# Optimization design method for Constrained Damping layer’s noise reduction based on the Hoff sandwich plate theory

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## ABSTRACT

Based on Hoff sandwich plate theory, an analytical method is proposed for predicting the loss factors of constrained damping layer. The application of the current approach in the engineering application of acoustic radiation optimization is examined. Numerical examples regarding isotropic sandwich plate with simply supported boundary conditions are carried out to demonstrate the accuracy and reliability of the current approach. It is shown that the current method has a good accuracy in determining loss factors of constrained damping layer. On this basis, optimization design of constrained damping layer is implemented. The effects of main parameters of damping layer on the loss factors are investigated. Then, the current approach is utilized to application of vibration and noise reduction for gearbox housing. Results show that parameters of damping layer have different effect on the vibration and noise reduction. The optimum design parameter of damping layer can be obtained through comprehensive comparison analysis. This investigation can provide an efficient design scheme of acoustic protection with damping layer for vibration and noise reduction.

Keywords: sandwich plate theory, Acoustic protection, damping layer, loss factors I-INCE Classification of Subjects Number(s): 38.3

## 1. INTRODUCTION

According to the damping theory, the most common method is surface damping treatment technology, which is mainly used by small bending vibration or thickness of the member-based thin-walled parts.

Guangli Cheng[1], used Statistical Energy Analysis (SEA) to study the method to calculate the loss factor, and illustrated the Statistical Energy Analysis with the application and limitations on its dissemination. SEA in the high frequency is widely used to predict vibration and noise transfer of the complex structure. By simulating frequency dependent of the viscous-elastic sandwich material properties with a sandwich complex modulus model, Zhigang Ren[2] and a group of people proposed the use of the modal strain energy iterative method and complex eigenvalues iterative method for solving complex meets of the natural frequencies and loss factor in sandwich structures and for example, in composite sandwich beam a comparative study and optimization design analysis of analytical solutions and numerical solutions of the theory are carried out. Chao Xu[3] and some people studied a finite element method analysis for natural frequency and loss factor embedded in multi-constrained damping viscous-elastic film laminated beam structure. The traditional method of their calculation and the experimental data were compared to show that the method had advantages of both a high accuracy and low computation cost.

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Based on Hoff sandwich plate theory, an analytical method which is verified by simply supported isotropic sandwich plate structure is proposed for calculating the loss factors of constrained damping layer, and its practical application in optimization measures of noise radiation is discussed in the essay; on this basis, the design of the constrained damping layer structure is optimized, and the influence of the main parameters of the constrained damping layer on loss factors is discussed, and it is taken into application in optimal design of the actual vibration and noise reduction in cabinet structure.

## 2. Calculation method of constrained damping structural loss factor based on sandwich plate theory

Constrained damping structure model as shown in Figure 1, the structure with three sheets is made of two layers of hard material and small thickness of the surface layer and a layer of an isotropic soft isotropic sandwich material. Assuming the thickness of the surface layer is $t_1$, thickness of the lower surface is $t_2$ (the materials of the upper and lower surface are assumed to be the same in the essay), $E$, $\mu$, $\rho_0$ are Young's modulus, Poisson's ratio, and density of materials; $h$ is the thickness of the Intermediate damping layer, $E_c$, $\mu_c$, $\rho_c$ are Young's modulus, Poisson's ratio, and density of damping layer; $\eta$ is loss factors of damping layer.

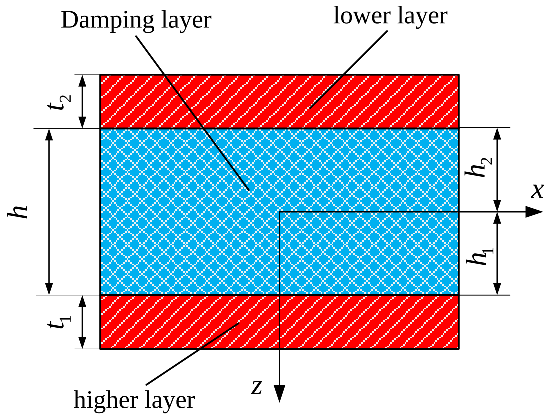



Figure 1 – Constrained damping structure

The basic hypotheses [4] of this theory used in the essay are:

(1) As the upper and lower surface is relatively thin, they can be considered as a sheet;

(2) The material of Intermediate damping layer is relatively softer than the hard material of the upper and lower surface, therefore the stress paralleling to the *xoy* plane in the damping layer can be ignored, which means $\sigma_x=\sigma_y=\tau_{xy}=0$;

(3) Since just considering antisymmetric deformation of constrained damping structure, it is assumed that in the damping layer, the strain $\varepsilon_z$ and stress of $z$ direction are so small that they can be ignored.

(4) There is no sliding between the ideal combinations of layers.

Assuming the displacement components of the substrate are $u^{\mathrm{I}}, v^{\mathrm{I}}, w^{\mathrm{I}}$, stress components are $\sigma_x^{\mathrm{I}}$, $\sigma_y^{\mathrm{I}}$, $\tau_{xy}^{\mathrm{I}}$; the displacement components of constrained layer are $u^{\mathrm{II}}, v^{\mathrm{II}}, w^{\mathrm{II}}$, stress components are $\sigma_x^{\mathrm{II}}, \sigma_y^{\mathrm{II}}, \tau_{xy}^{\mathrm{II}}$. According to figure 1, the distance from the neutral surface to adhesive surface of the substrate and damping layer is $h_1$, and the distance to adhesive surface of the constrained layer and damping layer is $h_2$, $\varphi_x$, $\varphi_y$ are respectively the angle in $xz$, $yz$ planes, which result in the connection deformation of corresponding points between the substrate plate and middle plane of constrained plate, the turning from $x$ axis and $y$ axis to $z$ axis are positive.

According to the first hypothesis, the displacement of the substrate and the constrained layer can be achieved

$$u^{\mathrm{I}} = -\left[h_1 + \frac{t_1}{2}\right]\varphi_x - \left[z - h_1 - \frac{t_1}{2}\right]\frac{\partial w}{\partial x} \tag{1}$$

$$v^{\mathrm{I}} = -\left[h_1 + \frac{t_1}{2}\right]\varphi_y - \left[z - h_1 - \frac{t_1}{2}\right]\frac{\partial w}{\partial y} \tag{2}$$

$$w^{\mathrm{I}} = w \tag{3}$$

$$u^{\mathrm{II}} = -\left[h_2 + \frac{t_2}{2}\right]\varphi_x - \left[z + h_2 + \frac{t_2}{2}\right]\frac{\partial w}{\partial x} \tag{4}$$

$$v^{\mathrm{II}} = -\left[h_2 + \frac{t_2}{2}\right]\varphi_y - \left[z + h_2 + \frac{t_2}{2}\right]\frac{\partial w}{\partial y} \tag{5}$$

$$w^{\mathrm{II}} = w \tag{6}$$

Due to the displacement continuity conditions, the displacement of its connected surface with substrate is

$$-\left[h_1 + \frac{t_1}{2}\right]\varphi_x + \frac{t_1}{2}\frac{\partial w}{\partial x}; \left[h_1 + \frac{t_1}{2}\right]\varphi_y + \frac{t_1}{2}\frac{\partial w}{\partial y}; w \tag{7}$$

The displacement of its connected surface with constrained layer is

$$\left[h_2 + \frac{t_2}{2}\right]\varphi_x - \frac{t_2}{2}\frac{\partial w}{\partial x}; \left[h_2 + \frac{t_2}{2}\right]\varphi_y - \frac{t_2}{2}\frac{\partial w}{\partial y}; w \tag{8}$$

According to the second and third hypotheses, the lateral displacement of the damping layer is the liner function of $z$, and it has the same deflection with substrate (or constrained layer). Expressions are as follows:

$$u^{\mathrm{e}} = -z\left[\frac{h + \frac{t_1 + t_2}{2}}{h}\varphi_x - \frac{t_1 + t_2}{2h}\frac{\partial w}{\partial x}\right] \tag{9}$$

$$w^{\mathrm{e}} = w \tag{10}$$

$$v^{\mathrm{e}} = -z\left[\frac{h + \frac{t_1 + t_2}{2}}{h}\varphi_y - \frac{t_1 + t_2}{2h}\frac{\partial w}{\partial y}\right] \tag{11}$$

For steady-state sinusoidal vibration, viscous-elasticity has a corresponding relationship with elasticity, when the viscous-elastic shear modulus is plural, its stress-strain relationship and elastic stress-strain relationship are consistent in form.

In the case of considering the lateral vibration only, the natural vibration equation of constrained damping layer structure is

$$D\left(\frac{\partial^2 \varphi_y}{\partial y^2} + \frac{1-\mu}{2}\frac{\partial^2 \varphi_y}{\partial x^2} + \frac{1+\mu}{2}\frac{\partial^2 \varphi_x}{\partial x \partial y}\right) + C\left(\frac{\partial w}{\partial y} - \varphi_y\right) = 0 \tag{12}$$

$$C\left(\nabla^2 w - \frac{\partial \varphi_x}{\partial x} - \frac{\partial \varphi_y}{\partial y}\right) - D_f \nabla^2 \nabla^2 w - \rho\omega^2 w = 0 \tag{13}$$

where $\omega$ is the natural frequency of the constrained damper plate.

The methods for the simplification of formula (13) can be consulted in the references, the introduction of the function $\omega$ and f makes the above equation reduce to two basic equations without coupling. But one point must be claimed that for most detail questions, in the expression of boundary conditions, $\omega$ and f are coupling. Therefore, they need a simultaneous solution. In the boundary conditions of an individual question, $\omega$ and f will not appeared at the same time, the question will be greatly simplified, and $\omega$ and f can be solved individually. Therefore, the angle and the deflection are expressed by$\omega$and f as below:

$$\varphi_x = \frac{\partial \omega}{\partial x} + \frac{\partial f}{\partial y}, \varphi_x = \frac{\partial \omega}{\partial y} - \frac{\partial f}{\partial x}, w{=}\omega - \frac{D}{C}\nabla^2 \omega \tag{14}$$

Substituting formula (14) into (13) to get

$$\frac{D}{2}(1-\mu)\nabla^2 f + Cf = 0 \quad (15a)$$

$$D\nabla^4 w + (1-\frac{D}{C}\nabla^2)(D_f\nabla^2\nabla^2 w - \rho\omega^2 w) = 0 \quad (15b)$$

where, $\rho=\rho_f(t_1+t_2)+\rho_c h$

On Base of the corresponding boundary condition, the natural frequency $\omega$ of constrained damping structure which is stated by plural can be achieved by solving formula (15); the natural frequency and loss factors of constrained damping structure can be achieved by solving formula (16).

$$\eta^2 = \frac{\mathrm{Im}(\omega^2)}{\mathrm{Re}(\omega^2)} \qquad f = \frac{\sqrt{\mathrm{Re}(\omega^2)}}{2\pi} \quad (16)$$

In the above process, its location parameter $h_1$ and $h_2$ with middle plane is uncertain. According to $\sum F_x=0$ and $\sum F_y=0$, there comes

$$\frac{E}{1-\mu^2}\left[-(h_2+\frac{t_2}{2})t_2+(h_1+\frac{t_1}{2})t_1\right](\frac{\partial\varphi_x}{\partial x}+\mu\frac{\partial\varphi_y}{\partial y})=0 \quad (17)$$

$$\frac{E}{1-\mu^2}\left[-(h_2+\frac{t_2}{2})t_2+(h_1+\frac{t_1}{2})t_1\right](\frac{\partial\varphi_y}{\partial y}+\mu\frac{\partial\varphi_x}{\partial x})=0 \quad (18)$$

When $\varphi_x$ and $\varphi_y$ value as random, formula (17) and (18) are both established. Therefore, there comes $-\left(h_2+\frac{t_2}{2}\right)t_2+\left(h_1+\frac{t_1}{2}\right)t_1=0$. Combining with geometric relation, $h_1+h_2=h$, $h_1$ and $h_2$ can be achieved.

$$h_1 = \frac{2ht_2 - t_1^2 + t_2^2}{2(t_1+t_2)} \; h_2 = \frac{2ht_1 + t_1^2 - t_2^2}{2(t_1+t_2)} \quad (19)$$

According to Mead & Markus[5] which makes the conclusions of the constrained damping loss factor structure on the impact of boundary conditions, changes in boundary conditions have no effect on the maximum loss factor of the structure, but it can make the natural frequency $f_{\max}$ of the maximum loss factor $\eta_{\max}$ move around. Assuming that boundary condition takes Simple Support Boundary Conditions, thus $f$=0, $\omega=\sum_m\sum_n A_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$, substituting them to equation (15) to get

$$\omega^2 = \frac{DK^2+\left(1+\frac{D}{C}K\right)D_f K^2}{\rho\left(1+\frac{D}{C}K\right)} \quad (20)$$

where, $K=\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2$, $a$ and $b$ are the length and width of the constrained damping structure, where $m$, $n$ =-1,2,…

Substituting $\omega^2$ to equation (15), the loss factor $\eta$ and natural frequency $f$ of the simply constrained damping structure of the subtense can be got.

## 3. The verification for the effectiveness of calculation methods

In order to verify the correctness of the calculation method which includes the loss factor of constrained damping structure, the essay chose an example given in the literature Johnson. It has an isotropic simply supported sandwich plate structure of 0.3048m×0.3480m, the thickness of upper and lower surface are both 0.762m, Elasticity is $E$ = 6.89×10$^4$ MPa, Poisson's ratio is $\mu$=0.30 , and density is $\rho$=2740 kg/m$^3$. The thickness of middle layer is 0.254mm, Shear modulus is 0.896MPa, Poisson's ratio is $\mu$=0.49 , density is $\rho$=999 kg/m$^3$ and loss factor is $\eta$=0.5.

The chart below gives the calculation results of the method proposed in this part, and the result listed in the references. In comparison, these two results comparatively match.

Table 1 The current results contrast with the FEM-based ABAQUS results

| Modal $m$, $n$ | Solution of theoretical literature | | Finite element solution literature | | the analytical solution in this paper | |
|---|---|---|---|---|---|---|
| | Frequency /Hz | Modal loss factor $\eta$ | Frequency /Hz | Modal loss factor $\eta$ | Frequency /Hz | Modal loss factor $\eta$ |
| (1, 2) | 60.3 | 0.190 | 57.4 | 0.176 | 60.2 | 0.189 |
| (1, 2) | 115.4 | 0.203 | 113.2 | 0.188 | 115.4 | 0.203 |
| (2, 1) | 130.6 | 0.199 | 129.3 | 0.188 | 130.2 | 0.199 |
| (2, 2) | 178.7 | 0.181 | 179.3 | 0.153 | 178.4 | 0.181 |
| (1, 3) | 195.7 | 0.174 | 196.0 | 0.153 | 195.5 | 0.174 |

From the results in Table 1 it can be seen that through Hoff sandwich plate theory the calculation method on natural frequency and the loss factor of constrained damping structure is simple, and it also has a high accuracy. Its results are in good agreements with the results calculated by software ABAQUS and related in literatures, so this method can be applied to the design and calculation of constrained damping plate structure.

## 4. The optimization design of constrained damping layer structure

The structural dynamics shows that, increasing the damping of box structure will reduce the vibration of it, which decreases radiated noise of the box structure. Once the material of the structure has been chosen, the only way to increase the damping of the structure is to set a constrained damping layer.

According to the given constrained damping layer, the thickness of damping layer can be changed, while Young's modulus of it is the initial value, $E = 8\times10^{6}$ Pa and the other parameters are constant. Research on the influence on the structure damping noise reduction with the thickness of damping layer structure is carried out and results are shown in the figure 2.

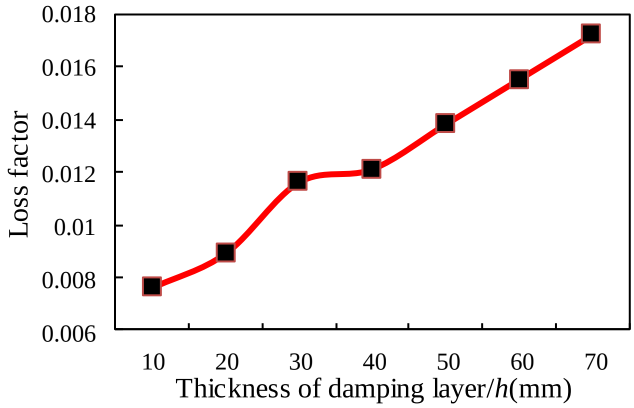



Figure 2 – The curve of loss factor changing with the thickness of damping layer

As it is shown in Figure 2, changes on the thickness of damping layer structure have some influence on loss factor of the constrained damping structure. The thicker the damping layer is, the greater the loss factor of the structure is, but the change is not very obvious, basically a linear growth. That’s to say, in the premise of reducing the structural radiation sound pressure. The thickness of damping layer should not be too bid. Whether to manufacture easily depends on the processing technology.

The thickness of damping layer is still the original value, 30mm, and other parameters remain unchanged. Research on the influence on loss factor of the constrained damping structure with the Young's modulus is carried out and results are shown in the figure 3.

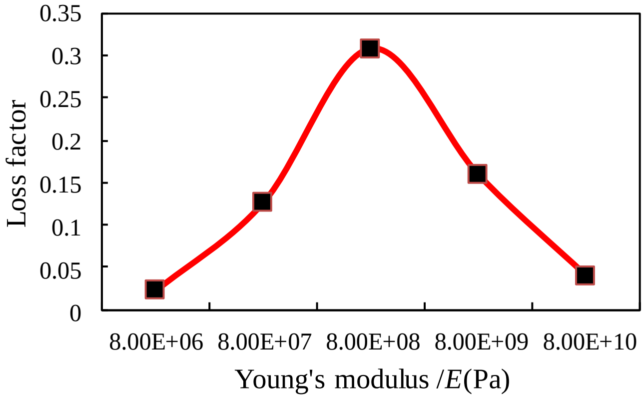



Figure 3 – The relationship between Yang's modulus and loss factor

From Figure 3: The increase of Yang's modulus has an effective influence on the loss factor of constrained damping structure. When Yang's modulus $E<1\times10^9$ Pa, loss factor of constrained damping structure increases exponentially as Yang's modulus increasing. When Yang's modulus $E>1\times10^9$ Pa, loss factor starts to decrease gradually as Yang's modulus increasing. Therefore, it is supposed to choose Yang's modulus at the range from $E=8\times10^8$ to $8\times10^9$ Pa.

Figure 4 shows that in the case of two kinds of Yang's modulus, the value of the total size of the radiation noise vibration contrast histogram of box structure. According to Figure 4: when Yang's modulus of damping layer $E=8\times10^8$Pa, the total radiation SPL of box structure is 35.94 dB; when Yang's modulus of damping layer $E=8\times10^6$Pa, the total radiation SPL of box structure is 42.71 dB. Thus, the changing of Yang's modulus has a big influence on the total radiation SPL. As a conclusion, when considering the damping noise reduction of the box structure, it can be focus on the Young's modulus of damping layer to do the constrained damping structure design.

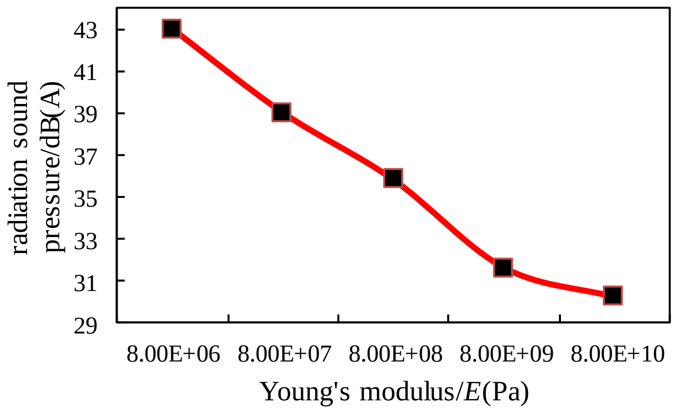
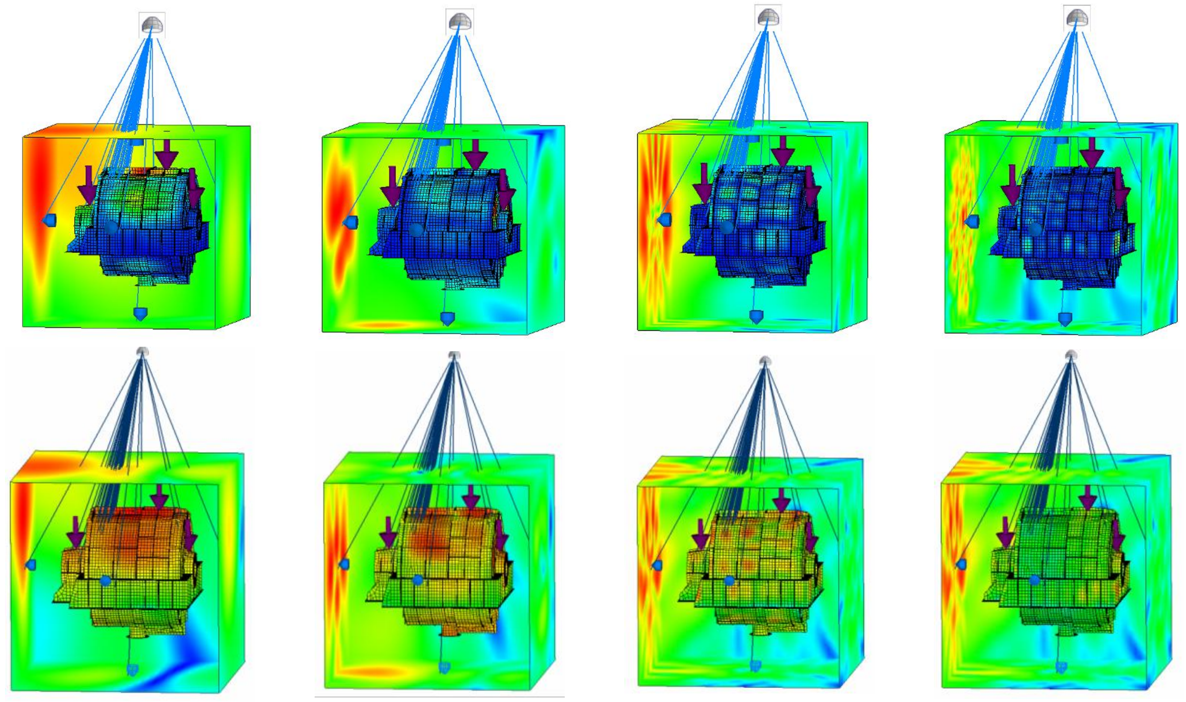



Figure 4 – Influence on radiation sound pressure of the box structure with changes of Young's modulus

Contrasting calculation results above, changes of Young's modulus of damping layer have more significant influence on loss factor than changes of thickness of damping layer. If doing damping noise reduction design, Young's modulus of damping layer should be considered firstly, approximately chosen at the range of $E$=8e8~8e9Pa. Selecting the thickness of the damping layer is not primarily from the aspect to increase loss factor, but more from other considerations, for example, processing craft. Therefore, we suggest that the thickness of damping layer can be maintained at the original level, 30mm.

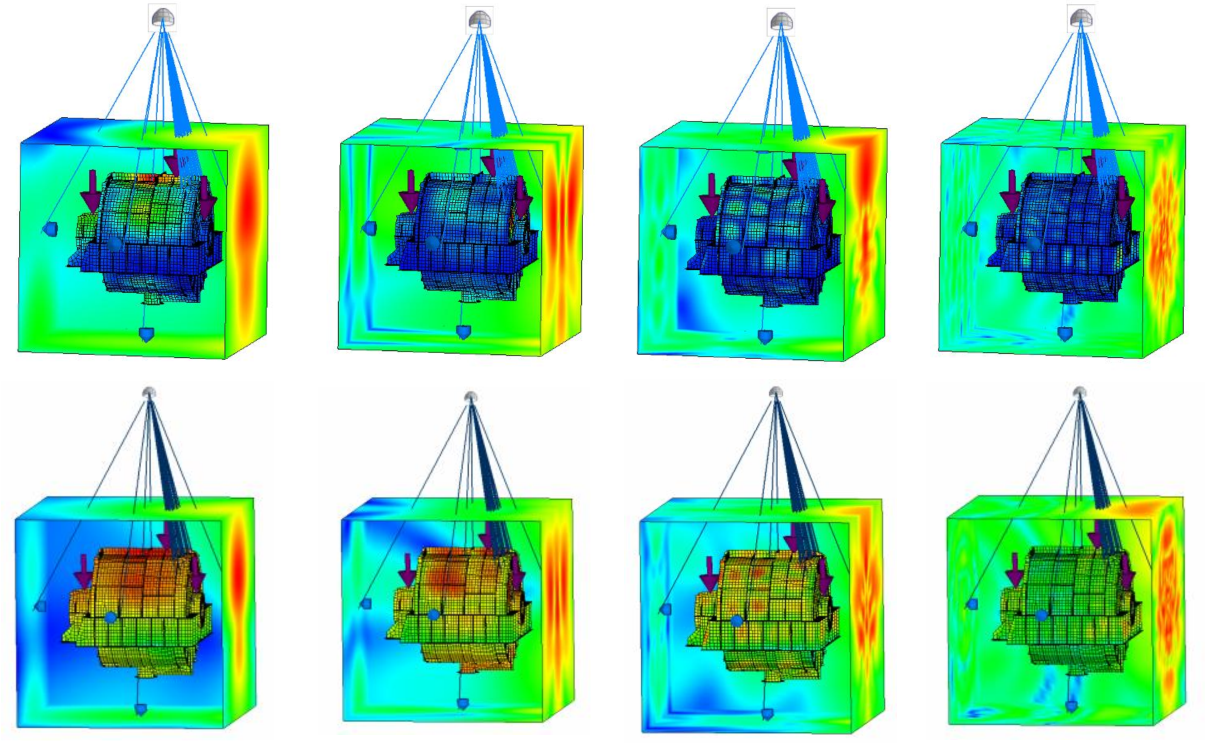
On base of loss factor of constrained damping structure, taking gear unit housing as an example, the comparison with results before and after the optimized design is made.

After optimized design, in the case of typical incentive, the sound radiation of back face and front face are shown in the pictures below.



(a) *f*=200 Hz (b) *f* =400 Hz (c) *f* =800 Hz (d) *f* =1600 Hz

Figure 5 – Under typical incentive, the sound radiation of back face before and after the optimized design



(a) *f*=200 Hz (b) *f* =400 Hz (c) *f* =800 Hz (d) *f* =1600 Hz

Figure 6 – Under typical incentive, the sound radiation of front face before and after the optimized design

Contribution of radiation sound pressure of each part on the surface of box structure is shown in figures 7-8. As it is seen from the figure, in the low frequency part, the front end face and the large circumferential plate have relatively large radiation noise compared to other parts; however, in the high frequency part, the radiation noise of the aft end face and small circumferential plate starts to increase, gradually exceeds that of the front end face and the large circumferential plate and becomes the dominant component of the radiation noise. Vibration characteristics of Box structure show significant changes after acoustic protection. Compared with results before the protection, total sound pressure has a decreasing of 15.9dB and the optimized design of noise radiation is effective.

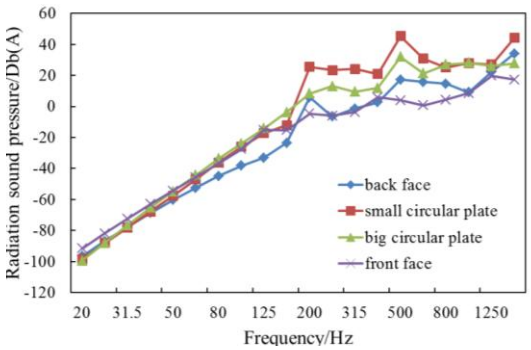



Figure 7 – Comparison on radiation sound pressure of each part under different frequencies before the optimized design

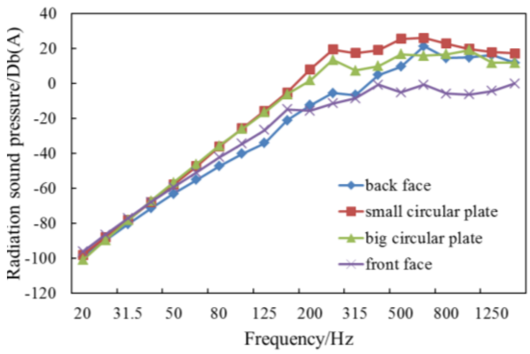



Figure 8 – Comparison on radiation sound pressure of each part under different frequencies after the optimized design

## 5. Conclusion

According to Hoff sandwich plate theory, a method of analyzing and calculating the loss factor of constrained damping layer is proposed in the essay, and its application in the optimized measure of the noise radiation is discussed; on the base, it is supposed to make the optimized design of constrained damping layer structure and also the analysis of the impact that the main parameters of the damping layer making on the loss factor of constrained damping layer, and it is taken applied in optimal design of the actual vibration and noise reduction of box structure. Therefore, the conclusions are:

(1) The analysis method of the loss factor of the constrained damping layer based on Hoff sandwich plate theory, has high calculation accuracy.

(2) Parameters of constrained damping layer structure make differently on the actual effect of vibration and noise reduction, it is possible to get the best damping layer parameters to optimize the design through comprehensive comparison, which provides a fast and effective forecast scheme about acoustic protection of constrained damping layer for vibration and noise reduction of the actual project.

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