

Source identification of a vibrating plate using phase conjugation and interior boundary element method

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ABSTRACT

A method combining phase conjugation with interior boundary element method is developed for the identification of the pressure and normal velocity distribution of a vibrating plate. An interior problem is formed by enclosing the phase conjugation array plane and the plate surface. The pressures at the array elements are phase-conjugated as the specified pressure boundary condition. The impedance relationship between the surface pressure and the surface normal velocity of the plate is utilized as a specified impedance boundary condition. The interior boundary element method is applied to solve the interior problem. The identification of the surface pressure and normal velocity distribution is studied numerically. The numerical results show that with the array located in the near field the proposed method achieves subwavelength focusing to identify the surface pressure and normal velocity distribution and clearly shows the response shapes.

Keywords: source identification, phase conjugation, boundary element method, interior problem, response shape, subwavelength focusing I-INCE Classification of Subjects Number(s): 75.7, 74.6

1. INTRODUCTION

The identification of sound sources plays an important role in the effective noise control. Time reversal can be used to focus sound and is also called phase conjugation (PC) in the frequency domain. The equivalence between phase conjugation in the frequency domain and time reversal in the time domain is established by Jackson and Dowling [1]. Due to its focusing property, the phase conjugation arrays could be used to build the image of a noise source and for source identification. However, the spatial resolution of the focused field of a classical PC array has a half wavelength limit because of diffraction. de Rosny and Fink [2] first showed that this limitation can be overcome by an acoustic sink. Fink et al [3] also showed that the subwavelength focusing can be achieved inside a microstructured medium. Conti et al [4] demonstrated the subwavelength focusing could be obtained without a priori knowledge of the source by a near-field time reversal procedure. de Rosny and Fink [5] investigated three species of the time reversal arrays in the near field of the initial source and concluded that only the dipole time-reversal array leads to subwavelength focusing. Liu et al [6] investigated the source identification of complex sources by phase conjugation arrays.

The boundary element method (BEM) has been used extensively in acoustic radiation from bodies with known velocity, pressure, or impedance distribution. Typical problem types include interior and exterior problems [7]. For an interior problem, the objective is to solve the Helmholtz equation in a cavity of finite dimensions with cavity boundary surface. For an exterior problem, the objective is to solve the Helmholtz equation in an unbounded fluid domain due to the acoustic radiation from a vibrating structure with boundary surface.

In this paper, a method combining PC with BEM is developed for the identification of the pressure and normal velocity distribution of a vibrating plate. In the proposed method, the measured pressure at the PC array is phase-conjugated as the specified pressure boundary condition of an interior problem, the a priori knowledge of the impedance relationship between the surface pressure and the surface normal velocity of the plate is utilized as the specified impedance boundary condition. With the measured pressure by the array and the a priori knowledge of the impedance relationship on the plate surface, the surface pressure and velocity distribution of the plate could be identified by an interior BEM. The results of the numerical simulation show the proposed method achieves a resolution higher than half a wavelength and obtains subwavelength focusing.

2. THEORY

2.1 Forward problem of sound radiation

For structural acoustic radiation problems in free space, the boundary element formulation is based on the Helmholtz integral equation

$$C(P)p(P) = \int_{S} \left(\frac{\partial G(Q, P)}{\partial n} p(Q) - G(Q, P) \frac{\partial p(Q)}{\partial n} \right) dS(Q), \qquad (1)$$

where P is the complex amplitude of the acoustic pressure and has a harmonic time dependency of $e^{i\omega t}$, in which $i = (-1)^{1/2}$ and ω is the circular frequency of excitation; S denotes the surface of the structure; $G(Q, P) = e^{-ikR}/4\pi R$ is the free-space Green's function in which R = |Q - P|, Q is any point on S, and P maybe in the acoustic domain or on S; $k = \omega/c$ is the wavenumber, c is the speed of sound, and n is the outward unit normal on S. The coefficient C(P) is equal to 1 for P in the acoustic domain, and it is equal to 1/2 for P on a smooth S. For P on a boundary that is not smooth, such as an edge or a corner,

$$C(P) = 1 + \int_{S} \frac{\partial}{\partial n} \left(\frac{1}{4\pi R(P,Q)} \right) dS(Q), \quad P \in S.$$
⁽²⁾

The discretization of the surface Helmholtz integral equation $(P \in S)$ leads to a matrix equation

$$\mathbf{E}\mathbf{p} = \mathbf{D}\mathbf{v}_{\mathbf{n}} \quad , \tag{3}$$

where **P** and \mathbf{v}_n are the vectors consisting of the field values for the surface acoustic pressure and the surface normal velocity at the nodal locations of a grid defining the surface of the structure, **E** and **D** are the assembled coefficient matrices.

If a planar surface extends over an infinite half-space, the acoustic pressure at any field point P according to the Rayleigh integral can be described as follows

$$p(P) = i\omega\rho \int_{S} e^{-ikR} v_n(Q) / 2\pi R dS , \qquad (4)$$

where p(P) is the acoustic pressure at the field point P, $v_n(Q)$ is the normal velocity of the vibrating surface at a point Q on the plate surface, S is the plate surface. Discretisizing the plate surface into elements and interpolating the structural normal velocity and surface pressure over each element allow Eq. (4) ($P \in S$) to be written in terms of the nodal normal velocity \mathbf{v}_n and surface pressure \mathbf{p} as

$$\mathbf{p} = \mathbf{H}\mathbf{v}_{\mathbf{n}} \,. \tag{5}$$

For the acoustic radiation from a vibrating structure under excitation without consideration of acoustic fluid-structure interaction, the velocity \mathbf{v} could be obtained by the finite element method (FEM). The vector of normal velocity \mathbf{v}_n is related to the vector of the structural velocity \mathbf{v} by a transformation matrix \mathbf{G}

$$\mathbf{v}_{\mathbf{n}} = \mathbf{G}^{\mathrm{T}} \mathbf{v} \,. \tag{6}$$

Then the surface pressure \mathbf{p} could be obtained by Eq. (3) or Eq. (5).

2.2 Backward problem of sound radiation - sound identification using PC arrays

The reason that time-reversed sound waves travel backwards is a direct consequence of the lossless linear wave equation for the acoustic pressure p(r,t)

$$\nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0.$$
⁽⁷⁾

This equation is time-reversal invariant because it contains only second-order derivatives with respect to time. Equation (7) ensures that if p(r,t) is a solution then p(r,-t) is too. Thus, if p(r,t) represents sound waves expanding away from a sound source, then p(r,-t) represents sound waves converging toward the same source. In the frequency domain p(r,t) and p(r,-t) could be replaced by $p(r,\omega)$ and $p^*(r,\omega)$ respectively, where P has a harmonic time dependency of $e^{i\omega t}$ and superscript * designates complex conjugation.

For a perfect PC array, both the original field P and its normal derivative $\frac{\partial p}{\partial n}$ should be recorded to serve as the weighting factors for the arrays of monopole and dipole sources, the phase-conjugated field is given by ^[1]

$$p_{PCP}(P) = \int \left[G(Q, P) \frac{\partial p^{*}(Q)}{\partial n} - p^{*}(Q) \frac{\partial G(Q, P)}{\partial n} \right] dS'(Q), \qquad (8)$$

where S' denotes the surface of the PC array. The realistic PC arrays are discrete and have N array elements, the output of various discrete PC arrays may be defined as follows

$$p_{PCP}(P) = \sum_{n=1}^{N} \left[\left(G(Q_n, P) \frac{\partial p^*(Q_n)}{\partial n} - p^*(Q_n) \frac{\partial G(Q_n, P)}{\partial n} \right) \times S_n \right].$$
(9)

The phase-conjugated field p_{PCP} is based on both the pressure and pressure gradient measurement and made of both monopole transceivers and dipole transceivers to reverse sound backwards.

The phase-conjugated field by the array made of monopole transceivers based on the pressure measurement is

$$p_{PCM}(P) = \sum_{n=1}^{N} \left[\left(G(Q_n, P) p^*(Q_n) \right) \times S_n \right].$$
(10)

The phase-conjugated field by the array made of dipole transceivers based on the pressure gradient measurement is

$$p_{PCD}(P) = \sum_{n=1}^{N} \left[\left(\frac{\partial G(Q_n, P)}{\partial n} \frac{\partial p^*(Q_n)}{\partial n} \right) \times S_n \right].$$
(11)

In the following numerical analysis, the pressure calculated at the array element based on Section 2.1 is used as the measurement pressure in the above equations. A double layer of array elements is used to provide the pressure and the pressure gradient. That is, the pressure p = (p1 + p2)/2 and the pressure gradient $\frac{\partial p}{\partial n} = (p2 - p1)/\Delta$, where Δ is the separation distance.

Using Eqs. (9)-(11), the surface pressure can be identified and then the surface normal velocity can also be recovered from Eq. (3) or Eq. (5).

2.3 Sound identification using PC and interior BEM

As mentioned in 2.2, when the original field P and its normal derivative $\frac{\partial p}{\partial n}$ are measured with the PC array, the pressure on the source surface could be identified by using Eqs. (9)-(11) using only P or $\frac{\partial p}{\partial n}$ or both. It is clear that the above PC method only uses the information measured by PC arrays. As we know, usually the relationship between the acoustic pressure and normal velocity on the

arrays. As we know, usually the relationship between the acoustic pressure and normal velocity on the surface of the source is known before source identification. How to use this relationship in the source identification with PC array? Here we propose a method which combines PC with interior BEM for the identification.

For an interior problem with BEM, the objective is to solve the Helmholtz equation in a cavity of finite dimensions. In the proposed method, PC array is modeled as a surface with known pressure p^* , the source surface is modeled as a surface with known impedance, and these two surfaces are enclosed by fictitious medium surfaces with characteristic impedance of the medium to form an interior

problem. For demonstration, an interior problem of a fictitious box is formed and shown in Fig. 1. The source structure is a finite flat plate on one side (z = 0), the PC array on the opposite side (z = d) is a plane array that has the same dimension as the plate surface, and these two surfaces are enclosed by four fictitious medium surfaces between them to form the box.



FIG. 1. Schematics for the interior problem with source surface and PC array surface

For the interior problem, the Helmholtz integral equation is

$$C^{0}(P)p(P) = \int_{S} \left(\frac{\partial G(Q, P)}{\partial n} p(Q) - G(Q, P) \frac{\partial p(Q)}{\partial n} \right) dS(Q), \qquad (12)$$

where

$$C^{0}(P) = \int_{S} \frac{\partial}{\partial n} \left(\frac{1}{4\pi R(P,Q)} \right) dS(Q), \quad P \in S.$$
(13)

Then the link between the acoustic pressure and normal velocity on the surface can be obtained by the discretization of the surface Helmholtz integral equation (12) $(P \in S)$ and written as

$$\mathbf{p} = \mathbf{Z}\mathbf{v}_{\mathbf{n}} \,. \tag{14}$$

The boundary conditions for the solution of Eq. (14) are as follows:

(1) A pressure boundary condition $p = p^*$ is specified for the PC array plane (z = d),

$$\mathbf{p}_1 = \mathbf{p}^*. \tag{15}$$

(2) An impedance boundary condition $\mathbf{p} = \mathbf{Z}' \mathbf{v}_n$ is obtained by Eq. (5) and specified for the source surface (z = 0),

$$\mathbf{p}_2 = \mathbf{H}\mathbf{v}_{\mathbf{n}2} \,. \tag{16}$$

(3) A characteristic impedance boundary condition $\frac{p}{v_n} = \rho c$ is specified for the fictitious surfaces between the array surface and the source surface,

$$\mathbf{p}_3 = \rho \mathbf{c} \mathbf{I} \mathbf{v}_{n3} \, .$$

Among the above boundary conditions, Equation (16) can not be directly used for solving Eq. (14), we rewrite Eq. (16) as

$$\begin{cases} \mathbf{p}_{1} \\ \mathbf{p}_{2} \\ \mathbf{p}_{3} \end{cases} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{p}_{1} \\ \mathbf{v}_{n2} \\ \mathbf{p}_{3} \end{bmatrix} .$$
(18)

Then, Eq. (16) is related to Eq. (14) as

$$\begin{cases} \mathbf{p}_{1} \\ \mathbf{p}_{2} \\ \mathbf{p}_{3} \end{cases} = \begin{bmatrix} \mathbf{Z} \\ \mathbf{v}_{n2} \\ \mathbf{v}_{n3} \end{cases} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{cases} \mathbf{p}_{1} \\ \mathbf{v}_{n2} \\ \mathbf{p}_{3} \end{cases} .$$
 (19)

With the other two boundary conditions Eq. (15) and Eq. (17), Eq. (19) can be arranged in such a way that all the boundary unknowns \mathbf{v}_n stay on the left-hand side. Denote the unknown vector by \mathbf{x} , Eq. (19) becomes,

(17)

(20)

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$
.

Eq. (20) can be solved by a standard complex matrix solver. Once the v_n are solved, the pressure and other quantities can be calculated.

3. NUMERICAL RESULTS

The dimensions of the plate are $0.8m \times 0.6m \times 0.005m$, the plate material is steel ($\rho_s = 7833 \text{ kg/m}^3$, E = 210 GPa, $\nu = 0.3$). The geometric center of the plate is the origin of the rectangular coordinate system. The plate is simply supported in a baffle. The acoustic fluid is air with density $\rho = 1.21 \text{ kg/m}^3$ and speed of sound c = 350 m/s. The excitation is a transverse point force of magnitude 1 N located at (-0.2, -0.1, 0). The finite element method is used to get the natural frequencies and the surface normal velocity of the plate. The plate is discretized into 192 four-node elements and 221 nodes for finite element analysis. The natural frequencies of the steel plate are listed in Table 1. The surface pressure and the field point pressure at 500Hz (corresponding to the (5, 1) mode) are calculated by the Rayleigh integral based on the surface normal velocity obtained by finite element analysis. The Rayleigh integral is made by discretisizing the plate surface into the same mesh as plate finite element mesh. The surface normal velocities calculated by FEM are shown in Fig. 2. The surface pressure and m/s for the normal velocity. The calculated field pressures at the array elements are used as the measurement pressures of the PC arrays.

TABLE 1.	The natural frequencies of the steel	plate
Mode type	Natural frequency	(Hz)
(1, 1)	52.0	
(2, 1)	108.1	
(3, 1)	201.5	
(4, 1)	332.0	
(5, 1)	499.7	



FIG. 2. The normal velocity calculated by FEM



FIG. 3. The surface pressure calculated by Rayleigh integral

3.1. $d = 0.1\lambda$

The PC array is a plane array that has the same dimension as the plate surface, the number of the uniform distributed array elements is 221, the array element spacing is $0.05m = 0.07\lambda$. The surface pressures obtained by PC arrays using Eqs. (9)-(11) are shown in Figs. 4-6. The surface normal velocities obtained by PCD array are shown in Fig. 7. The surface normal velocities and the surface pressures obtained by interior BEM are shown in Fig. 8 and Fig. 9. The surface of the box is discretized into 776 four-node elements and 778 nodes in the boundary element analysis and the mesh on the plate

and the PC array plane are the same 16×12 of 192 elements and 221 nodes as in the above analysis. It can be seen that the results by the interior BEM clearly show the response shapes of the surface pressure and the surface normal velocity and the spatial resolution achieved by the interior BEM is at least $0.2m = 0.29\lambda$ which is smaller than half a wavelength (The half wavelength $0.5\lambda = 0.35m$).



 $(d = 0.1\lambda)$



FIG. 4. The surface pressure obtained by PCP FIG. 5. The surface pressure obtained by PCM $(d = 0.1\lambda)$

PRESSURE(Pa): 0.0005 0.001 0.0015 0.002 0.0025 0.003 0.0035 0.004 0.0045



FIG. 6. The surface pressure obtained by PCD FIG. 7. The normal velocity obtained by PCD $(d = 0.1\lambda)$ $(d = 0.1\lambda)$

3.2 $d = 3\lambda$

We only change the distance between the PC array and the plate surface from $d = 0.1\lambda = 0.07m$ to $d=3\lambda=2.1m$. The mesh on the plate and the PC array plane are the same as before, only the elements and nodes of the other four sides of the box are increased with increasing distance, and the surface of the box is discretized into 1504 four-node elements and 1506 nodes for interior BEM. The surface pressures obtained by the PCD and the interior BEM are shown in Fig. 10 and Fig. 11. We can see that with the array not located in the near field, both the interior BEM and the PCD fail to get high resolution to identify the exact response shape.

3.3 Comparison

To show the result more clearly, the normalized amplitudes of the surface pressures at y = -0.1m obtained by the Rayleigh integral, the PCD and the interior BEM are shown in Fig. 12. It can be seen more clearly that the interior BEM ($d = 0.1\lambda$) gives the best results of the response shape and the spatial resolution achieved by the interior BEM is less than $0.2m = 0.29\lambda$.



FIG. 8. The normal velocity obtained by interior BEM ($d = 0.1\lambda$)



FIG. 10. The surface pressure obtained by PCD ($d = 3\lambda$)



FIG. 12. The normalized pressure amplitudes at y = -0.1m



FIG. 9. The surface pressure obtained by interior BEM ($d = 0.1\lambda$)



FIG. 11. The surface pressure obtained by interior BEM ($d = 3\lambda$)



FIG. 13. The surface pressure calculated by Rayleigh integral for excitation location (-0.25, -0.25, 0)

3.4 Different excitation location

A different excitation location (-0.25, -0.25, 0) is considered for the identification of the response shape. The surface pressures obtained by the Rayleigh integral, interior BEM and PCD array are shown in Figs. 13-15. The surface normal velocities obtained by the FEM, interior BEM and PCD array are shown in Figs. 16-18. It is also clear that from these figures the interior BEM identifies the response shape excited from a different location by the force.



FIG. 14. The surface pressure obtained by interior BEM ($d = 0.1\lambda$) for excitation location (-0.25, -0.25, 0)



FIG. 15. The surface pressure obtained by PCD $(d = 0.1\lambda)$ for excitation location (-0.25, -0.25, 0)



FIG. 16. The normal velocity calculated by FEM for excitation location (-0.25, -0.25, 0)



FIG. 17. The normal velocity obtained by interior BEM ($d = 0.1\lambda$) for excitation location (-0.25, -0.25, 0)

4. CONCLUSIONS

The phase conjugation arrays could be used to build the image of a noise source and for source identification. However, the a priori knowledge of the impedance relationship between the surface pressure and the surface normal velocity of the original source is not utilized in the identification of the surface pressure. In this paper, the phase conjugation has been combined with the interior boundary element method to identify the sound sources. A plate subject to a point force is involved to verify the effectiveness of the method. An interior problem is formed by enclosing the PC array plane and the plate surface. The interior boundary element method is applied to solve the interior problem with the a priori knowledge of the impedance relationship of the plate as an impedance boundary condition and

the phase-conjugated pressure at the array plane as a pressure boundary condition. The identification of the surface pressure and normal velocity is studied numerically. The numerical results show that with the arrays in the near field the proposed method achieves a resolution higher than half a wavelength to identify the surface pressure and normal velocity distribution and clearly shows the response shapes.



FIG. 18. The normal velocity obtained by PCD ($d = 0.1\lambda$) for excitation location (-0.25, -0.25, 0)

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