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A power constrained algorithm for multi-zone sound reproduction

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ABSTRACT

In the process of multi-zone sound reproduction, large output power may be required because of the non-robust for the reproduction system, which will cause loudspeaker failure. To overcome this problem, Least Squares matching approach mostly be adopted to limit total output power. But this method may not assure every loudspeaker power is within the limit. So a power constrained optimization algorithm is proposed to make every loudspeaker power being constrained. Considering power constraint for multi-zone sound reproduction as a convex optimization problem, the primal-dual interior point algorithm is introduced. The simulation performance of reproducing multi-zone sound field in the reverberant room is shown to reach the goal of single power constraint. Therefore, each loudspeaker signal can be realized while minimizing the error between the reproductive and desired sound field.

Keywords: Multi-zone sound reproduction, Power constrained optimization algorithm, Primal-dual interior point algorithm I-INCE Classification of Subjects Number(s): 51.1

1. INTRODUCTION

Currently, vehicle interior sound quality evaluation is mainly based on ear-phone playback system, where the judgments of the subjects just depend on hearing sense. This hearing sense may be lack of stereoscopic impression and comfort by wearing ear-phones for long time[1]. Therefore, multi-zone 3D sound reproduction by loudspeaker array has been widely concerned. Compared to single zone 3D sound reproduction, multi-zone reproductive system is more non-robust，where the large reproduction errors may result from small perturbations of the acoustic transfer functions from the loudspeakers to the reproduction zones. Recently, the common method for improving the robustness is limiting the total output power by regularization[2-3], but this method may not ensure the power of each loudspeaker is beyond the threshold.

Thus, a pressure matching approach is presented to realize that the power of each loudspeaker is constrained. Primal-dual interior point algorithm[4-5] is applied for determining loudspeaker weights, and the multi-zone 3D sound reproductive simulation result in the reverberant room by this proposed algorithm is compared with regularized least squares method in the section 4.

2. PROBLEM STATEMENT

As Fig.1 shows, multi-zone sound fields are reconstructed in the reverberant room using L loudspeakers, where $D_1, D_2 \cdots D_N$ represent N_{th} reproductive region and an array of M microphones is distributed in the each zone. $\left[\mathbf{H}_1(f)\right]_{ml}$ denotes the transfer function from the l_{th} loudspeaker to the m_{th} microphone in the zone 1, so the total transfer function of all regions is $\mathbf{H}_{NM\times L} = \left[\mathbf{H}_1 \quad \mathbf{H}_2 \quad \cdots \quad \mathbf{H}_N\right]^T$.

The problem is to determine the weights of loudspeakers which minimizes the squared error between the desired sound field and the reproductive one over the pressure matching points, subject to a limit on each loudspeaker power:

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$$\begin{aligned} &\min_{\mathbf{G}(f)} \left(\left\| \mathbf{H}(f)\mathbf{G}(f) - \mathbf{P_d}(f) \right\|_2^2 \right) \\ &s.t. \quad g_i^H(f)\, g_i(f) \le b \quad for\, i = 1, 2 \cdots L \end{aligned} \tag{1}$$

where $\mathbf{G}(f) = \left[g_1(f) \quad g_2(f) \quad \cdots \quad g_L(f) \right]^T$, the vector of loudspeakers' excitations, $\mathbf{P_d}(f) = \left[P_{d1}(f) \quad P_{d2}(f) \quad \cdots \quad P_{d(NM)}(f) \right]^T$, the vector of desired sound pressures, ^{H} is the complex conjugate operator, $g_i^H(f)\, g_i(f)$ is assumed to the single loudspeaker power, and b is the power upper threshold.

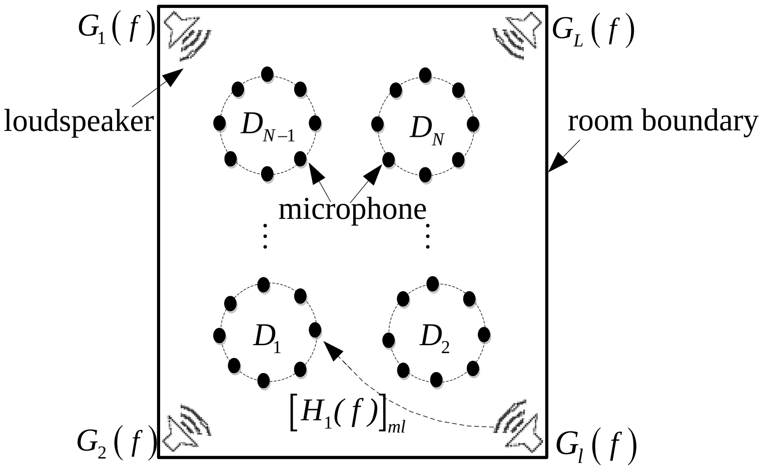

Figure 1 – Multi-zone reproduction system in the reverberant room

3. PRIMAL-DUAL INTERIOR POINT ALGORITHM

The optimized problem in section 2 is a Quadratically Constrained Quadratic Program (QCQP), which can be solved as a convex optimization problem[6]. Firstly, the following quantities are defined as:

$$\begin{aligned} &\mathbf{X} = \left[\mathrm{Re}\{\mathbf{G}\} \right], \mathbf{Y} = \left[\mathrm{Im}\{\mathbf{G}\} \right], \mathbf{Z} = \left[\mathrm{Re}\{\mathbf{G}\} \quad \mathrm{Im}\{\mathbf{G}\} \right]^T \\ &\mathbf{C}_R = \left[\mathrm{Re}\left\{ \mathbf{H}^H \mathbf{P}_d \right\} \right], \mathbf{C}_I = \left[\mathrm{Im}\left\{ \mathbf{H}^H \mathbf{P}_d \right\} \right], \mathbf{C} = \left[\mathbf{C}_R \quad \mathbf{C}_I \right]^T \\ &\mathbf{H}_R = \left[\mathrm{Re}\left\{ \mathbf{H}^H \mathbf{H} \right\} \right], \mathbf{H}_I = \left[\mathrm{Im}\left\{ \mathbf{H}^H \mathbf{H} \right\} \right], \mathbf{H}_Z = \begin{bmatrix} \mathbf{H}_R & -\mathbf{H}_I \\ \mathbf{H}_I & \mathbf{H}_R \end{bmatrix} \end{aligned} \tag{2}$$

So the Eq.1 can be reposed as:

$$\begin{aligned} &\min_{\mathbf{X}} \left(\mathbf{Z}^T \mathbf{H}_z \mathbf{Z} - 2\mathbf{C}^T \mathbf{Z} \right) \\ &s.t. \quad \mathbf{W} = b\mathbf{e}_L - \mathrm{diag}(\mathbf{X})\mathbf{X} - \mathrm{diag}(\mathbf{Y})\mathbf{Y} \\ &\mathbf{W} = \left[w_1 \quad w_2 \quad \cdots \quad w_L \right]^T, \quad w_i \ge 0 \ for\, i = 1, 2 \cdots L \end{aligned} \tag{3}$$

where $\mathbf{W}$ is the vector of the slack variables, $\mathbf{e}_L$ denotes the $L \times 1$ matrix where all elements are 1.

Then the Lagrangian form can be described as:

$$\Gamma(\mathbf{Z}, \boldsymbol{\lambda}) = \mathbf{Z}^T \mathbf{H}_z \mathbf{Z} - 2\mathbf{C}^T \mathbf{Z} + \mathrm{diag}(-\mathbf{W})\boldsymbol{\lambda} \tag{4}$$

where $\boldsymbol{\lambda} = \left[\lambda_1 \quad \lambda_2 \quad \cdots \quad \lambda_L \right]^T$ and the multipliers $\lambda_i \ge 0$.The optimal solution is found by setting the derivative of $\Gamma(\mathbf{Z}, \boldsymbol{\lambda})$ with respect to $\mathbf{Z}$ to be zero:

$$\frac{\partial \Gamma}{\partial \mathbf{Z}} = 2\left\{ \mathbf{H}_z \mathbf{Z} - \mathbf{C} + \mathrm{diag}(\mathbf{Z}) \left[\boldsymbol{\lambda}^T \quad \boldsymbol{\lambda}^T \right]^T \right\} = \mathbf{0} \tag{5}$$

Informed by primal-dual principle[7], any minimization problem corresponds to a maximization problem, thus, the Wolfe dual problem corresponding to Eq.1 is formed as follows:

$$\max_{\boldsymbol{\lambda}} \left(-\mathbf{C}^T \mathbf{Z} - b{\mathbf{e}_L}^T \boldsymbol{\lambda} \right) \tag{6}$$

Summarizing the above equations, along with the Karush-Kuhn-Tucker (complementarity) condition, $\mathrm{diag}(\boldsymbol{\lambda})\mathbf{W} = \mu \mathbf{e}_L$ (μ is the complementarity condition), the solution vectors will satisfy:

$$\begin{cases} \mathbf{H}_R \mathbf{X} - \mathbf{H}_I \mathbf{Y} - \mathbf{C}_R + \mathrm{diag}(\mathbf{X})\boldsymbol{\lambda} = \mathbf{0} \\ \mathbf{H}_I \mathbf{X} + \mathbf{H}_I \mathbf{Y} - \mathbf{C}_R + \mathrm{diag}(\mathbf{Y})\boldsymbol{\lambda} = \mathbf{0} \\ \mathrm{diag}(\mathbf{X})\mathbf{X} + \mathrm{diag}(\mathbf{Y})\mathbf{Y} + \mathbf{W} - b\mathbf{e}_L = \mathbf{0} \\ \mathrm{diag}(\boldsymbol{\lambda})\mathbf{W} = \mu \mathbf{e}_L \end{cases} \tag{7}$$

Eq.7 can be solved by iterative method[8]. Substituting $\mathbf{X} + \varDelta\mathbf{X}$ for $\mathbf{X}$, $\mathbf{Y} + \varDelta\mathbf{Y}$ for $\mathbf{Y}$, $\mathbf{W} + \varDelta\mathbf{W}$ for $\mathbf{W}$ and $\boldsymbol{\lambda} + \varDelta\boldsymbol{\lambda}$ for $\boldsymbol{\lambda}$ into Eq.7:

$$\begin{bmatrix} \mathbf{H}_R + \mathrm{diag}(\boldsymbol{\lambda}) & -\mathbf{H}_I & \mathbf{0} & \mathrm{diag}(\mathbf{X}) \\ \mathbf{H}_I & \mathbf{H}_R + \mathrm{diag}(\boldsymbol{\lambda}) & \mathbf{0} & \mathrm{diag}(\mathbf{Y}) \\ 2\,\mathrm{diag}(\mathbf{X}) & 2\,\mathrm{diag}(\mathbf{Y}) & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathrm{diag}(\boldsymbol{\lambda})^{-1}\mathrm{diag}(\mathbf{W}) \end{bmatrix} \begin{bmatrix} \varDelta\mathbf{X} \\ \varDelta\mathbf{Y} \\ \varDelta\mathbf{W} \\ \varDelta\boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{bmatrix} \tag{8}$$

where
$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{bmatrix} = \begin{bmatrix} -\mathbf{H}_R \mathbf{X} + \mathbf{H}_I \mathbf{Y} + \mathbf{C}_R - \mathrm{diag}(\mathbf{X})\boldsymbol{\lambda} - \mathrm{diag}(\varDelta\mathbf{X})\varDelta\boldsymbol{\lambda} \\ -\mathbf{H}_I \mathbf{X} - \mathbf{H}_R \mathbf{Y} + \mathbf{C}_I - \mathrm{diag}(\mathbf{Y})\boldsymbol{\lambda} - \mathrm{diag}(\varDelta\mathbf{Y})\varDelta\boldsymbol{\lambda} \\ b\mathbf{e}_L - \mathbf{W} - \mathrm{diag}(\mathbf{X})\mathbf{X} - \mathrm{diag}(\mathbf{Y})\mathbf{Y} - \mathrm{diag}(\varDelta\mathbf{X})\varDelta\mathbf{X} - \mathrm{diag}(\varDelta\mathbf{Y})\varDelta\mathbf{Y} \\ \mu\,\mathrm{diag}(\boldsymbol{\lambda})^{-1}\mathbf{e}_L - \mathbf{W} - \mathrm{diag}(\boldsymbol{\lambda})^{-1}\mathrm{diag}(\varDelta\boldsymbol{\lambda})\varDelta\mathbf{W} \end{bmatrix} \tag{9}$$

From the Eq.8, $\varDelta\mathbf{W}$ and $\varDelta\boldsymbol{\lambda}$ can be eliminated as follows:

$$\begin{cases} \varDelta\mathbf{W} = \gamma - 2\,\mathrm{diag}(\mathbf{X})\varDelta\mathbf{X} - 2\,\mathrm{diag}(\mathbf{Y})\varDelta\mathbf{Y} \\ \varDelta\boldsymbol{\lambda} = \mathrm{diag}(\mathbf{W}^{-1})\,\mathrm{diag}(\boldsymbol{\lambda})(\delta - \varDelta\mathbf{W}) \end{cases} \tag{10}$$

Substituting Eq.10 into the Eq.8, $\varDelta\mathbf{X}$ and $\varDelta\mathbf{Y}$ can be solved as follows:

$$\left(\mathbf{H}_z + \begin{bmatrix} \mathrm{diag}(\boldsymbol{\lambda}) & \mathbf{0} \\ \mathbf{0} & \mathrm{diag}(\boldsymbol{\lambda}) \end{bmatrix} + 2 \begin{bmatrix} \mathrm{diag}(\mathbf{X}) \\ \mathrm{diag}(\mathbf{Y}) \end{bmatrix} \mathrm{diag}(\mathbf{W})^{-1}\mathrm{diag}(\boldsymbol{\lambda}) \left[\mathrm{diag}(\mathbf{X}) \quad \mathrm{diag}(\mathbf{Y}) \right] \right) \begin{bmatrix} \varDelta\mathbf{X} \\ \varDelta\mathbf{Y} \end{bmatrix}$$
$$= \begin{bmatrix} \alpha \\ \beta \end{bmatrix} + \begin{bmatrix} \mathrm{diag}(\mathbf{X}) \\ \mathrm{diag}(\mathbf{Y}) \end{bmatrix} \mathrm{diag}(\mathbf{W})^{-1}\mathrm{diag}(\boldsymbol{\lambda})(\gamma - \delta) \tag{11}$$

Firstly, the initial numbers of solution vectors $\mathbf{X}$, $\mathbf{Y}$, $\mathbf{W}$ and $\boldsymbol{\lambda}$ are to be set, and the nonlinear terms in the right hand side of Eq.11 are ignored. Therefore, $\varDelta\mathbf{X}$ and $\varDelta\mathbf{Y}$ can be calculated by inversion of Eq.11 and $\varDelta\mathbf{W}$ and $\varDelta\boldsymbol{\lambda}$ can be calculated by Eq.10.

Then $\varDelta\mathbf{X}$, $\varDelta\mathbf{Y}$, $\varDelta\mathbf{W}$ and $\varDelta\boldsymbol{\lambda}$ are added to $\mathbf{X}$, $\mathbf{Y}$, $\mathbf{W}$ and $\boldsymbol{\lambda}$ by a factor ς to ensure that $\mathbf{W}$ and $\boldsymbol{\lambda}$ remain non-negative. The constraint ς [8]requires:

$$\varsigma = \min\left(0.95 \middle/ \max\left(\frac{-\varDelta w_i}{w_i}, \frac{-\varDelta v_i}{v_i} \right), 1 \right), i = 1, 2 \cdots L \tag{12}$$

The value μ of the next iteration is set as Eq.13:

$$\mu = \frac{\boldsymbol{\lambda}^T \mathbf{W}}{2(L+NM)}\left(\frac{\varsigma-1}{\varsigma+10}\right)^2 \tag{13}$$

Finally, the iteration is terminated when the difference between the primal objective Eq.3 and dual objective Eq.6 is below the threshold.

4. SIMULATION RESULT AND DISCUSSTION

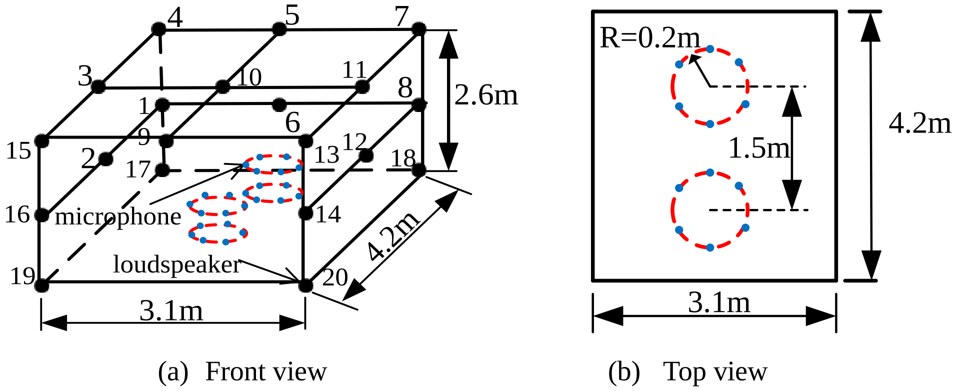

Figure 2 – The configurations of loudspeakers and microphones

3D sound fields of two-zones are reconstructed[9] based on single loudspeaker power constrained algorithm and regularized least squares method[10] desperately at 400Hz in the room of dimensions $3.1\times4.2\times2.6m$. The desired sound fields of two-zones are the same and the arrays of loudspeakers and microphones are shown in Fig.2. Each region is consisted with two circles and distributed by 12 microphones.

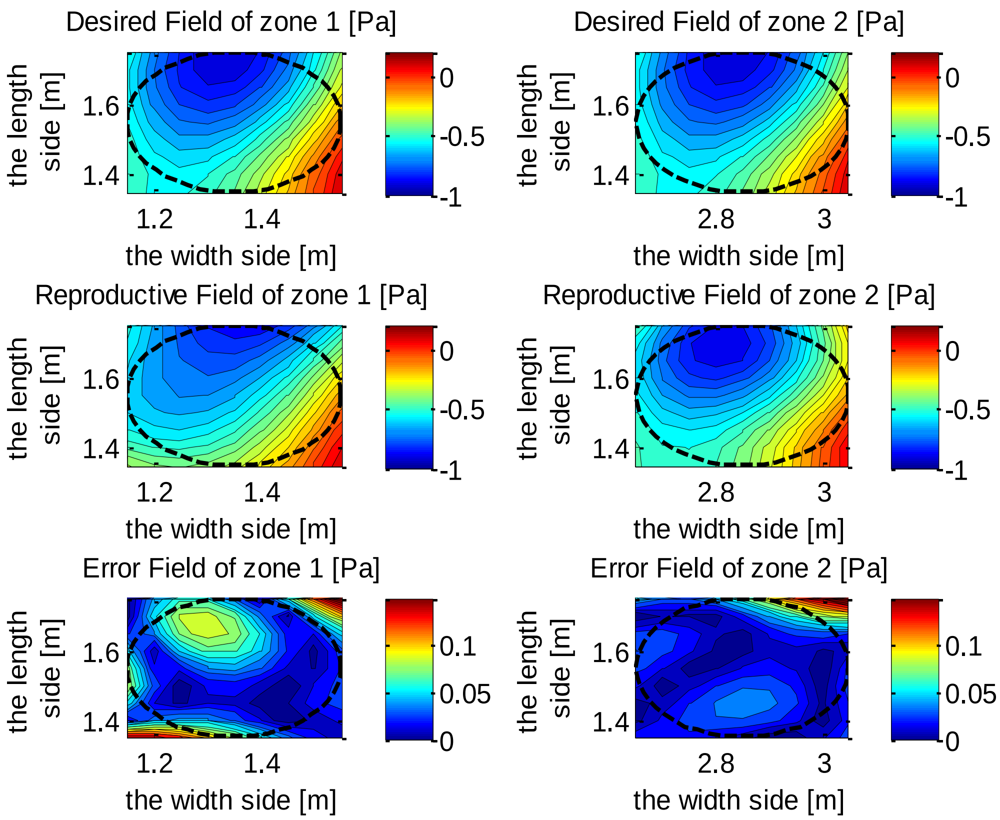

Figure 3 – Real part of reproductive results based on the single loudspeaker power constrained algorithm

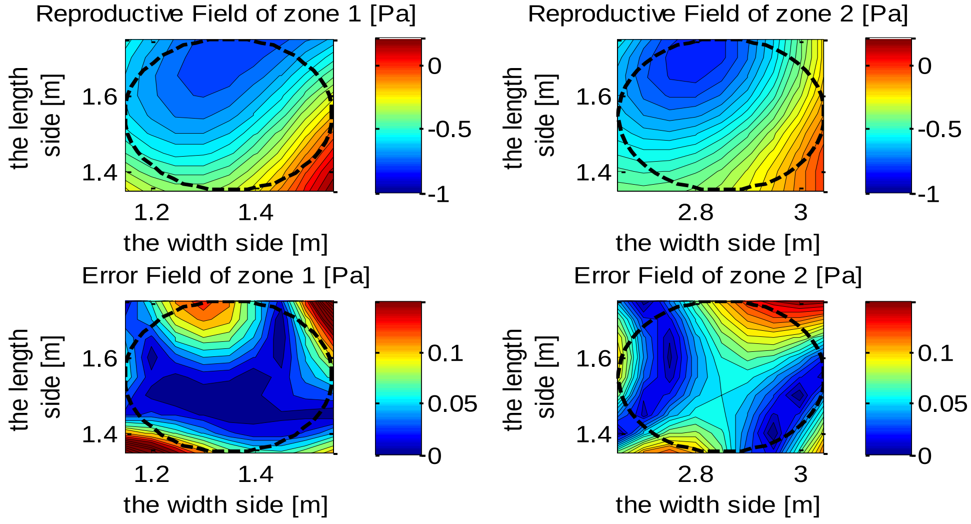

Figure 4 – Real part of reproductive results based on the regularized least squares method

Selecting single loudspeaker power upper threshold $b = 8$, the reproductive results based on the single loudspeaker power constrained algorithm are shown in Fig.3. The comparison surface in the figure is the middle cross section of the reconstructed region. The mean reproductive errors in the zone 1 and 2 are respectively 6.12% and 4.36%. Fig.4 shows the reproductive results based on regularized least squares approach and the mean reproductive errors in the zone 1 and 2 are 6.45% and 9.44% respectively. Thus, it can be found that the error of the former method is smaller. The required loudspeaker power based on the above two methods are compared in Fig.5, in which each loudspeaker power based on the former method is all constrained to the upper limit, the latter method may finally cause loudspeaker failure.

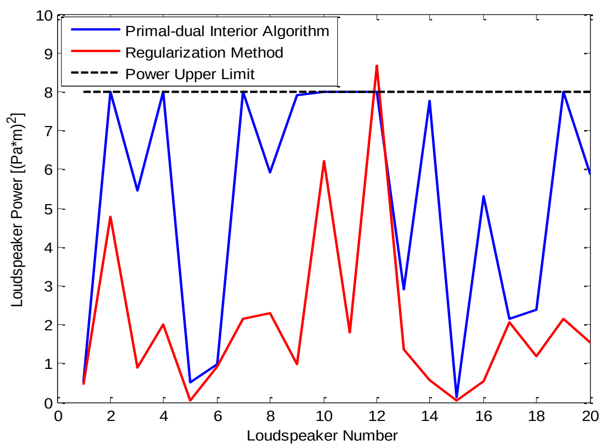

Figure 5 – Comparison of single loudspeaker power based on two methods

5. CONCLUSIONS

To improve the poor robustness in the multi-zone sound reproduction, the optimization approach is proposed, where the power of each loudspeaker is constrained and the error between the reproductive and desired sound field is minimized. Primal-dual interior point method is applied for solving this optimization problem. Then the 3D multi-zone sound field is reconstructed based on the power of single loudspeaker constrained algorithm, by comparison with the reproductive field based on regularized least squares matching approach. The result shows that the former constrained algorithm provides less error than the latter one.

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REFERENCES

1. Liao, X.N., Zheng, S. F., et al., A playback system for sound quality evaluation of interior noise, Proceedings of 21th International congress on Sound and Vibration, ICSV 2014.Beijing, China, 14-17 July, (2014).
2. Poletti M. An investigation of 2-D multizone surround sound systems[C]. Audio Engineering Society Convention 125. Audio Engineering Society, 2008.
3. Radmanesh N, Burnett I S. Reproduction of independent narrowband soundfields in a multizone surround system and its extension to speech signal sources[C]. Acoustics, Speech and Signal Processing (ICASSP), 2011 IEEE International Conference on. IEEE, 2011: 461-464.
4. Teal P D, Betlehem T, Poletti M A. An algorithm for power constrained holographic reproduction of sound[C]. Acoustics Speech and Signal Processing (ICASSP), 2010 IEEE International Conference on. IEEE, 2010: 101-104.
5. Vanderbei R J. LOQO: An interior point code for quadratic programming[J]. Optimization methods and software, 1999, 11(1-4): 451-484.
6. Betlehem T, Teal P D. A constrained optimization approach for multi-zone surround sound[C]. Acoustics, Speech and Signal Processing (ICASSP), 2011 IEEE International Conference on. IEEE, 2011: 437-440.
7. Liu M, Tso S K, Cheng Y. An extended nonlinear primal-dual interior-point algorithm for reactive-power optimization of large-scale power systems with discrete control variables[J]. Power Systems, IEEE Transactions on, 2002, 17(4): 982-991.
8. Mehrotra S. On the implementation of a primal-dual interior point method[J]. SIAM Journal on optimization, 1992, 2(4): 575-601.
9. Allen J B, Berkley D A. Image method for efficiently simulating small-room acoustics[J]. The Journal of the Acoustical Society of America, 1979, 65(4): 943-950.
10. Poletti M. An investigation of 2-D multizone surround sound systems[C]. Audio Engineering Society Convention 125. Audio Engineering Society, 2008.