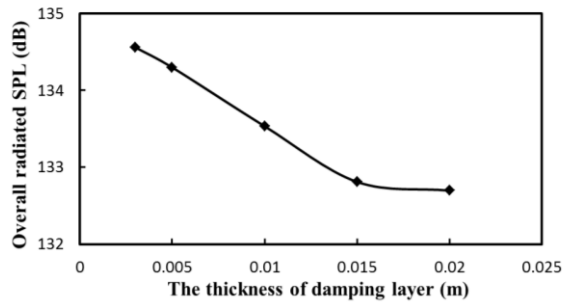
3.2 The Influence of Characteristics Parameters on the Vibroacoustic Characteristics

3.2.1 The Influence of Thickness of Damping Layer

The damping layer thicknesses of stiffened composite plate are defined as $h=0.003$ m, $h=0.005$ m, $h=0.01$ m, $h=0.015$ m, $h=0.02$ m, respectively. The modal loss factors of damping layer of typical thicknesses are frequency dependent and calculated by using the above principle. Figure 3 shows the radiated sound pressure level (SPL) of flow field source under different frequency corresponding to typical thicknesses of damping layer.



(a) 1/3 Oct curve of radiated SPL of stiffened composite plate



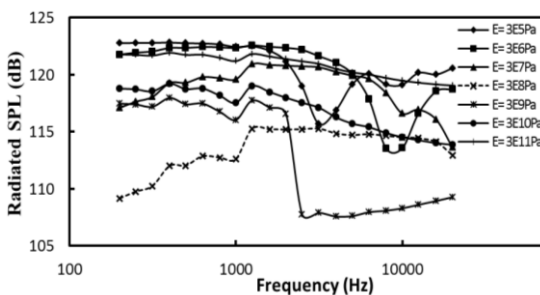
(b) Overall radiated SPL of stiffened composite plate

Figure 3 –Radiated SPL of typical thickness of damping layer frame

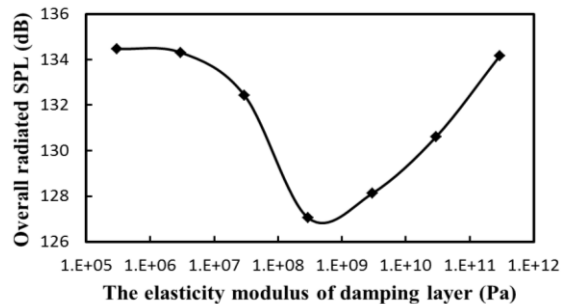
Figure 3 (a) shows that the thickness of damping layer change has little influence on radiated SPL in the low-middle frequency stage (200 ~ 1 kHz). When the frequency is 1 k ~ 3 kHz, with increasing the damping layer thickness, radiated SPL are declining. When the frequency is greater than 3 kHz, its influence law is complex. From the Figure 3 (b), with the increase of damping layer thickness, overall radiated SPL of stiffened composite plate structure decreases.

3.2.2 The influence of elasticity modulus of damping layer

The chosen damping layer elasticity modulus of stiffened composite plate are $E=3 \times 10^5$ Pa, $E=3 \times 10^6$ Pa, $E=3 \times 10^7$ Pa, $E=3 \times 10^8$ Pa, $E=3 \times 10^9$ Pa, $E=3 \times 10^{10}$ Pa, $E=3 \times 10^{11}$ Pa. The modal loss factors in dependence on frequency of typical elasticity modulus of damping layer are calculated. The results of radiated SPL are drawn in Figure 4.



(a) 1/3 Oct curve of radiated SPL of stiffened composite plate



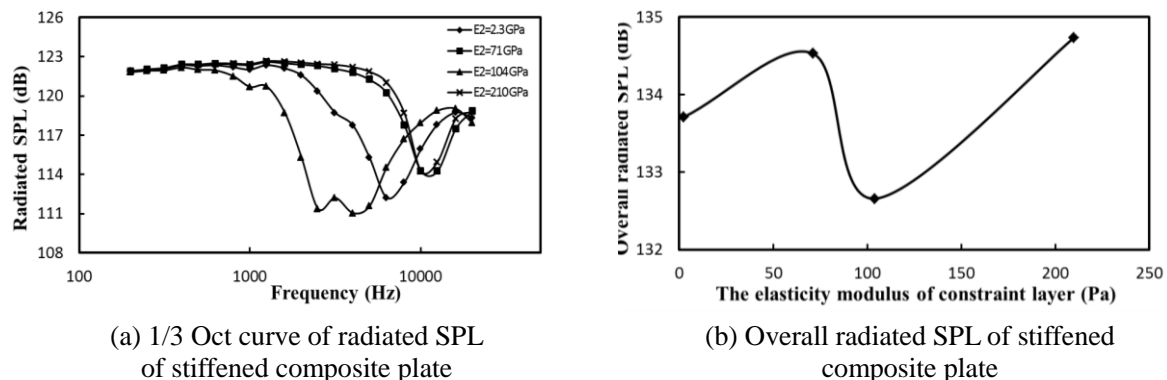
(b) Overall radiated SPL of stiffened composite plate

Figure 4 –Radiated SPL of typical elastic modulus of damping layer frame

Figure 4 (a) shows that when the frequency is located in 200 ~ 1 kHz, radiated SPL with elastic modulus $E=3 \times 10^8$ Pa is the lower, while radiated SPL with elastic modulus $E=3 \times 10^9$ Pa is significantly smaller with the frequency ranging from 2k Hz to 20k Hz. From the Figure 4 (b), with the increase of damping layer elastic modulus, overall radiated SPL of stiffened composite plate shows parabola changes. When elasticity modulus of damping layer is among $1 \times 10^8 \sim 1 \times 10^9$ Pa, overall radiated SPL is smaller.

3.2.3 The Influence of Elasticity Modulus of Constraint Layer

The chosen materials are Hard Rubber, Aluminum, Brass, and Steel as constraint layer material. The corresponding elasticity modulus are $E_2=2.3\text{G Pa}$, $E_2=71\text{G Pa}$, $E_2=104\text{G Pa}$ and $E_2=210\text{G Pa}$. The modal loss factors in dependence on frequency of typical elasticity modulus of constraint layer are calculated. The results of radiated SPL are drawn in Figure 5.

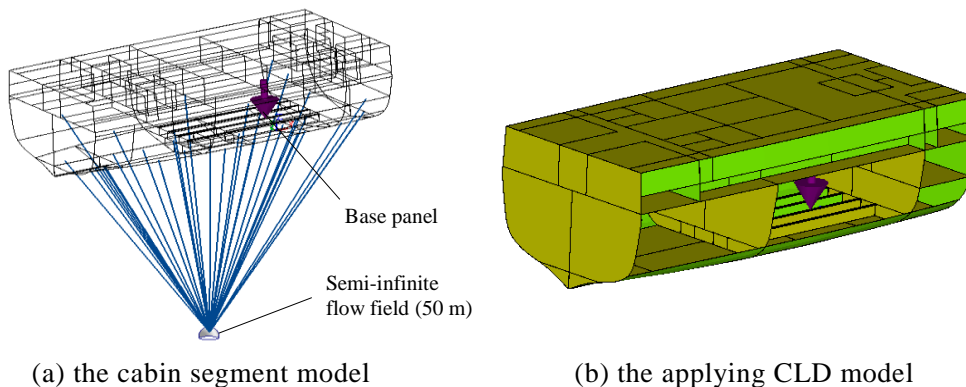


(a) 1/3 Oct curve of radiated SPL of stiffened composite plate
 (b) Overall radiated SPL of stiffened composite plate
 Figure 5 –Radiated SPL of typical elastic modulus of constraint layer frame
 Figure 5 (a) shows that the elasticity modulus of constraint layer has a little effect on radiated SPL in the frequency stage of 200 ~ 1 kHz. When the frequency is 1 k ~ 5 kHz, the radiated SPL of stiffened composite plate with $E_2 = 104\text{G Pa}$ is significantly smaller than others. As can be seen from the Figure 5 (b), when elastic modulus of the constraint layer is 104G Pa, overall radiated SPL of stiffened composite plate is smaller significantly.

4. APPLICATION CONSTRAINED LAYER DAMPING IN REAL CABIN

4.1 The Establishment of Cabin Segment Model

The goal of research on vibro-acoustic characteristic of stiffened composite plate is applied in practice. The cabin segment of three compartments, which includes the engine room, the room before and after it, is adopted to verify the noise reduction of typical parameters. Concentrated force (1N) is loaded on the base panel. The semi-infinite flow field with the radius of 50 m is established. The range of frequency is 200 ~ 20 kHz 1/3 octave. The SEA model of cabin segment and the way to lay CLD (all decks, transverse bulkheads, base web and panel) are shown in Figure 6.



(a) the cabin segment model
 (b) the applying CLD model
 Figure 6 –The SEA model of cabin segment
 The fit parameters for damping layer are given as $\rho=1100\text{Kg}/\text{m}^3$, $E=3\times 10^9\text{Pa}$, $\mu=0.49$, $\eta=0.5$. The thickness of damping layer is $h=0.01\text{m}$. The chosen parameters for constraint layer material are $E=210\text{GPa}$, $\mu=0.3$, $\rho=7800\text{Kg}/\text{m}^3$ and the thickness of constraint layer is 0.005 m.

Although the loss factor for viscoelastic materials in general is frequency dependent, it is assumed here to have a constant value of 1 for convenience. Note that this will not be an issue use of this method since the emphasis here is to note the difference among these results. After calculated, flow field source overall radiated SPL of naked cabin segment is 83.33 dB, while it falls to 64.62dB after laying CLD. The outcome of radiated SPL is shown in Figure 7. The reduction of overall radiated SPL is up to 20 dB.

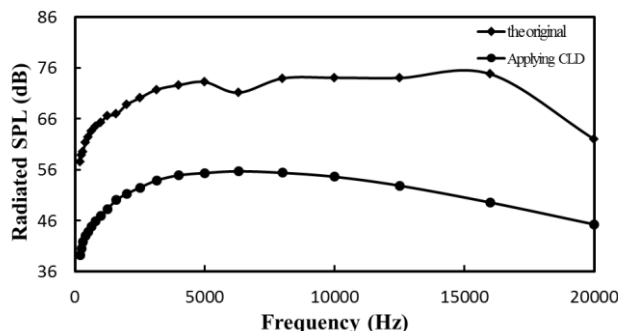


Figure 7 –Flow field source radiated SPL of cabin segment before and after applying CLD

4.1.1 The Influence of Thickness of Damping Layer

For studying the influence of thickness of damping layer on radiated SPL, the thickness of damping layer is changed from 0.01 m to 0.02 m. Overall radiated SPL corresponding to 0.02 m is 63.24 dB. The comparative condition is shown in Figure 8.

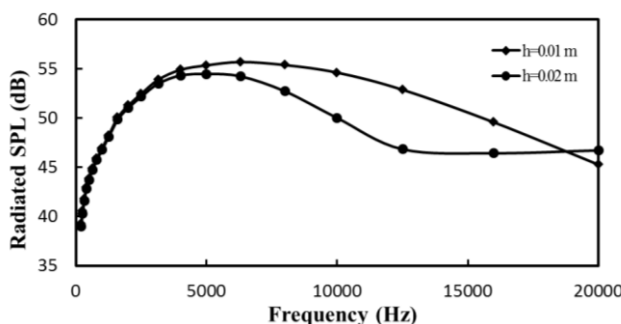


Figure 8 –Flow field source radiated SPL of cabin segment with different thickness of damping layer

Figure 8 shows that when the frequency is 200 Hz ~ 5k Hz, the thickness of damping layer has little influence on radiated SPL. When the frequency is 5k Hz ~ 18k Hz, radiated SPL of 0.02 m is less than that of 0.01 m obviously. The corresponding overall radiated SPL declines from 64.62 dB to 63.24 dB and decreases about 1.4 dB.

4.1.2 The Influence of Elasticity Modulus of Damping Layer

The elasticity modulus of damping layer is changed from $E=3 \times 10^9$ Pa to $E=3 \times 10^8$ Pa. The overall radiated SPL of flow field source corresponding to $E=3 \times 10^8$ Pa is 62.99 dB. The comparative condition is shown in Figure 9.

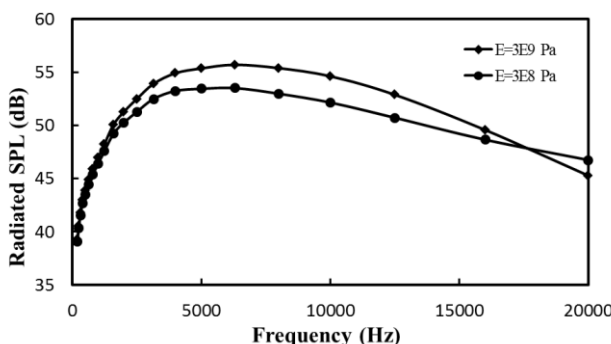


Figure 9 –Flow field source radiated SPL of cabin segment with different elasticity modulus of damping layer

In the Figure 9, when the frequency is 200 Hz ~ 18k Hz, radiated SPL of $E=3 \times 10^8$ Pa is less than that of $E=3 \times 10^9$ Pa clearly. The corresponding overall radiated SPL declines from 64.62 dB to 62.99 dB and decreases about 1.6 dB.

4.1.3 The Influence of Elasticity Modulus of Constraint Layer

The elasticity modulus of constraint layer is changed from $E_2 = 210$ G Pa to $E_2 = 105$ G Pa. The

overall radiated SPL corresponding to $E_2 = 105 \text{ G Pa}$ is 63.07 dB. The comparative condition is shown in Figure 10.

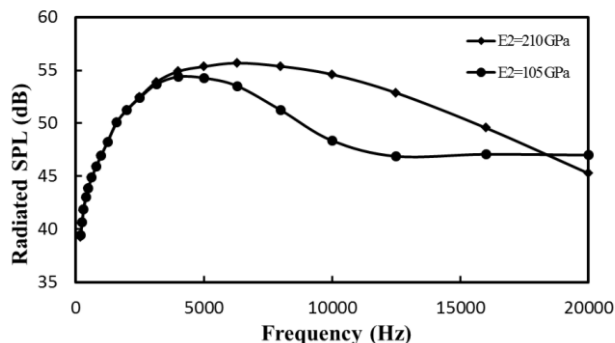


Figure 10 –Flow field source radiated SPL of cabin segment with different elasticity modulus of constraint layer

Figure 10 shows that when the frequency is 200 Hz ~ 4k Hz, the elasticity modulus of constraint layer has little influence on radiated SPL. When the frequency is 4k Hz ~ 18k Hz, radiated SPL of $E_2 = 105 \text{ G Pa}$ is less than that of $E_2 = 210 \text{ G Pa}$. The corresponding overall radiated SPL declines from 64.62dB to 63.07dB and decreases about 1.6 dB.

4.2 Vibroacoustic Reduction Effect of CLD In Different Parts

The vibro-acoustic of structure is closely related to the modal loss factor. It is beneficial to reduce radiated pressure level with increasing the modal loss factor. The plate modal loss factors of different location and thickness can change with the elastic modulus of damping layer. The average modal loss factors corresponding to the different elastic modulus ($E = 3 \times 10^9 \text{ Pa}$, $E = 3 \times 10^{10} \text{ Pa}$) of damping layer are shown in Table 2.

Table 2 –Structural modal loss factors of different thickness

Different thickness plates	The modal loss factor ($E = 3 \times 10^9 \text{ Pa}$)	The modal loss factor ($E = 3 \times 10^{10} \text{ Pa}$)
0.003	0.21	0.05
0.004	0.24	0.06
0.005	0.24	0.07
0.006	0.22	0.05
0.008	0.23	0.08
0.009	0.23	0.09
0.01	0.2	0.16
0.016 (the base web)	0.09	0.14
0.028 (the base panel)	0.01	0.04

It can be seen from Table 2 that except for the thickness of 1.6 cm, 2.8 cm plate (base web and base panel), the modal loss factors of ship composite plate with other thicknesses and locations are much smaller than the originals. The radiated SPL of these plates are greater than the originals, while those of base web and base panel are smaller. The overall radiated SPL of cabin segment is 57.57dB and the comparative condition is shown in Figure 11.

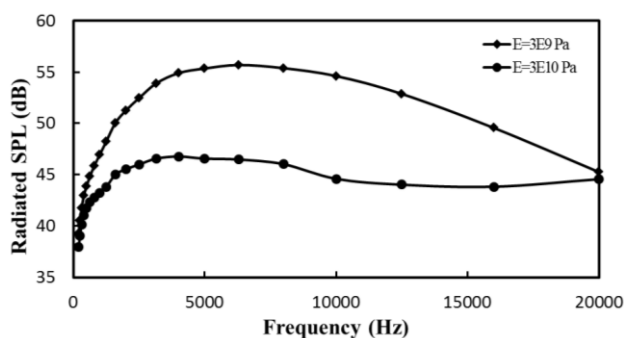


Figure 11 –Flow field source radiated SPL of the cabin segment with different frequencies

Except for the base web and panel, the radiated SPL of other location plates are larger than the originals. But at last, the overall radiated SPL is still decreased up to 7 dB. Therefore, we can conclude that the contribution of base web and panel vibration to the radiated SPL is bigger than others. So the conclusion can be summarized that in excitation source (such as engine base) laid CLD, can be better to achieve the effect of acoustic radiation reduction.

5. CONCLUSIONS

Based on the statistical energy analysis method, via using SEA software to establish base panel and cabin model, the influence laws of parameters of damping and constraint layer on vibro-acoustic characteristics of stiffened composite plate structure were studied. The laws are verified with cabin segment.

- (1) With the increase thickness of damping layer, overall radiated SPL of stiffened composite plate structure decline gradually. With the increase of damping layer elastic modulus, overall radiated SPL of stiffened composite plate shows parabola changes. There is the elasticity modulus of damping layer corresponding to the minimum of overall radiated SPL. The elasticity modulus of constraint layer corresponding to the minimum and maximum of overall radiated SPL is exerted.
- (2) The effect of laying CLD on reducing overall radiated sound pressure is significantly. The reduction of overall radiated SPL of cabin segment could be up to 20 dB. The thickness of damping layer is changed from 0.01m to 0.02m. The corresponding overall radiated SPL declines from 64.62 dB to 63.24 dB and decreases about 1.4 dB. The elasticity modulus of damping layer is changed from $E=3\times 10^9$ Pa to $E=3\times 10^8$ Pa. The corresponding overall radiated SPL declines from 64.62 dB to 62.99 dB and decreases about 1.6 dB. The elasticity modulus of constraint layer is changed from $E_2=210$ G Pa to $E_2=105$ G Pa. The corresponding overall radiated SPL declines from 64.62 dB to 63.07 dB and decreases about 1.6 dB.
- (3) The laying location of CLD has a great impact on the acoustic radiation reduction of cabin segment. In excitation source (such as engine base) laid CLD, it can be better to achieve the effect of acoustic radiation reduction.

ACKNOWLEDGEMENTS

This paper is funded by the International Exchange Program of Harbin Engineering University for Innovation-oriented Talents Cultivation. The work also acknowledges National Natural Science Foundation China (No.51209052), Heilongjiang Province Natural Science Foundation (Grant No.QC2011C013), Harbin Science and Technology Development Innovation Foundation of youth (2011RFQXG021), Fundamental Research Funds for the Central Universities (HEUCF40117), High Technology Ship Funds of Ministry of Industry and Information Technology of P.R. China, Opening Funds of State Key Laboratory of Ocean Engineering of Shanghai Jiaotong University (No. 1307).

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