Generation of localized sound using speaker array

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ABSTRACT
This paper presents a methodology of generating a sound field in which the sound energy around the speakers is high while that in the other region is relatively low. Although directional loudspeaker that delivers sound energy only for the specific direction has been discussed in the past literature, such a method leaks sound to outside of the specific area because of reflection of the sound wave. To overcome this problem, a novel methodology of generating localized sound is presented which is based on the phenomena that radiated sound from vibration of a rectangular plate under the specific conditions decays drastically more than normal distance decay effect. First, spatial distribution of a rectangular plate vibration at a frequency where acoustic power becomes extremely low is verified, it being clarified localized sound is generated by coupling certain modes each other. Next, a matrix-type speakers array is used to imitate the spatial distribution that generates localized sound in order to achieve generating a sound field not only at a single frequency but in band-limited spectra. As a result, efficacy of speakers array to generate of localized sound is revealed.

Keywords: Localized sound, Speaker array, Cancellation mechanism
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1. INTRODUCTION
Radiated sound from speakers for communications are indispensable tool for people. However, the sound that has unnecessary information for listeners is able to be noise. Because there is no room to choose the sound as far as listeners are in the range the sound reaches, the healthy situation is the sounds which take a role as communications are able to be accurately delivered for people who need the sounds, while it is not delivered for people who do not need. Recently, studies about localization of sound attract attention to improve the sound environment. For example, Itou et al. presented the method to generate the evanescent wave using speaker array (1), and Nagao and Naemura presented directional loud speaker that radiates the sound for only specific area (2).

This study proposes new speaker system method that localizes the sound. This method based on the vibration phenomena of a rectangular plate. By coupling of the odd-order modes, spatial distribution whose volume velocity is set to zero is excited, and then acoustic power is decreased by cancellation phenomena of sounds (3). Then radiated sound of the plate decays extremely. Focusing on this phenomena, localized sound is realized by applying the amplitude of spatial distribution that the volume velocity is set to zero by coupling of the odd-order modes to matrix-type speaker array. In this study, for the purpose of illustration, the spatial distributions of the plate that the volume velocity is set to zero by coupling of the odd-order modes are called the volume-velocity cancelled distribution (VVC distribution). The composition of this paper is as follows. In section 2, basic theory about sound radiation of a rectangular plate is clarified. In section 3, it is verified by computer simulation that sound radiation becomes localized sound at specific
frequencies. Method of generating the local sound field using speaker array and the computer simulation are presented in section 4. Finally, we examine widening bandwidth of localized sound in section 5.

2. THEORY OF THE ACOUSTIC POWER OF RECTANGULAR PANEL

In this section, the expression for vibration radiated sound of a simply-supported rectangular plate that is embedded in infinite baffle is developed. The vibration velocity of a plate \( v(x,y) \) must be revealed for the purpose of developing the expression for vibration radiated sound. First, equation of motion of a rectangular plate is described as Eq.(1).

\[
DV^2 \nabla^2 w(x,y,t) + \rho_s h \frac{\partial^2 w(x,y,t)}{\partial t^2} = f(x,y,t)
\]

(1)

\[
D = \frac{Eh^3}{12(1-\nu^2)}
\]

(2)

Here, \( w(x,y,t) \) is displacement of the plate, \( D \) is flexural rigidity, \( \rho_s \) is density, \( E \) is Young’s modulus, \( \nu \) is Poisson’s ratio, \( h \) is panel thickness, \( \nabla^2 \) is Laplacian, \( f \) is external force. In the case where harmonic force acts upon \( (x,y) = (\xi_1, \eta_1) \), external force \( f \) is described as Eq.(3), where \( \delta \) is the Dirac’s delta function, \( f_1 \) is amplitude of the external force. Then, displacement of the plate is described in a similar way as follows.

\[
f(x,y,t) = f_1 \delta(x-\xi_1) \delta(y-\eta_1) e^{j\omega t}
\]

(3)

\[
w(x,y,t) = w(x,y) e^{j\omega t}
\]

(4)

Next, substituting Eq.(3) and (4) into Eq.(1), equation of motion is described as Eq.(5), and the solution of Eq.(5) is vibration displacement of the plate. Therefore, vibration velocity can be written by Eq.(7).

\[
DV^2 \nabla^2 w(x,y) - \omega^2 \rho_s hw(x,y) = f_1 \delta(x-\xi_1) \delta(y-\eta_1)
\]

(5)

\[
w(x,y) = \sum_{m,n=1}^{\infty} \varphi_{mn}(x,y) \varphi_{mn}(\xi_1, \eta_1) \left( \omega_{mn}^2 - \omega^2 \right)
\]

(6)

\[
v_s(x,y) = j\omega w(x,y)
\]

(7)

Here, \( \varphi_{mn} \) is eigenfunction of the \( (m, n) \) mode and \( \omega_{mn} \) is the corresponding natural angular frequency.

\[
\varphi_{mn}(x,y) = \frac{2}{\sqrt{L_x L_y L_z}} \sin \frac{m\pi x}{L_x} \sin \frac{n\pi y}{L_y}
\]

(8)

\[
\omega_{mn} = \sqrt{\frac{D}{\rho_s h}} \sqrt{\frac{m^2}{L_x^2} + \frac{n^2}{L_y^2}}
\]

(9)

From the above, vibration displacement and velocity of a simply-supported rectangular plate are calculated.

Next, sound radiation of a rectangular plate is considered. Radiated sound pressure from an arbitrary point \( (x_i,y_i) \) of the plate to an observation point \( (x,y,z) \) is described as Eq.(10) with vibration velocity \( v_s(x_i,y_i) \) and area \( dA_s \) of small elements.

\[
AP_i(x,y,z) = \frac{jo\omega}{2\pi r} v_s(x_i,y_i) dA_s e^{j\theta} e^{j\omega t}
\]

(10)

\[
r = \sqrt{(x-x_i)^2 + (y-y_i)^2 + z^2}
\]

(11)

Therefore, sound pressure observed at \( (x,y,z) \) is described as Eq.(12) by totalizing the sound pressure from each elements.
\[ \Delta p(x, y, z, t) = \frac{J_0(\omega_0 r)}{2\pi} \int \frac{v_{\tilde{x}}(\tilde{x}, \tilde{y}) e^{i\omega t}}{r} \, d\tilde{x} d\tilde{y} \]  

\[ r = \sqrt{(x - \tilde{x})^2 + (y - \tilde{y})^2 + z^2} \]  

Next, radiated acoustic power \( P_W \) from simply-supported rectangular plate is developed. Acoustic power is calculated by integration of acoustic intensity on the surface of the hemisphere in far field. Fig.1 shows the coordinate system for calculating the acoustic power of the rectangular plate. When a coordinate system that an origin is center of the plate is defined as the right-handed Cartesian coordinate system, a tilt angle from the z-axis is \( \theta \), a tilt angle from the x-axis is \( \Psi \). Then, an arbitrary point \( r \) on the surface of the hemisphere with radius \( r \) from the origin can be written by \( r = (r, \theta, \Psi) \), and the sound pressure and the particle velocity on the arbitrary point are defined as \( \Delta p(r) \) and \( v(r) \), respectively. The particle velocity can be expressed by using the sound pressure in far field, hence the acoustic intensity is described as Eq.(14). Therefore, radiated acoustic power is expressed as Eq.(15).

\[
I_a = \frac{\text{Re}}{2} \left[ \frac{\Delta p(r)}{2\rho_c c} \right] v'(r) \]

\[
P_W = \int_{\theta=0}^{2\pi} \int_{\varpi=0}^{\pi/2} \left[ \frac{\Delta p(r)}{2\rho_c c} \right] r'^2 \sin\theta \, d\varpi \, d\theta \]

3. GENERATION OF LOCALIZED SOUND

In this section, the method of generating localized sound is clarified. The vibration and acoustic characteristics of the rectangular plate in which the arbitrary odd-order modes are excited are examined by computer simulation. The rectangular plate of an examination object is 0.575 m in width, 0.945 m in length and 0.02 m in thickness. Loss factor is \( \eta = 0.001 \) and the boundary condition is simply-supported.

First, in order to observe only the odd modes, the exciting force acts upon a central point of the plate. Figure 2 shows the frequency characteristics of the acoustic power and the volume velocity in the above condition. In the calculation of acoustic power, the velocity distribution of the plate is regarded as the set of point sources, and hence the plate is divided into a finite number of small elements. The volume velocity is calculated by integrating vibration velocity over the whole plate. From Fig. 2, decreasing of acoustic power and volume velocity after the second peak is observed. In other frequencies, it is observed that acoustic power and volume velocity is decreased at the same frequencies. It is considered that the acoustic radiation efficiency is lowered by the anti-resonance phenomenon (4).

Next, to consider detail of the phenomena, frequency characteristics of acoustic power and volume velocity is calculated by using only the \((1, 1)\) and the \((1, 3)\) modes. Figure 3 shows the numerical result of acoustic power and volume velocity. The \((1, 1)\) and the \((1, 3)\) modes are excited respectively at frequency of the first and second peaks, and a notch of acoustic power and volume velocity at 77 Hz indicates that the coupling of the \((1, 1)\) and the \((1, 3)\) modes causes the decrement of acoustic radiation efficiency. Figure 4
Figure 2 – Frequency characteristics of the acoustic power and volume velocity of the plate. (All odd mode)

Figure 3 – Frequency characteristics of the acoustic power and volume velocity of the plate. ($m=1, n=1,3$)

Figure 4 – Vibration displacement distribution at 64 Hz. (the (1, 3) mode)

Figure 5 – Vibration displacement distribution at 77Hz. (the (1, 1) and the (1, 3) modes are coupled)

Figure 6 – Radiated sound pressure distribution from the plate at 77Hz. ($m=1, n=1,3$)

Figure 7 – Radiated sound pressure distribution from the plate at 77Hz. ($m=1, n=3$)
shows vibration displacement distribution of the (1, 3) mode that is observed at second peak, and Figs. 5 and 6 show vibration displacement distribution and radiated sound pressure distribution at 77 Hz. From the vibration displacement distribution, it is verified that volume velocity becomes notch at 77 Hz by increasing amplitude around the center of the plate. From the radiated sound pressure distribution shown in Fig. 6, the radiated sound pressure decay over 30 dB within the range of 1 m from the plate. Figure 7 shows the radiated sound pressure distribution at 77 Hz only by using the (1, 3) mode. Comparing Fig. 6 with Fig. 7, it is seen that radiated sound of the VVC distribution do not propagate in far field by canceling each other. Therefore, it is verified that localized sound is generated by coupling of the odd-order modes.

4. LOCALIZED SOUND BY SPEAKER ARRAY

In the previous section, it is verified that radiation sound from the rectangular plate decays drastically by the coupling of the (1, 1) and the (1, 3) modes. However, in natural phenomena, the existence of the even-order and the higher order modes makes it difficult to excite only the (1, 1) and the (1, 3) modes. Therefore, as a method of generating the localized sound at any frequency by keeping the coupled distribution that is indicated Fig. 5, using matrix-type speaker array, the localized sound which is generated by imitating vibration of the plate is examined. The speaker array is composed of twenty one speakers arranged like a 3-by-7 matrix. Each speaker is embedded in enclosure, and its radius is 0.0275 m. Figure 8 shows illustration of speaker array with its dimensions. The plate vibration is imitated by speaker array in the following way. On the assumption that the plate of the same size as speaker array is vibrated by the VVC distribution of the (1, 1) and the (1, 3) modes, and representative vibration velocity of plate small elements which are in the corresponding location to each speaker are totalized, and then it is applied to velocity amplitude of speakers. Furthermore, to eliminate the differences of vibrational amplitude between frequencies of vibrating the plate, amplitude of each speaker is normalized by maximum amplitude. The simulation result of the sound pressure distribution of speaker array that is vibrated by imitating the VVC distribution at 100 Hz is shown in Fig. 9. The black spots mean the center of the speakers. From the simulation result, it shows that similar attenuation to the sound radiation of plate is caused, and it clarifies that sound field which is similar to sound radiation of the plate can be imitated by using speaker array that imitates the spatial distribution of the plate. The frequency characteristics of acoustic power by speaker array that imitates the VVC distribution is shown in Fig. 10, and Fig. 11 shows the radiated sound pressure distribution of speaker array which is vibrated by coupled mode at 400 Hz. These simulation results show that as vibration frequency becomes higher, the more acoustic power is increased, and the attenuation of sound near the speaker array is lessened. It is considered that this enhancement of acoustic radiation efficiency is caused by strong directivity of sound wave. The strong directivity results from the fact that wavelength of sound becomes shorter than that of bending wave which is imitated by speaker array.

![Figure 8](image1.png)  
**Figure 8** – Schematic diagram of a speakers array.

![Figure 9](image2.png)  
**Figure 9** – Normalized sound pressure distribution of the speakers array at 100 Hz.
5. WIDENING BANDWIDTH OF LOCALIZED SOUND

The localized sound that the speaker array imitating the VVC distribution of the (1, 1) and the (1, 3) modes generates is effective at low bandwidth up to 200 Hz, however, at higher bandwidth over 400 Hz, the radiation efficiency is increased due to the wavelength of sound shorter than that of bending waves, and thus the sound attenuation is lessened as a result. To widen bandwidth of localized sound, the wavelength of the bending wave should be shortened. Therefore, the method is proposed which utilizes the coupling of the higher odd modes, generating the localized sound at higher frequencies. Figure 12 shows the frequency characteristics of acoustic power and volume velocity for the case of coupling between the (1, 5) mode and the (1, 7) mode. From the volume velocity and acoustic power in Fig.12, the decrement of radiation efficiency at 470 Hz is observed. The displacement distribution and the radiated sound pressure distribution at 470 Hz are shown in Fig.13 and Fig.14, respectively. From the displacement distribution of the plate, the amplitude becomes smaller as it goes to the edge of the plate from center, and it is verified that the total of vibrational displacement becomes zero for the case of coupling between the higher-order odd modes. The radiated sound pressure distribution indicates that the radiated sound is attenuated significantly since the two odd modes are cancelled each other. Thus, the local sound field is generated at a frequency which the volume velocity is set to zero by coupling of the (1, 5) and the (1, 7) modes, and then efficacy of the coupling higher-order odd modes is verified.

Figure 10 – Frequency characteristics of the acoustic power of the speakers array. (the (1, 1) and the (1, 3) modes are coupled)

Figure 11 – Radiated sound pressure distribution from the speakers array at 400 Hz.

Figure 12 – Frequency characteristics of the acoustic power and volume velocity of the plate. \(m=1, n=5,7\)

Figure 13 – Vibration displacement distribution at 460Hz. (the (1, 5) and the (1, 7) modes are coupled)
Next, Fig. 15 shows the acoustic power of speaker array imitating the VVC distribution of the (1, 5) and (1, 7) modes. Comparing with speaker array imitating the VVC distribution of the (1, 1) and the (1, 3) modes.

Figure 14 – Radiated sound pressure distribution from the plate at 470 Hz. \((m=1, n=5,7)\)

Figure 15 – Frequency characteristics of acoustic power of the speakers array. (the (1, 5) and the (1, 7) modes are coupled)

Figure 16 – Radiated sound pressure distribution from the speakers array.

Next, Fig.15 shows the acoustic power of speaker array imitating the VVC distribution of the (1, 5) and (1, 7) modes. Comparing with speaker array imitating the VVC distribution of the (1, 1) and the (1, 3) modes.
modes, the change of gradient from 200 Hz to 800 Hz is small, and the overall values is small. It is considered that shortening the wavelength of the bending wave expand the bandwidth that the radiation efficiency is low. Figure 16 shows the radiation sound pressure distribution of speaker array that is excited from 200 Hz to 800 Hz at intervals of 200 Hz. From Fig.16, radiation characteristics are different from the coupling of the (1, 1) and the (1, 3) modes, that is, attenuation effect concentrates near the radiation surface. Although radiation sound pressure by the coupling of the (1, 1) and the (1, 3) modes is raised drastically when frequency increases, radiation sound by the coupling of higher-order odd modes is localized in this frequency band.

**CONCLUSIONS**

Taking account of the phenomena in which the radiated sound from a rectangular plate is attenuated significantly by the coupling of the odd-order modes, a novel method that can be generated localized sound was proposed. The main results are summarized as follows.
(1) Imitating the vibration distribution of the plate by using a speaker array enables to reproduce the vibration radiated sound of plate. As a result, a novel speaker system for generating a local sound field was proposed.
(2) The coupling of the higher-order odd modes shorten the wavelength of the bending wave. Using this characteristics, the method for widening the bandwidth of localized sound was proposed, its validity being clarified by computer simulation.

**REFERENCES**

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