Tailoring Acoustic Metamaterials to Aeroacoustic Applications

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ABSTRACT

The advent of metamaterials in acoustics is shaking-up the research community. The theoretical models and the realization technologies developed during the last decade have suddenly made possible applications that were barely conceivable before. Acoustic invisibility, perfect acoustic mirrors and lenses or ideal wave guides are now realizable with devices exploiting the potential of acoustic metamaterials. The present paper aims to investigate the applicability of such technologies in the aeroacoustic domain. The final goal of the research is the disclosure of the tremendous potential of metamaterial technologies in the abatement of civil aviation noise. The acoustic metamaterial theory developed so far is based on the assumption that the compressible medium and the scattering obstacles are at rest. This is clearly a limitation not compatible with aeroacoustic applications, where aerodynamic convection plays a fundamental role in the propagation and scattering patterns. The paper presents an original approach to extend the existing theory to the analysis of moving media and obstacles. Attention is focused on the design of a cloaking device, i.e. a metamaterial cover able to cancel the scattering of the cloaked object. Two formulations are presented, both based on the extension of the classical coordinate transformation approach to consider the existence of the mean flow. Preliminary numerical results are presented to demonstrate the effectiveness of both the approaches.

Keywords: aircraft noise, sound quality, noise annoyance, multi-objective optimization

I-INCE Classification of Subjects Number(s): 21.5, 23.6, 35, 38.5.2, 75.5

1. INTRODUCTION

In 2006, Leonhardt (1) and Pendry, Schurig and Smith (2) independently published in the same issue of *Science* (no. 5781, vol. 312) two papers on the possibility of designing metamaterials to manipulate electromagnetic fields through the application of an appropriate conformal mapping to the governing equations. These papers have opened up what is currently one of the most effervescent research topics in the field of applied physics, with possible engineering applications spreading from optical invisibility to electromagnetic super antennas. In addition, the basic idea has rapidly contaminated other branches of physical science, and has been almost immediately ported from the realm of electromagnetism to the land of mechanical vibrations. Milton *et al.*(3) analyzed the general properties of the formal changes of elastodynamic equations under curvilinear transformations, whereas Cummer and Shurig (4) have disclosed the path to acoustic cloaking (*i.e.*, the possibility of cancelling the scattering response of an obstacle impinged upon by an acoustic perturbation with a suitable metamaterial cover) using the same approach. From that point on, the number of scientific papers published on the subject has grown exponentially and it is almost impossible to keep track of all the contributions produced so far. Here, we'd like to mention at least the work of Sanchez-Dehesa and his collaborators (*e.g.*, (5, 6, 7, 8, 9)) mainly focused on the actual realization of metamaterial-based acoustic devices, as well as the key paper published by Andrew Norris in 2008 (10), where the fundamental theory of the acoustic cloaking via coordinate transformation has been formalized for the first time. In that interesting paper, Norris has proposed an approach to circumvent the most penalizing limitation of the so–called *inertial acoustic cloaking*: a metamaterial designed through coordinate transformation would produce devices having infinite mass. Indeed, the propagation of an acoustic perturbation involves the local motion of material particles, and thus the inertial properties of the medium play a fundamental role. Focusing on the constitutive equation of the hypothetical material, rather than simply on the mass/momentum conservation laws, would

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help in overcoming this drawback. This achievement is particularly appealing in the present context, as this paper deals with the analysis of the potential of acoustic metamaterials in aeroacoustics, with particular attention paid to aeronautical applications, for which weight is a key aspect. Unfortunately, this is not the only technical problem which must be addressed. The other limitation of the approaches presented above lies in the assumption that the acoustic phenomenon takes place in a medium at rest, in presence of motionless obstacles. This is clearly never true in the real word, where sources, obstacles and media are typically in relative motion, in some case with speed comparable to that of the acoustic perturbation. In particular, this is the case for commercial aircraft, for which the Mach number could range from 0.2 at take–off and landing, up to 0.9 in cruise. At these speeds, all the predictions made using the classic coordinate transformation approach would fail, making the metamaterial design completely useless. Indeed, the aspect which makes the extension of the coordinate transformation approach to this specific case not exactly straightforward resides in the fact that the conformal mapping required to rewrite the convected wave equation in the form of a generalized d'Alembertian operator necessarily mixes space and time. In Visser (11) it has been noticed how this class of transformations yields a relativistic metric, with the consequence that the appropriate mathematical tools must be used to obtain suitable transformations. This concept has been applied in Garcia–Meca *et al.*(12) to show its effectiveness in handling simple aeroacoustic problems. Here, the same approach is used to reinterpret some classic coordinate transformations used in acoustics within this framework. Eventually, a possible strategy to derive a metamaterial tailored to the aeroacoustic problem from Taylor's transform is presented. The method suggested is based on the matching of the wave vectors of the incoming wave and the wave propagating inside the cloak at the interface between the two media. The aim of the paper is to provide to the aeroacoustic community an approach to the problem using familiar tools. The final goal of the research is the design of metamaterials specifically tailored to the development of a new generation of devices for the abatement of the noise produced by commercial aircraft. Here, we address the analysis of the well–known Taylor transformation (13) within this framework, including some preliminary numerical results, obtained with the boundary integral formulation presented in Iemma and Burghignoli (14).

2. TAYLOR'S TRANSFORMATION

This transformation was first introduced by Taylor (13) to analyze the effect of a uniform main stream on the propagation of acoustic perturbations in the far field, and provide a tool for the interpretation of low–speed wind–tunnel aeroacoustic experiments, and the correction of static measurements. The transformation is based on the approximation of the convected wave equation to order $\mathscr{O}(M_0)$, M_0 being the asymptotic Mach number of a steady homentropic potential flow,

$$\left(\frac{\partial^2}{\partial t^2} + 2\nabla\Phi\cdot\nabla\frac{\partial}{\partial t} - c_0^2\nabla^2\right)\phi = S(\boldsymbol{x},t), \tag{1}$$

where c_0 is the speed of sound at infinity, t the time, Φ is the aerodynamic velocity potential, ϕ is the acoustic velocity potential generated by the source distribution S in the rectangular cartesian coordinates x,y,z. The condition for ϕ on the boundary $\Gamma(\boldsymbol{x})$ of an acoustically rigid object is

$$\nabla\phi\cdot\nabla\Gamma = 0 \qquad \text{on} \qquad \Gamma(\boldsymbol{x}) = 0, \tag{2}$$

$$\text{and} \qquad \phi = \frac{\partial\phi}{\partial t} = S = 0 \qquad \text{when} \qquad t < t_0 - \frac{\Phi}{c_0^2}. \tag{3}$$

In the case of aerodynamic impermeability of the boundary Γ the condition on Φ (which satisfies the Laplace equation, $\nabla^2\Phi = 0$) is

$$\nabla\Phi\cdot\nabla\Gamma = 0 \quad \text{on} \quad \Gamma(\boldsymbol{x}) = 0, \tag{4}$$

$$\text{and} \quad \Phi \approx x \quad \text{as} \quad r \to \infty. \tag{5}$$

where $r \equiv |\boldsymbol{x}| \equiv \sqrt{x^2+y^2+z^2}$. Taylor's transformation consists in applying a time shift which is not uniform in space, due to the presence of the potential of the mean flow. The presence of a space-dependent variable in the time term introduces new terms in the operation of the gradient and in the Laplacian. Applying the transformation, Equation 1 is recast in the form of the classic wave equation. Indeed, let's assume

$$(\boldsymbol{X},T) = \left(\boldsymbol{x}, t + \frac{\Phi}{c_0^2}\right), \qquad \varphi(\boldsymbol{X},T) = \phi(\boldsymbol{x},t), \tag{6}$$

where $\boldsymbol{X}$ represents a point in the transformed space. This space–time mixing is the reason for the need for a more sophisticated approach to the transformation acoustics, that will be examined later. The relationships between the differential operators in the two spaces are

$$\frac{\partial}{\partial t}=\frac{\partial}{\partial T}, \quad \nabla=\nabla_X+\nabla\Phi\frac{1}{c_0^2}\frac{\partial}{\partial T}, \quad \nabla^2=\nabla_X^2+2\nabla\Phi\cdot\nabla_X\frac{\partial}{c_0^2\partial T}+\frac{|\nabla\Phi|^2}{c_0^4}\frac{\partial^2}{\partial T^2} \tag{7}$$

where the subscript X indicates differentiation with respect t $\boldsymbol{X}$. Clearly, Eqs. 1 and 6 yields, to the order M_0,

$$\left(\frac{\partial^2}{\partial T^2}-c_0^2\nabla_X^2\right)\varphi=S(\boldsymbol{X},t), \tag{8}$$

i.e., the convected wave equation in the physical space is transformed into the classic wave equation for a medium at rest.

3. TRANSFORMING A CLOAKED FIELD

Taylor has shown in (13) how the transformation presented can be used to find the relationship between the static acoustic potential φ_s and that propagating in the presence of an aerodynamic background velocity. The results presented in Taylor's work is

$$\varphi(\boldsymbol{x},t)=\varphi_s(\boldsymbol{x},t)\,e^{-i\,KM_0\,\Phi(\boldsymbol{x})/v_0} \tag{9}$$

As we have seen in the previous section, the correction stems from the fact that the coordinate transformation recast the convected wave equation to the standard static wave operator, under the assumption of low Mach number. In the present work, the correction 9 has been applied to the static cloaking of a sound–hard cylinder. Figure 1 depicts the geometry of the problem, which represents a classic benchmark first introduced in Pendry *et al.*(2). The static solution has been calculated using the approach presented in Iemma and Burghignoli

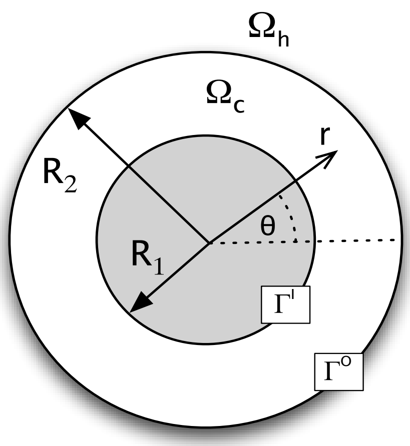
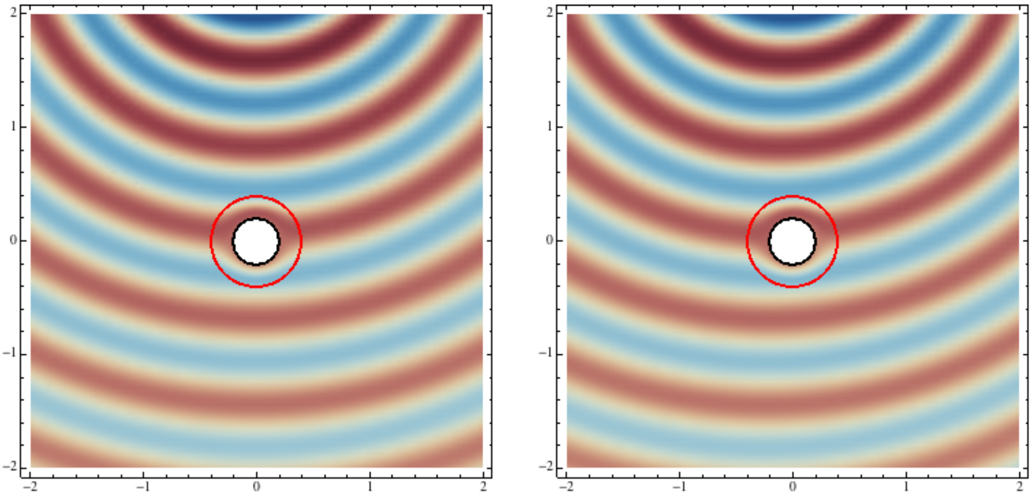


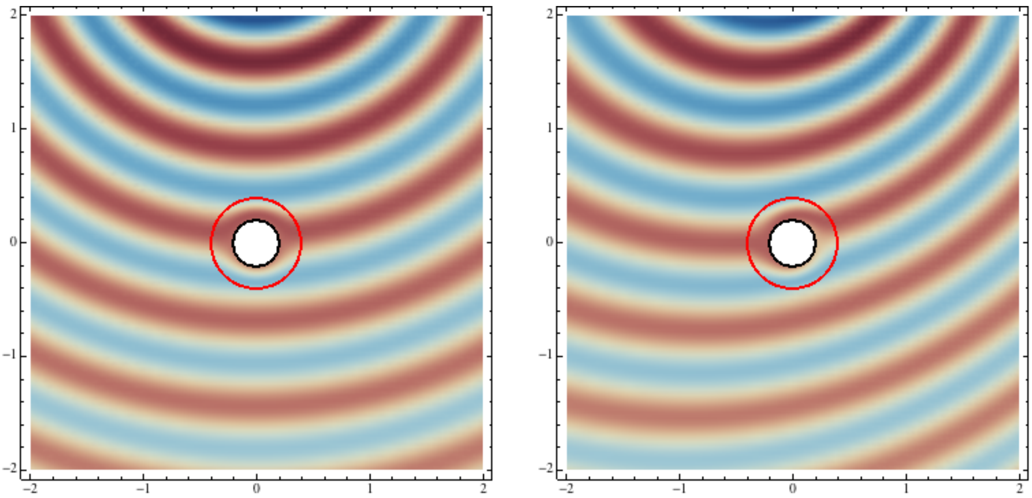
Figure 1 – Acoustic cloaking of a circular cylinder. Ω_h is the domain occupied by the hosting medium, whereas Ω_c is the portion of the space filled with the metamaterial.

(14) for an incident field produced by a monopole located at $\boldsymbol{x}_s=(0.0,\,3.0)$, pulsating at a frequency varying from 300 to 600 Hz. The source is assumed to be co–moving with the cloaked obstacle, with velocity aligned with x-axis. The background aerodynamic potential Φ is given by the classic analytical solution around a circle of radius a, $\Phi(R,\Theta)=v_0\,R\,cos\Theta\,\left(1+a^2/R^2\right)$. Figure 2 shows the effect of the correction for $M_0=0.05$. The motion of the object is, in this case, perfectly compatible with the low Mach number assumption on the basis of Taylor's transformation. The effect of the coordinate mapping is definitely satisfactory. The field in the hosting medium appears to be affected by the Doppler effect, and the propagation within the cloak is distorted accordingly. The latter is more evident in Figures 3 and 4, where the motion is at $M_0=0.2$ and $M_0=0.3$, respectively. Increasing the Mach number beyond the limit for which the transformation is valid, the effectiveness of the transformed clock is maintained also at higher frequencies, even if the the background incompressible potential solution used is not consistent anymore with the underlying physics (Figures 5,6 and 7). These results demonstrate how a classical coordinate transformation, widely used in aeroacoustics, can be helpful in the design of metamaterials suitable for the masking of objects moving in the low subsonic regime.



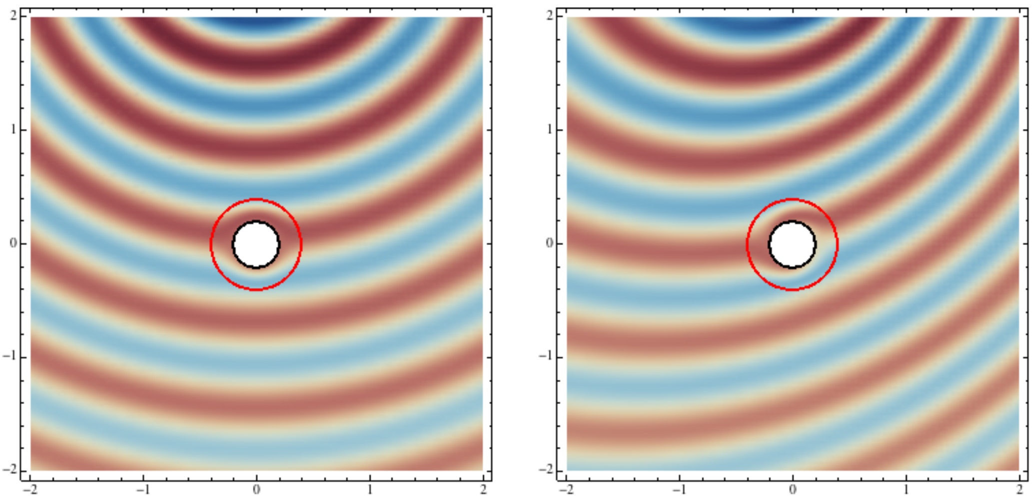
(a) Static solution (b) Transformed solution

Figure 2 – Cloaking of a cylinder moving at $M_0 = 0.05$, $f = 450$ Hz.



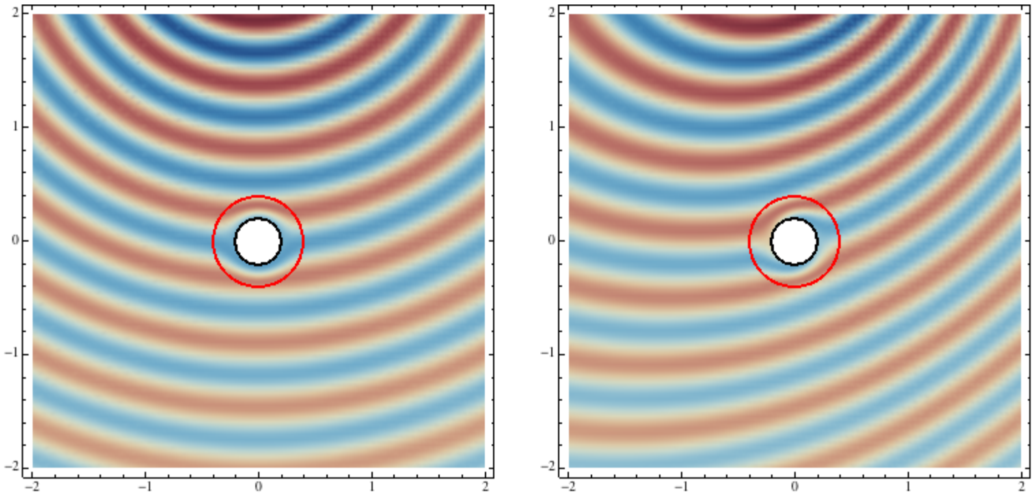
(a) Static solution (b) Transformed solution

Figure 3 – Cloaking of a cylinder moving at $M_0 = 0.2$, $f = 450$ Hz.



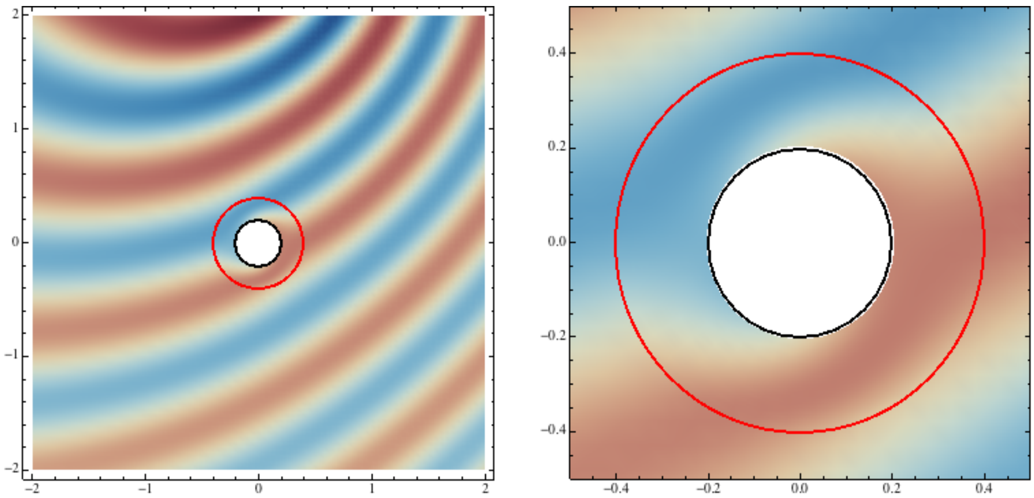
(a) Static solution (b) Transformed solution

Figure 4 – Cloaking of a cylinder moving at $M_0 = 0.3$, $f = 450$ Hz.



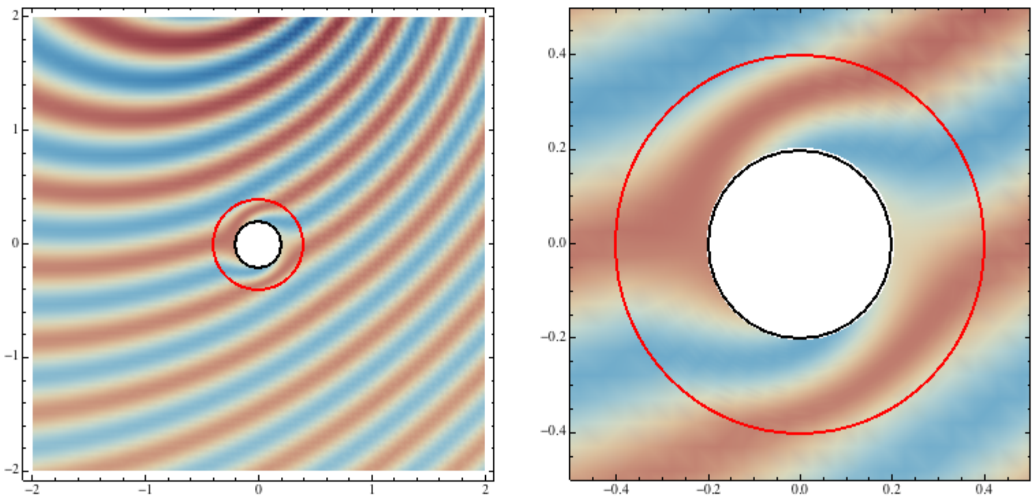
(a) Static solution (b) Transformed solution

Figure 5 – Cloaking of a cylinder moving at $M_0 = 0.3$, $f = 600$ Hz.



(a) Static solution (b) Transformed solution

Figure 6 – Cloaking of a cylinder moving at $M_0 = 0.5$, $f = 300$ Hz.



(a) Static solution (b) Transformed solution

Figure 7 – Cloaking of a cylinder moving at $M_0 = 0.5$, $f = 600$ Hz.

4. TRANSFORMING THE CLOAK

The analysis presented in Section 3 must be considered as a test of the applicability of the Taylor's transformation, but it has only a marginal significance in view of the design of a suitable metamaterial. Indeed, no modifications of the inertial and elastic properties of the metamaterial have been derived so far. Nevertheless, the properties of the coordinate transformation analysed and the presented results legitimate the expectation that a superposition of the classic coordinate mapping used to build the static cloak (for the present applications, see Cummer and Schurig (4)) and Taylor's time shift could provide a suitable modification of the bulk modulus and the inertial tensor. The major difficulty here resides in the fact that the Taylor transformation (like any other coordinate transformation used by aeroacousticians) is meant to be applied to the classic form of the convected wave equation, which is not form invariant with respect to mappings mixing space and time. Indeed, form invariance is the pillar of matamaterial design based on coordinate transformation, and this limitation strongly restricts the application of the tools developed by the aeroacoustic community. The remarkable works published by Visser (15, 11) address this problem from a relativistic viewpoint, pointing out that the propagation of acoustic perturbations in the presence of a background flow (of any kind) exhibits a Lorentzian structure, and that a form–invariant formulation of the equation can be obtained using appropriate tools. These results have been applied in Garcia–Meca *et al.*(12). It is worth mentioning that in Huang *et al.*(16), the problem of cloaking of a moving object is addressed in a way closer to a classic aeroacoustic approach, but without analyzing the relationship with the approach presented in Garcia–Meca *et al.*(actually, it is only mentioned in the paper). Anyway, both papers include noticeable simulations, although limited to very low Mach number and simple boundary geometries. Aim of our research is an attempt in interpreting the classic aeroacoustic transformations within this context.

Recalling the general form of the Laplacian operator in curvilinear (spatial) coordinates, the wave equation in a medium at rest can be written in the form

$$\frac{1}{\sqrt{g}}\frac{\partial}{\partial x_i}\left(\sqrt{g}\,g^{ij}\frac{\partial\varphi}{\partial x_j}\right)-\frac{1}{c_0^2}\frac{\partial^2\varphi}{\partial t^2}=0 \tag{10}$$

where g is the determinat of the (Riemannian) metric tensor and g^{ij} are the components of the inverse metric. It can be shown easily that Eq. 10 has the same structure under any transformation of coordinates involving only spatial variables. This is no longer true if the coordinate transformation mixes space and time. The metric tensor has, in such a case, a *Lorentzian* signature $(3,1)$ or $(1,3)$[1] (depending on the sign convention used), and thus a Riemanninan metric can no longer be used. Rather, the generalized d'Alembertian on a manifold having a Lorentzian structure has the form (we address the reader to Visser (15, 11), Garcia–Meca *et al.*(12, 17) for details).

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x_\alpha}\left(\sqrt{-g}\,g^{\alpha\beta}\frac{\partial\varphi}{\partial x_\beta}\right)=0 \tag{11}$$

where $01\leq\alpha,\beta\leq 3$, and $(x_0,x_1,x_2,x_3)\equiv(t,x,y,z)$. Visser demonstrated that the propagation of an acoustic perturbation within a moving medium can be recast in this form (see (15)). The Analogue Transformation Acoustics (ATA, see (12, 17)) approach exploits this evidence to formalize a procedure to derive the metamaterial properties also in the cases of coordinate mapping involving space and time mixing. Reinterpreting Taylor's transformation within this context can give an interesting insight on its direct applicability to the tailoring of aeroacoustic matematerials. Let's consider, for simplicity, the case where $\Phi(\boldsymbol{x},t)=v_0 x$, *i.e.*, a uniform mean flow aligned with the x–axis. In such a case, and using the space–time notation introduced, transformation 6 can be rewritten as

$$(X_0,X_1,X_2,X_3)=\left(x_0+\frac{X_1}{c_0^2},x_1,x_2,x_3\right) \tag{12}$$

The application of the ATA approach yields

$$g^{\alpha\beta}=\frac{1}{\rho_0\,c_0}\begin{pmatrix}-(1-M_0^2) & -v_0 & 0 & 0\\ -v_0 & c_0^2 & 0 & 0\\ 0 & 0 & c_0^2 & 0\\ 0 & 0 & 0 & c_0^2\end{pmatrix},\qquad \sqrt{-g}=\frac{\rho_0^2}{c_0} \tag{13}$$

[1]The signature (p,n) of a non–degenerate tensor represents the number of positive and negative eigenvalues, respectively.

which has clearly a Lorentzian signature. Equation 11 and 13 yields (indicating, for compactness, with subscript α the partial derivative w.r.t. x_α)

$$\frac{1}{c_0^2}\,\varphi_{00}\left(1-M_0^2\right)+2\,\frac{M_0}{c_0}\,\varphi_{01}-\varphi_{11}-\varphi_{22}-\varphi_{33}=0 \tag{14}$$

which, to the order M_0 and under the assumptions made, is identical to the convected wave equation, Eq. 1. Having said this, the application to the cloaked static solution appears to be legitimate, but it must be formulated so as to have some useful hint about the metamaterial properties required. To this aim, the matching of the pressure and acoustic velocity at the interface between the cloak and the hosting medium can be used, starting from the Fourier transforms of the corrected fields $\tilde{\varphi}(\boldsymbol{x})=\tilde{\varphi}_s(\boldsymbol{x})\,e^{-i\,K M_0\,\Phi(\boldsymbol{x})/v_0}$ and $\tilde{p}(\boldsymbol{x})=\rho_0 c_0\,(i K-M_0\nabla\Phi(\boldsymbol{x})\cdot\nabla)\,\tilde{\varphi}(\boldsymbol{x})$. The anisotropic nature of the metamaterial yields different values of the speed of sound in the radial and azimuthal directions, thus requiring the definition of appropriate correction factors in the two directions. Indeed, the speeds of sound in the cloak are given by $c_c^r=\sqrt{\mathscr{B}/\rho_\nabla}$ and $c_c^\theta=\sqrt{\mathscr{B}/\rho_\theta}$, where $\mathscr{B}$ is the bulk modulus and $\rho_{r,\theta}$ are the radial and azimuthal components of the unsteady inertia of the cloak. The matching of the field at the interface can be expressed in terms of coincidence of local wave vectors. In case of a uniform background velocity field $\boldsymbol{v}_0$ it yields

$$K\,\left(\hat{\boldsymbol{K}}+\boldsymbol{M}_0\right)\cdot\boldsymbol{x}=\boldsymbol{k}_c\cdot\boldsymbol{x}_c \qquad \forall\boldsymbol{x}\in\Gamma^o \tag{15}$$

where the hat indicates unit vectors, $\boldsymbol{k}_c$ is the wave vector observed at the cloak interface, and Γ^o is the outer boundary of the cloak. Decomposing the previous equation along the radial and azimuthal directions, the inertia tensor of the metamaterial in presence of a uniform flow, $\boldsymbol{\rho}^M$, can be obtained from the static one as

$$\boldsymbol{\rho}^M=\begin{pmatrix}\alpha^r\rho^r & 0\\ 0 & \alpha_\theta\,\rho^\theta\end{pmatrix}. \tag{16}$$

Using the relationship between speed of sound, bulk modulus and inertial coefficients, we obtain for the correction factors

$$\alpha^r=\frac{(K^r-M_0^r)^2}{c_0^2 K^{r2}}\,\frac{\Delta^2}{R_2^2},\qquad \alpha^\theta=\frac{\left(K^\theta-M_0^\theta\right)^2}{c_0^2 K^{\theta 2}}\,\frac{\Delta^2}{R_2^2} \tag{17}$$

where the superscripts r and θ indicate the radial and azimuthal components of the vectors in Eq. 15, Δ is the thickness of the cloaking layer, and r_2 its external radius. It is worth noting how the correction becomes isotropic if the incident field is a plane wave aligned with a uniform background aerodynamic velocity, in agreement with what obtained by other authors. The formulation is currently being applied to the classic Cummer-Schurig problem to be validated and compared to other results available in the literature.

5. CONCLUSIONS

An approach to the design of acoustic metamaterials for the cloaking of a moving object has been presented. The approach is based on the application of the coordinate transformation introduced by Taylor, originally developed for the correction of static acoustic measurements and the interpretation of wind–tunnel aeroacoustic experiments. The correction is valid for a background potential velocity field at low Mach number. The transformation has been applied to the static cloaked solution, showing satisfactory results within the applicability limits. An interpretation of the approach adopted within the framework of analogue transformation acoustics has been outlined, as well as a possible strategy to derive the mechanical properties required to tailor the metamaterial to the application at hand. The correction factors are here derived for a uniform stream, but the procedure can be extended to a generic background velocity. The suggested corrections are in course of validation, and numerical simulation will be presented at the conference to corroborate the theoretical derivation.

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