Measurement of Structural Intensity Using an Angular Rate Sensor

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ABSTRACT

Measuring or calculating structural intensity allows us to understand the propagation paths of vibration and the amount of transmitted power. In this paper, a new structural intensity measurement method is proposed, in which an angular rate sensor and an accelerometer are used. The results obtained by the new method are compared numerically and experimentally with those from the 2-point method, which is commonly used measurement method using two accelerometers. However the finite difference calculation of displacements at two points for angular displacement cause an error in the process of the 2-point method. On the other hand, the new method using the angular rate sensor directly measure the angular rate, so it does not cause such errors. Effectiveness of the new method is shown through numerical simulation that the new method can measure the structural intensity in a beam structure at least as accurately as the 2-point method. Also, sensitivity against sensor noise is examined; the simulation results show that the new method and the 2-point method are both robust enough against regular sensor noise. Furthermore, structural intensity measurements on a simple supported acrylic beam were performed experimentally. The results show that new methods could measure structural intensity with similar accuracy as 2-point method.

Keywords: Structural intensity, Measurement, Simulation
I-INCE Classification of Subjects Number(s): 43.5

1. INTRODUCTION

The values of mechanical products are evaluated not only by performance, durability, and strength, but comfort, low vibration, and low noise have become important index. Development of designing methods for silent and low vibration products has become a trend in recent years. Especially, designing desirable structural vibration is more important because most of sound is radiated from mechanical structure vibration. Here, roughly two approaches are usually performed to handle structural vibration; directly cut down the vibration at source, or indirectly take measures on propagation path of vibration. In either ways, understanding the propagation accurately with the view of reduce vibration and noise economically and efficiently. Put simply, establishment of analysis and measuring technique of noise and vibrations is important to develop valuable mechanical systems.

In these procedures, the measurement or calculation of Structural Intensity (SI) is essential. SI is defined as “the amount of power flow per unit width of cross section perpendicular to the direction of the flow” and was proposed by Noiseux in 1970 (1) to visualize the flow of vibration energy. Then, Pavic proposed the measurement method of SI using the finite difference approximation in 1976 (2) and studies of SI followed by many researchers, including some Japanese.

The first major measurement method was named the 4-point method. It requires four accelerometers

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to determine the definitional equation of SI precisely in the case where the bending vibration of a uniform beam is targeted. Because of its complicacy and time consuming process, the 4-point method didn't spread around; a fast, simple, and accurate measurement method is required. Subsequently, the 2-point method using two accelerometers was proposed as a way of simplifying the measuring process; this method is effective when the near field can be ignored.

Measurement of traditional SI is carried out by the 2-point method. However, the 2-point method contains an error because the angular displacement is calculated by finite difference approximation of the two sensor signals. Generally, if two sensors' distance is smaller than bending wavelength, the influence of the approximation is small. Nevertheless, it is necessary to reduce the error as much as possible (3) (4) (5), or new measurement method without finite difference approximation is required.

In this study, a new SI measurement method using an angular rate sensor is proposed. A major advantage of the new method is that displacement and angular displacement are measured directly with an accelerometer and an angular rate sensor so that no finite difference calculation is operated. The measurement accuracy of the 2-point method and the new method is discussed through numerical simulation and experiment; a vibration analysis of a uniform beam is considered in this case. First, using numerical simulation, the superiority of new method to the 2-point method is shown and the influence of the finite difference error is clarified. Also, the influence of sensor error is discussed. Furthermore, experiments show that the new method shows gives results that are similar to those obtained using the 2-point method.

2. MEASUREMENT OF STRUCTURAL INTENSITY

2.1 Basic Equations of SI

SI is defined as energy which flows through unit width in unit time in the structure. In a straight uniform beam as one dimensional bending, vibration of beam element has anti-plane wave and plane wave. One dimensional SI is subject to bending wave only because anti-plane wave generally: bending wave especially, is dominant. In a straight uniform beam, the SI spectrum, $I(x;\omega)$ at a point x on the beam is given by

$$I(x;\omega)=\frac{1}{2}\mathrm{Re}\left[-j\omega\left(Q\cdot\ \zeta^{*}+M\cdot\ \theta^{*}\right)\right], \tag{1}$$

where ω, ζ, θ, Q and M are the frequency, bending displacement (translatory displacement), angular displacement, shear force of vibration, and bending moment, respectively. Re, j, and * represent the real part, the complex unit, and complex conjugate, respectively. As B is the bending stiffness of the beam, the angular displacement, the shear force, and the bending moment are given by

$$\theta=\frac{\partial\zeta}{\partial x}, \tag{2}$$

$$Q=B\frac{\partial^{3}\zeta}{\partial x^{3}}\ \text{ and} \tag{3}$$

$$M=-B\frac{\partial^{2}\zeta}{\partial x^{2}}. \tag{4}$$

If the near field can neglect, shear force member equals bending moment member. Therefore, the far field to remove over approximately 3/4 of a wavelength from boundary of the vibration source can be simplified to

$$\frac{\partial^{2}\zeta}{\partial x^{2}}=-k^{2}\zeta \tag{5}$$

.

Here, k is the wave number, which is given by

$$k^4 = \frac{\rho bh}{B}\omega^2 \quad , \tag{6}$$

where ρ, b, and h are the mass density and the width and the thickness of the beam, respectively.

Therefore, by equations (1) and (5) the SI spectrum can be expressed as a function of the bending displacement:

$$I(x;\omega) = -\frac{Bk^2}{2}\mathrm{Re}\left[j\omega\left(\frac{\partial\zeta}{\partial x}\zeta^* + \zeta\frac{\partial\zeta^*}{\partial x}\right)\right]. \tag{7}$$

2.2 SI Measurement by the 2-point method

The sensor location for the 2-point method is shown in Figure. 1. The translatory displacement and the angular displacement are measured by displacements ζ_1, ζ_2 of 2 points separated by a distance δ across the measurement point x. The displacement of the measurement point x is the average of the two displacements. The angular displacement is calculated from the finite difference of two translatory displacements as shown by

$$\zeta = \frac{\zeta_1 + \zeta_2}{2} \quad \text{and} \tag{8a}$$

$$\frac{\partial\zeta}{\partial x} = \frac{\zeta_1 - \zeta_2}{\delta} \quad . \tag{8b}$$

Here, acceleration is equivalent to displacement in frequency function by using the equation $\ddot{\zeta} = -\omega^2\zeta$. Then SI can be rewritten for practical use by substituting equations (8*a*) and (8*b*) into equation (7).

$$I(x;\omega) = \frac{\sqrt{B\rho bh}}{\delta\omega^2}\mathrm{Im}\left[\ddot{\zeta}_1\ddot{\zeta}_2^*\right] = \frac{\sqrt{B\rho bh}}{\delta\omega^2}\mathrm{Im}\left[G_{12}\right] \tag{9}$$

Here, Im[] is the imaginary part and G_{12} is the cross spectrum between the two acceleration signals.

2.3 SI Measurement by the New Method

In this section, equation of SI with a term of angular rate is derived from equation (7). The sensor location for the new method is shown in Figure. 2. The 2-point method contains truncation error because of using the finite difference method to calculate the angular displacement. On the other hand, the new method has the benefit not to contain truncation error of finite difference method in order to locate the angular rate sensor and the accelerometer on the same measurement point.

Given acceleration $\ddot{\zeta} = -\omega^2\zeta$ and angular rate $\dot{\theta} = j\omega\theta$, Eq.(7) can be transformed to

$$I(x;\omega) = -\frac{\sqrt{B\rho bh}}{\omega}\mathrm{Re}\left[\dot{\theta}\cdot\ddot{\zeta}^*\right] = -\frac{\sqrt{B\rho bh}}{\omega}\mathrm{Re}\left[G_{ra}\right], \tag{10}$$

where G_{ra} is the cross spectrum between the angular rate and acceleration.

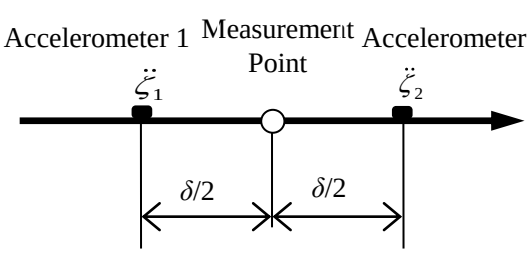


Figure. 1 - Sensor locations for the 2-point method

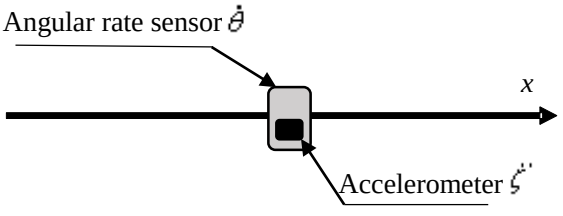


Figure. 2 - Sensor locations for the new method

3. NUMERICAL SIMULATION

3.1 Conditions

The 2-point method and the new method are compared through numerical simulation. Discussion is focused on approximately 3/4 wavelength away from boundary of excitation source to neglect near field.

The analyzed model is a simply supported uniform acrylic beam having a rectangular cross section (length 1.0 m, width 15 mm, thickness 15 mm, modulus of longitudinal elasticity 2.1 GPa, density 7834 kg/m^3, viscous damping coefficient 0.003 Ns). Experimental model is shown in figure. 3. Multiple sine wave excitation force is input on the beam in the bending direction at 0.4m from the left end; every 10 Hz from 10Hz to 1000Hz with maximum magnitude F=1N. The bending displacement ζ of the beam is calculated from equation (13) using natural angular frequencies ω_n and normal modes φ_n:

$$\omega_n = \left(\frac{n\pi}{\ell}\right)^2 \sqrt{\frac{B}{\rho A}} \tag{11}$$

$$\phi_n(x) = \sqrt{\frac{2}{M}} \sin\left(\frac{n\pi}{\ell} x\right) \tag{12}$$

$$\varsigma = \sum_n^N \frac{F\phi_n(x_F)}{\omega_n^2(1+j\eta_n)-\omega^2}\phi_n(x). \tag{13}$$

Here, n, ℓ, A, and ρ are mode order, length, mass, section area, and density, respectively. The maximum mode order N is set to 100 and 101 measurement points are set at intervals of 0.01 m.

3.2 Results of the 2-Point Method and the New Method

The theoretical value is given by equation (13), the result from the 2-point method is given by equation (9), and the result from the new method is given by equation (10). Because the 2-point method uses a finite difference approximation of the angular displacement, as shown in equation (8), this has an effect on the accuracy of the measurement difference interval (accelerometer interval) δ. The measurement error is small, being less than 6 % if δ/λ is less than 0.1. As an example of the results, the results for the 14th resonance (1010 Hz) with accelerometer distances of δ=0.01 m and 0.02 m are shown in Figure. 4. For δ=0.01m, δ/λ is 0.07 and the error in the 2-point method is 4.6% compared with theoretical value at the excitation force point (x=0.4 m). However, lower order resonance is analyzed at a high frequency in order to be sensitive to the phase error δ/λ is reduced. For δ=0.02 m, the accelerometer interval is 0.14 and the error is as large as 15.6%. The new method produces results with about 1.3% error compared to the theoretical predictions.

Figure.5 shows that the new method and the 2-point method of δ=0.01m and 0.02m establish absolute error of SI calculating result on measuring points from 0.5m to 0.9m and theoretical value result of equation(7). The 2-point method is clarified that distance of accelerometer interval is large by truncation error of the finite difference approximation.

Without containing sensor error, the new method can calculate SI actually in order to locate the angular rate sensor and the accelerometer on the same measurement point.

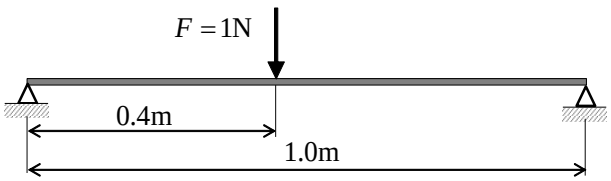


Figure. 3 - Test beam with simple supports

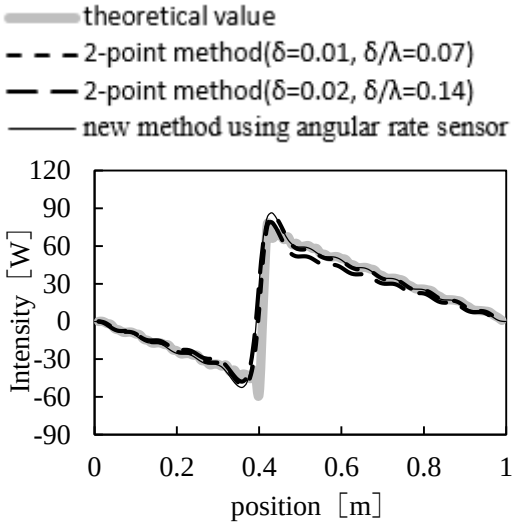


Figure. 4 - Comparison of structural intensity measurement accuracy

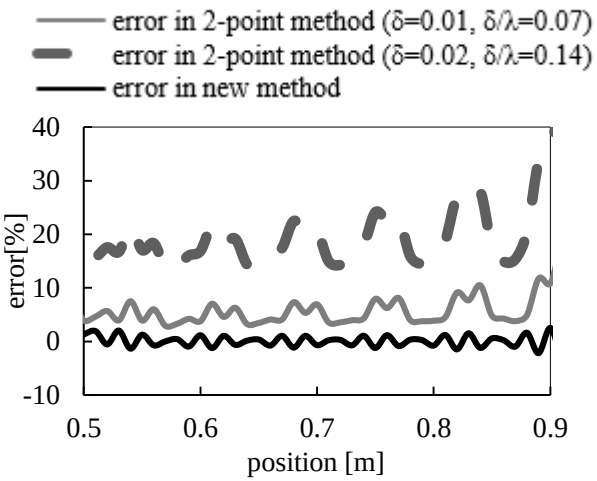


Figure. 5 - Comparison of structural intensity measurement error

4. INFLUENCE OF SENSOR NOISE

4.1 Overview of Simulation Considering Measurement Noise

Influence of sensor noise of the accelerometers and angular rate sensors is discussed in this section through numerical simulation. As mentioned above, SI calculation requires translatory displacement and angular displacement. Here, measuring acceleration is equivalent to measure translatory displacement in frequency function. Likewise, angular rate or its differential is equivalent to angular displacement. Angular rate sensor has been developed recently, so angular rate can be measured directly. Still finite difference method with two accelerometers are widely used to observe angular acceleration. Then, influence of two sensors' errors, phase errors of accelerometers and gain errors of angular rate sensors, is evaluated.

Acceleration response $\ddot{\zeta}_\varepsilon$ and angular rate response $\dot{\theta}_\varepsilon$ including sensor noise are expressed as follows.

$$\ddot{\zeta}_\varepsilon = \ddot{\zeta} \exp\left[\frac{\varepsilon_p}{180}\pi\right] \quad \text{and} \tag{14}$$

$$\dot{\theta}_\varepsilon = \dot{\theta}\varepsilon_g . \tag{15}$$

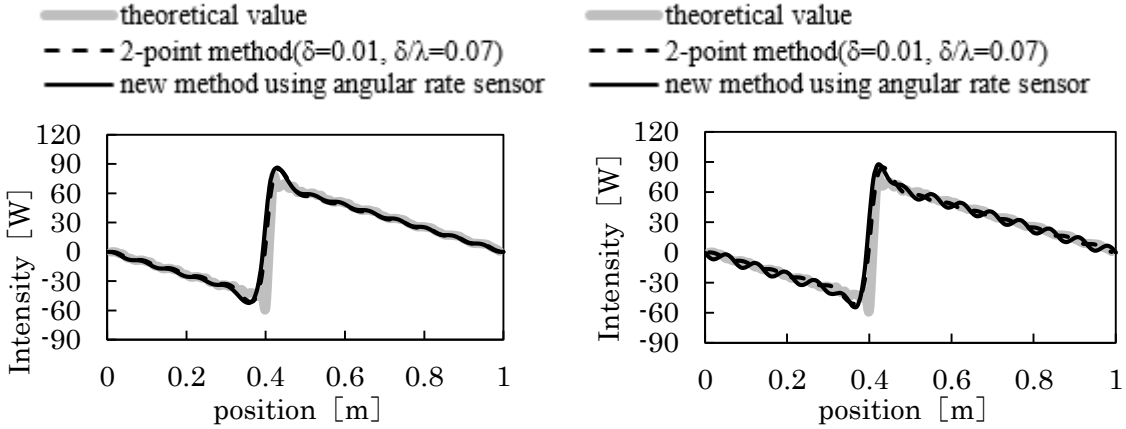
where phase error of an accelerometer is ε_p and the gain error of an angular rate sensor is ε_g. Other conditions of simulation are same as previous simulation. An acrylic beam is randomly excited and SI along the beam is calculated by two methods based on error included data.

4.2 Error of Accelerometers

The influence of the phased error in the accelerometers of the 2-point method and the new method is discussed. The phase error ε_p on acceleration is given a uniform random distribution with a maximum of 0.05 degrees or 0.2 degrees; two different levels of error is given to evaluate sensitivity of SI on phase error.

Figure 6 shows a comparison of the theoretical value (with no phase error) and the calculated SI, including two levels of phase error, with the 2-point method and the new method. In figure 6 (a), results of SI with two methods with a maximum phase error of 0.05 degrees and theoretical SI is compared. The average errors of the 2-point method and new method were 4.6% and 1.6%, respectively. This was qualitatively no difference to the result without sensor noise shown in Figure.4. Figure.6 (b) shows results of SI with two methods with a maximum phase error of 0.2 degrees and theoretical SI. The error of SI calculated by the new method increases as phase error increases. It

means that new method is more sensitive to accelerometers' phase error in theory. However, commonly used sensors contain less than 0.05 degree phase error, so that the influence of the phase error will be small enough in practical term.



(a) Error at maximum 0.05 degrees (b) Error at maximum 0.2 degrees

Figure.6 - Influence of accelerometer noise

4.3 Error in Angular Rate Sensors

The influence of errors of the angular rate sensor on the new method is discussed in this section. The proposed method measures SI with an accelerometer and an angular rate sensor (refer on section 2.3), so sensitivity of SI against sensor error is examined through numerical simulation; maximum sensor errors 0.15% and 20% are simulated and compared with theoretical value. Figure 7 shows comparison of the theoretical value and SI calculated by the new method including error. When the maximum error ε_g in the sensor was 0.15%, the average error in the SI derived from the new method was 1.6%. On the other hand, when maximum sensor error of 20% is included, the average error of SI becomes 15.3%. The error of SI measured with new method increases with increasing of sensor error. However, DTS ARS1500, angular rate sensors we assumed, contain less than 0.15% of gain error, so that the influence of the error will be small enough.

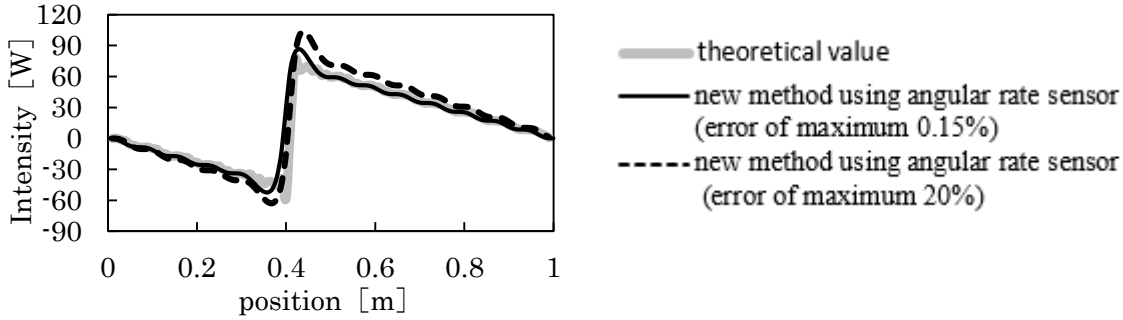


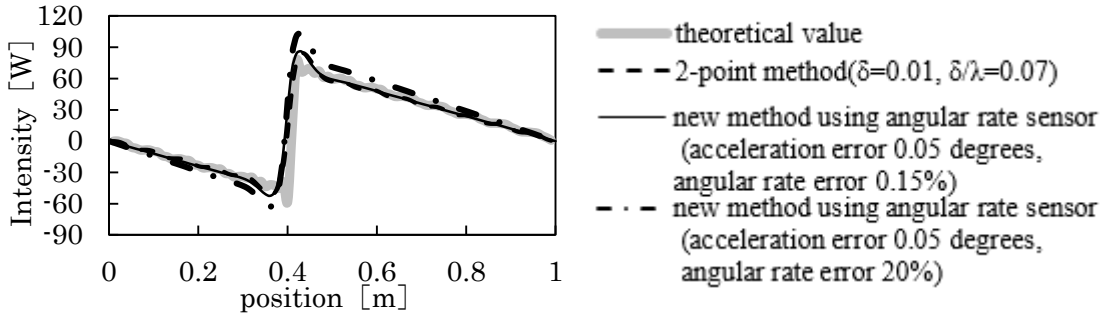
Figure.7 - Influence of angular rate noise

4.4 Combination of errors in accelerometers and angular rate sensors

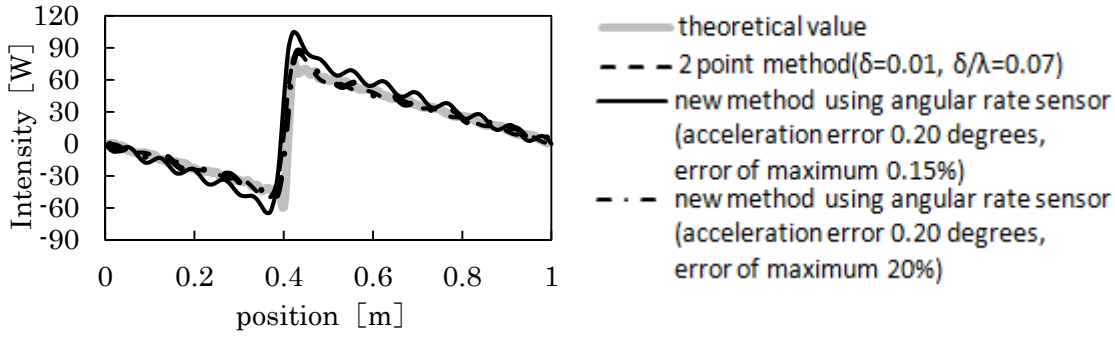
The influence of coupled errors, combination of accelerometers' phase error and angular rate sensors' gain error is discussed. Examined levels of errors are same as section 4.2 and 4.3; 0.05 degrees or 0.2 degrees to phase error ε_p, and maximum 0.15% or 20% to angular rate error ε_g. The results of SI calculated with combined errors are shown in Figure. 8; combination of 0.05 degree and 0.15%, 0.05 degree and 20% are shown in fig.8 (a); combination of 0.2 degree and 0.15%, 0.2 degree and 20% are shown in fig.8 (b). Bold gray lines represent theoretical values, and dash lines represent SI calculated with 2-point method; Thin solid lines represent SI calculated with the new method which contains 0.15% angular rate sensor error, and chain lines represent SI calculated with the new method

which contains 20% angular rate sensor error

By comparing (a) and (b), each sensor error affect SI measurement accuracy independently, and there is no coupling enhanced error. Note that, the new method is more affected by the accelerometer phase error than 2-point method, while the influence of the angular rate sensor is small.



(a) Acceleration error 0.05 degrees, angular rate error 0.15% , 20%



(b) Acceleration error 0.20 degrees, angular rate error 0.15% , 20%

Figure. 8 - Influence of combining accelerometer and angular rate noise

5. EXPERIMENT

5.1 Experimental Method and Device

The angular rate sensor used in the experiment, DTS ARS1500, is shown in figure 9. Figure 10 shows the whole experimental set up. The acrylic beam with a rectangular cross section (length 1000mm, width 15mm, thickness 15mm, Young's modulus 4.5 GPa) is supported in sand at both ends to neglect reflection wave from either edge. A shaker attached at 500 mm from the left end, and it excites the beam with a random signal. A force sensor is placed between the beam and the shaker to measure excitation force, acceleration, and input power.

Figure 11 shows the layout of the angular rate sensor and accelerometer to calculate SI by the new method. As a reference, SI was also calculated with the 2-point method with two accelerometers placed on both sides of the measurement points; the distance between the accelerometers was 20 mm. Measurement point of SI is 14 points, every 60 mm along the beam.

The calculated SI at every measurement points are also normalized; SI at each point is divided by the input power. Input power is derived from the following equation: by the cross spectrum of acceleration G_{FA}

$$P_{in} = -\frac{1}{2\omega}\mathrm{Im}\left[G_{FA}\right] \tag{16}$$

Figure. 9 - An angular rate sensor

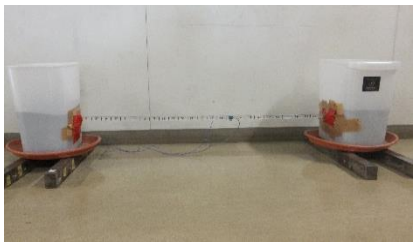


Figure.10 - Apparatus of experimental setup

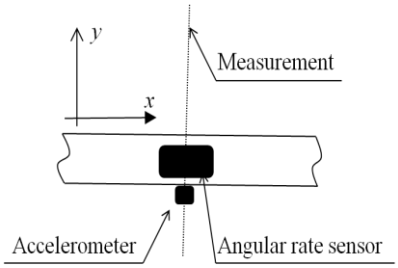


Figure.11 - side view of the beam

5.2 Comparison of the Angular Displacement

Before discuss about SI, measurement accuracy of the angular displacement is examined. The finite difference calculation with two accelerometers and data obtained with an angular rate sensor are compared. the angular displacement calculated from directly measured angular rate $\dot{\theta}$ with the following equation:

$$\theta = -j\frac{1}{\omega}\mathrm{Re}\left[\dot{\theta}\right] + \frac{1}{\omega}\mathrm{Im}\left[\dot{\theta}\right]. \tag{17}$$

In the experiments, angular displacement is derived from measuring the cross spectrum of angular rate and excitation force F, and is given by the following equation:

$$\frac{\theta}{F} = -j\frac{1}{\omega|F|^2}\left\{\mathrm{Re}\left[F \cdot \dot{\theta}\right]\right\} - \frac{1}{\omega|F|^2}\left\{\mathrm{Im}\left[F \cdot \dot{\theta}\right]\right\}. \tag{18}$$

Figure 12 shows real parts of angular displacements measured with two methods. More noise are observed on the angular rate sensor data especially in low frequency range compared with finite difference approximation. One possible reason of noise is that the angular rate sensor is in contact with the beam face; ideally there should be a point contact.

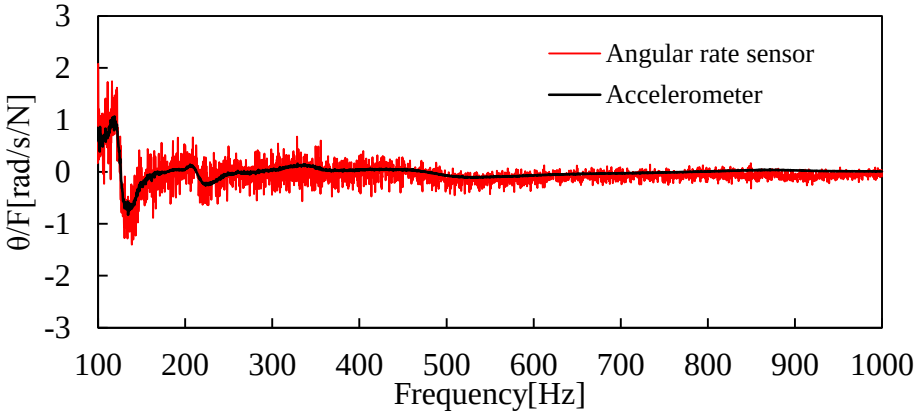


Figure. 12 - Comparison of angular displacements

5.3 Comparison of SI measured with two methods

Figure 13 shows comparison of normalized SI around 1000Hz obtained by the 2-point method and the new method. A sum of SI data for a range of ±20 Hz around a center of 1000Hz is calculated. Horizontal axis is the measurement location. At the 500 mm point, the beam is excited. SI results from the 2-point method and the new method are similar. However, there are differences due to noise in the angular rate sensor, as mentioned in Figure. 12.

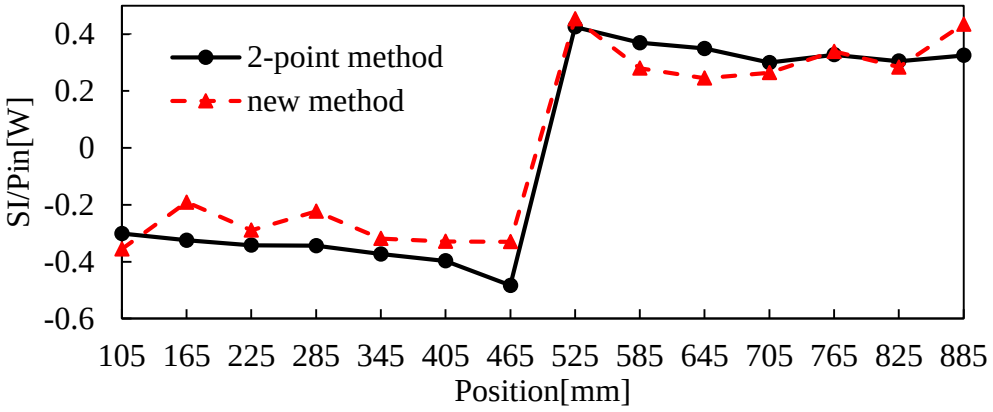


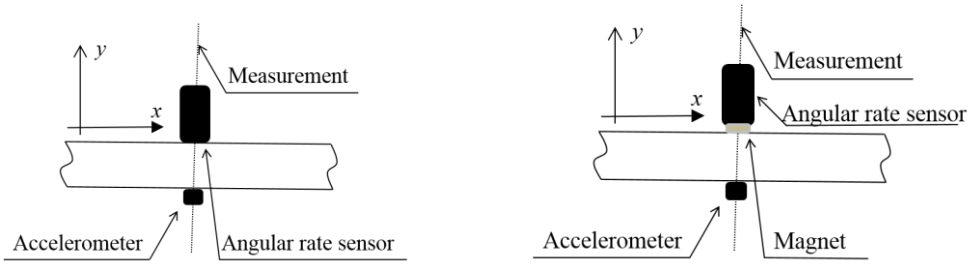
Figure. 13 - Comparing Structural Intensities from the new method and the 2-point method

5.4 The Influence in Location Method of Angular Rate Sensor

Additionally, the influence of sensor location on SI measurement accuracy is evaluated. Previous experiments were conducted with the angular rate sensor on the side of beam, as shown in Figure. 11. However, this location may be inappropriate for complex constructions. Alternative locations on the beam and through the magnet on the top are shown in Figure. 14 (a) and (b).

Figure 15 shows calculated comparison of SIs for the three locations at the 705 mm measurement point. It can be seen that sensor location does not affect SI measurement accuracy.

In the future, a study of complex constructions is planned.



(a) On the top of the beam (b) Through the magnet on the top of the beam

Figure. 14 - Angular rate sensor locations for the new method

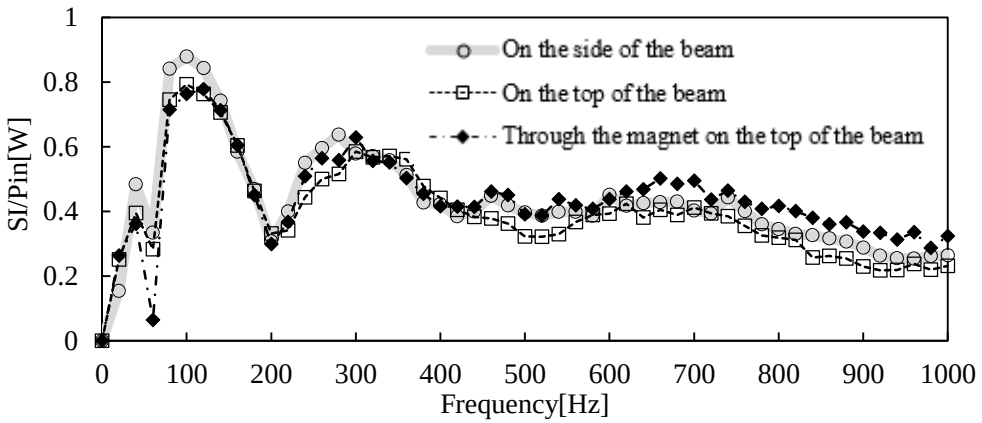


Figure. 15 - Comparing Structural Intensity for each location of the angular rate sensor

6. CONCLUSIONS

In this paper a new measurement method of SI using an angular rate sensor is proposed and its feasibility is discussed through numerical simulation and experiment target on acrylic beam; expected eliminate the error induced by a finite difference approximation. The results obtained in this study are the following.

1) Equations, which derive SI in far field using the angular rate sensor, is developed.
2) The new method is compared with the 2-point method through numerical simulation and its validity is shown. The new method have advantage over the 2-point method because it does not operate the finite difference approximation. However the influence of sensor error is similar on both methods.
3) Effectiveness of the new method is also examined through an experiment using an angular rate sensor. The result showed that the new method could measure SI at the same level of SI measured with the 2-point method.

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