High accuracy calculating model for sound field simulation with DFT-based FDTD on polar-quaternion-based axis towards craft restoration

Kota NAKANO[1]; Masato NAKAYAMA[2]; Takanobu NISHIURA[2]; Yoichi YAMASHITA[2]

[1] Graduate School of Information Science and Engineering, Ritsumeikan University

[2] College of Information Science and Engineering, Ritsumeikan Univeristy

ABSTRACT

In this paper, an approach is proposed to improve the numerical precision at the sound field simulation by the finite-difference time-domain (FDTD) method. The FDTD method is a numerical solution for the wave equation, proposed by R. Courant. Although it has achieved to simulate wave propagation by numerical solution for the wave equation, it causes numerical dispersions due to their approximated approach. Thus, we also studied to improve the precision of the FDTD method and have proposed a calculation model based on spatial spectrum. The improved method allows the FDTD to detach the approximations by DFT-based computation, and it has achieved to reduce numerical dispersion. However, the discrete Fourier transform(DFT)-based computation has caused wrong results, which are the wave spatially-wrapped propagation, due to the periodic extension of the DFT. Therefore, we have proposed a new approach to avoid the problems by employing polar-quaternion for the wave equation. In the approach, the wave-equation is represented in polar-coordinate with quaternion, and the DFT-based partial differential operator is applied on space-time angle (argument) spectrum. The argument domain is originally periodic, and the problems of periodic extension can be avoided by using the proposed approach. The conducted numerical experiments indicated that the approach successfully prevented the FDTD method from causing problems of DFT-based spatial extensions.

Keywords: Sound field simulation, FDTD, Quaternion, Spacetime
I-INCE Classification of Subjects Number(s): 74.3

1. INTRODUCTION

Space-time sound propagation can be presented by the wave equation[1]. Computer achieves to simulate the wave propagation by solving the wave equation, and several methods have been proposed to solve that. One of the methods, the finite-difference time-domain (FDTD) method[2, 3] is an effective method to solve the wave equation. The wave equation is approximated by the finite difference and deformed into a recurrence formula with the FDTD method, and the wave propagation achieves with iterative computation of the recurrence formula.

The conventional FDTD method has problems of numerical dispersion due to the approximation. Thus, we have studied to reduce numerical dispersions and have previously proposed the spatial spectrum-based approach, which does not require the numerical approximation.

However, the discrete Fourier transform (DFT) applies the periodic extension[4] to the target space. Accordingly, DFT-based computation depends on periodic convolutions, and the computation provides incorrect impulse responses by wrapped propagation. In this paper, a new approach is proposed to avoid the problem of wrapped propagation. In this approach, the sound pressure distribution is redefined from the spatial domain to the polar-quaternion domain. In the domain, the wave equation can be deformed by the absolute value (norm or Euclid-distance) and the arguments as quaternion's real-and-imaginary parts. Then, the wave equation is deformed into a recurrence formula for the real part calculation with the imaginary part domain instead of spatial domain. Each computation depends on only argument domain. In addition, the argument domain is covered by the trigonometric function, and the trigonometric function is originally wrapped. As a result, the proposed approach avoids the wrapped propagation. Several numerical experiments were conducted to evaluate the proposed approach. Their results indicated that neither numerical dispersion nor wrapped propagation was caused in the sound filed simulation.

2. DFT-BASED FDTD METHOD

We have previously proposed the DFT-based FDTD method to simulate wave characteristics without numerical dispersions, and that was achieved by using an improved FDTD method. By calculating in the frequency domain, the accuracy was improved[5]. Although reducing numerical dispersion was achieved, the spatially wrapped propagation was caused.

2.1 Approximated deformation of the wave equation for computation

The wave characteristic follows the wave equation which can be written as ordinary differential equation,

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}u(t,x) = \frac{\partial^2}{\partial x^2}u(t,x,y,z) + \frac{\partial^2}{\partial y^2}u(t,x,y,z) + \frac{\partial^2}{\partial z^2}u(t,x,y,z), \tag{1}$$

where t is a temporal variable, (x,y,z) are spatial variables, u is a function for sound pressure distributed in (t,x,y,z), and c is the sonic velocity. In addition, we define a spatial vector $\boldsymbol{v}$ which contains spatial elements (x,y,z) and we can redefine the u with the vector $\boldsymbol{v}$. Substituting the vector, we obtain the following.

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}u(t,\boldsymbol{v}) = \frac{\partial^2}{\partial \boldsymbol{v}^2}u(t,\boldsymbol{v}), \tag{2}$$

$$\boldsymbol{v} = (x,y,z). \tag{3}$$

The sound propagation can be simulated by solving these wave equations. However, the wave equation cannot be solved analytically for complex space-time such as real world. The wave equation is then usually solved numerically by using computers. The continuous partial differences can be approximated by finite differences,

$$\frac{\partial^2 u}{\partial t^2} \simeq \frac{[u(t+\Delta t,\boldsymbol{v}) + u(t-\Delta t,\boldsymbol{v}) - 2u(t,\boldsymbol{v})]}{\Delta t^2}, \tag{4}$$

$$\frac{\partial^2 u}{\partial \boldsymbol{v}^2} \simeq \frac{[u(t,x+\Delta x) + u(t,x-\Delta x) - 2u(t,x)]}{\Delta x^2} + \frac{[u(t,y+\Delta y) + u(t,y-\Delta y) - 2u(t,y)]}{\Delta y^2} + \tag{5}$$

$$\frac{[u(t,z+\Delta z) + u(t,z-\Delta z) - 2u(t,z)]}{\Delta z^2}, \tag{6}$$

where Δt and $(\Delta x, \Delta y, \Delta z($ are the intervals of the lattice points for sampled space and time, respectively. By substituting equivalent value to $(\Delta x, \Delta y, \Delta z)$ in the vector $\boldsymbol{v}$, we obtain the following.

$$\frac{\partial^2 u}{\partial \boldsymbol{v}^2} \simeq \frac{[u(t,\boldsymbol{v}+\Delta\boldsymbol{v}) + u(t,\boldsymbol{v}-\Delta\boldsymbol{v}) - 2u(t,\boldsymbol{v})]}{||\Delta\boldsymbol{v}||^2}, \tag{7}$$

where $||\cdot||$ is Euclid norm. A recurrence formula can be derived by deforming the wave equation with the finite differences,

$$u(t+\Delta t,x) = \frac{||\Delta\boldsymbol{v}||^2}{c^2\Delta t^2}||u(t,\boldsymbol{v}+\Delta\boldsymbol{v}) + u(t,\boldsymbol{v}-\Delta\boldsymbol{v}) - u(t,\boldsymbol{v}) - 2u(t-\Delta t,\boldsymbol{v}) + u(t,\boldsymbol{v}). \tag{8}$$

By iterative calculation for the obtained recurrence formula, the future $(t > \tau)$ sound pressure distribution $u(t+\Delta t,\boldsymbol{v})$ can be calculated from the "current $(t = \tau)$" and past $(t < \tau)$ sound pressure distributions $u(t,\boldsymbol{v})$ and $u(t-\Delta t,\boldsymbol{v})$ respectively. However, the transfer function of continuous difference and finite difference are not the same. The distribution function u can be transformed to the transfer function U by Fourier transform[4],

$$U(\omega_{\boldsymbol{v}}) = \mathscr{F}[u(\boldsymbol{v})], \tag{9}$$

where $\mathscr{F}[\cdot]$ means Fourier transform and ω_x is the frequency of spatial variable x. By Fourier transform, it is indicated that the transfer functions between continuous difference and finite difference are significantly different,

$$-4\pi^2 j\omega_{\boldsymbol{v}}{}^2 U(\omega_{\boldsymbol{v}}) = \mathscr{F}\left[\frac{\partial^2}{\partial \boldsymbol{v}^2}u(\boldsymbol{v})\right], \tag{10}$$

$$-(2-2\cos(\omega_{\boldsymbol{v}}))U(\omega_{\boldsymbol{v}}) = \mathscr{F}[u(\boldsymbol{v}+\Delta\boldsymbol{v}) + u(\boldsymbol{v}-\Delta\boldsymbol{v}) - 2u(\boldsymbol{v})]. \tag{11}$$

The difference between the transfer functions causes numerical dispersion, and the wave packet is disturbed. Figure 1 indicates the simulated sound propagation which is distorted by the numerical dispersion.

2.2 Calculation in spatial frequency domain

Equation (10) indicates the ideal transfer function of the continuous difference, which does not contain any approximations. Thus, accurate calculation can be achieved in the spatial frequency domain. Therefore, the DFT-based FDTD method has been used in this study to reduce numerical dispersion for the FDTD method. The studied method has utilizes the recurrence formula,

$$u(t+\Delta t,\boldsymbol{v}) = \frac{\Delta \boldsymbol{v}^2}{c^2\Delta t^2}\mathscr{F}^{-1}\left[-4\pi^2{\omega_{\boldsymbol{v}}}^2\mathscr{F}\left[u(t,\boldsymbol{v})\right]\right] - u(t-\Delta t) + 2u(t,\boldsymbol{v}), \tag{12}$$

where $\mathscr{F}^{-1}[\cdot]$ means inverse Fourier transform. Although the method has reduced numerical dispersion, the DFT-based FDTD method causes spatial-wrapped propagation which is physics-defying. Figure 2 indicates the sound pressure distributions and their differences in an open space. Figures 2(a) and 2(c) are sound pressure distributions. Figures 2(b) and 2(d) are second-order partial difference for the sound pressure distribution. Figure 2(d) indicates that the energies are wrapped over their boundaries although the sound pressure is distributed in open space. For accurate calculation, the wrapped propagation should be avoided.

3. CALCULATION OF POLAR-FORM SPACE-TIME

The problem of wrapped propagation depends on a periodic extension for DFT, and it is impossible for the DFT to be applied without a periodic extension. Therefore, a new calculation method, which basically depends on applying DFT to originally wrapped axis, is proposed in this section.

In this paper, the polar-form quaternion is used. The wave equation consists of second-order partial differences for the temporal domain and spatial domain. By using wave operator in Minkowski space-time [6], the temporal domain and spatial domain can be integrated. The sonic velocity c is used instead of the light velocity in the Minkowski space-time in this study. Accordingly, the wave equation can be deformed as,

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}u - \frac{\partial^2}{\partial \boldsymbol{v}^2}u = 0. \tag{13}$$

The sound pressure distribution can be defined as Eq. (14) for space-time domain as polar-form by using quaternion. The temporal variable t is a real part, and the spatial variables (x,y,z) are imaginary parts of the quaternion.

$$\mu\left(r(r,\boldsymbol{v}),\boldsymbol{\theta}(r,\boldsymbol{v})\right) = u(t,\boldsymbol{v}), \tag{14}$$

$$r(t,\boldsymbol{v}) = \sqrt{t^2+\boldsymbol{v}^T\boldsymbol{v}}, \tag{15}$$

$$\boldsymbol{\theta}(t,\boldsymbol{v}) = \frac{\boldsymbol{v}}{||\boldsymbol{v}||}\arctan\left(\frac{||\boldsymbol{v}||}{t}\right), \tag{16}$$

where Eq. (15) gives the distance from the origin of the coordinates, and Eq. (16) gives the argument for the polar-form represented space-time. The recurrence formula, similar to the standard FDTD method, is also defined for the distance from the origin as

$$\mu(r+\Delta r,\boldsymbol{\theta}) = g\left[\mu(r,\boldsymbol{\theta}),\mu(r-\Delta r,\boldsymbol{\theta})\right], \tag{17}$$

where g is a function to present the recurrence formula that requires the current $\mu(r,\boldsymbol{\theta})$ and previous $\mu(r,\boldsymbol{\theta}-\Delta\boldsymbol{\theta})$ for the next $\mu(r,\boldsymbol{\theta}+\Delta\boldsymbol{\theta})$. By iterative calculation of Eq. (17), the impulse responses from

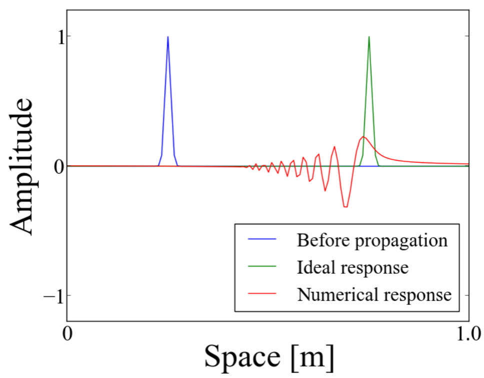
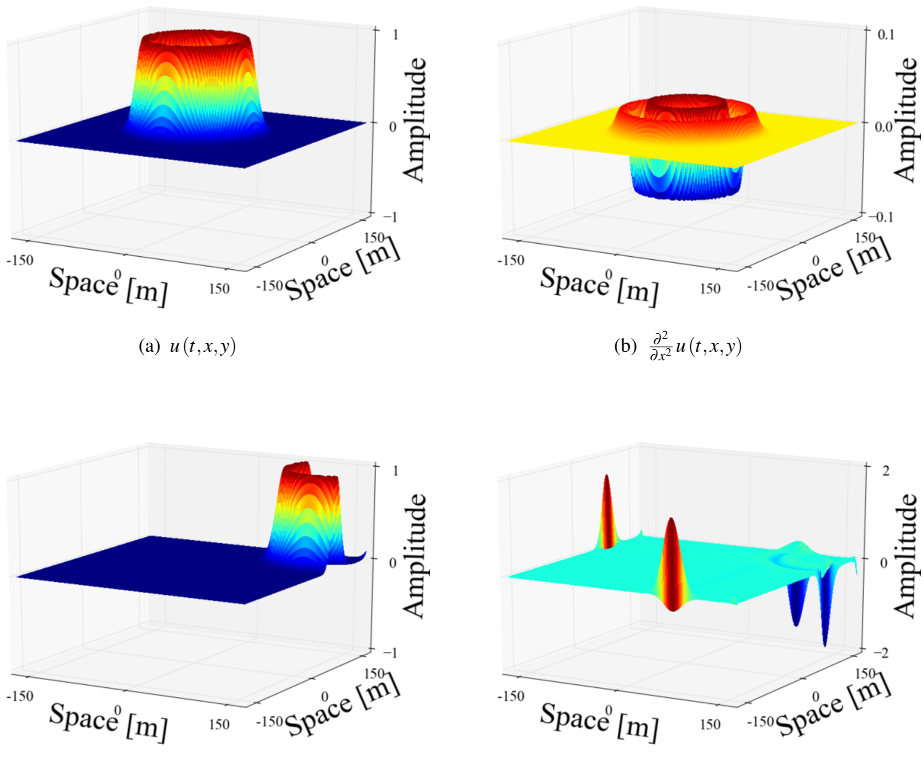


Figure 1 – Wave packet distorted by numerical dispersion in FDTD



(a) $u(t,x,y)$ (b) $\frac{\partial^2}{\partial x^2}u(t,x,y)$

(c) $u(t,x,y)$ (d) $\frac{\partial^2}{\partial x^2}u(t,x,y)$

Figure 2 – Sound pressure distributions and their differences

the origin (sound source position) can be calculated in polar-form represented space-time. Moreover, the impulse responses in cartesian space-time can be obtained by coordinate conversion from polar coordinate,

$$u(t,\boldsymbol{v}) = \mu\left[\sqrt{(t^2+\boldsymbol{v}^T\boldsymbol{v})}, \frac{\boldsymbol{v}}{||\boldsymbol{v}||}\arctan\left(\frac{||\boldsymbol{v}||}{t}\right)\right]. \tag{18}$$

Spatial sound pressure distribution $m_\tau(x)$ can be obtained by tracing the impulse responses in space-time along with the constant time $t=\tau$. Temporal impulse responses $h_\eta(t)$ can also be obtained by slicing the impulse responses in space-time along with the constant space $\boldsymbol{v}=\boldsymbol{\eta}$.

$$m_\tau(x) = u(\tau,x), \tag{19}$$
$$h_{\boldsymbol{\eta}}(t) = u(t,\boldsymbol{\eta}). \tag{20}$$

3.1 Formula development for calculation

In this section, the way to calculate the recurrence formula in space-time with polar coordinate is explained. The wave equation, Eq. (13), can be deformed into Eq. (21) with factorization.

$$\left(\frac{\partial^2 u}{\partial t^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial \boldsymbol{v}^2}\right) = \left(\frac{\partial u}{\partial t} + \frac{1}{c}\frac{\partial u}{\partial \boldsymbol{v}}\right)\left(\frac{\partial u}{\partial t} - \frac{1}{c}\frac{\partial u}{\partial \boldsymbol{v}}\right) = 0. \tag{21}$$

By applying Eqs. (15),(16) and the Jacobian value for coordinate conversion, Eq. (21) can be deformed to,

$$\begin{aligned}&\left(\sin\theta - \frac{1}{c}\cos\theta\right)\frac{\partial}{\partial r}\left(\sin\theta + \frac{1}{c}\cos\theta\right)\frac{\partial}{\partial r} + \left(\sin\theta - \frac{1}{c}\cos\theta\right)\frac{\partial}{\partial r}\frac{1}{r}\left(\cos\theta - \frac{1}{c}\sin\theta\right)\frac{\partial}{\partial \theta}\\ &+\frac{1}{r}\left(\cos\theta + \frac{1}{c}\sin\theta\right)\frac{\partial}{\partial \theta}\left(\sin\theta - \frac{1}{c}\cos\theta\right)\frac{\partial}{\partial r} + \frac{1}{r}\left(\cos\theta + \frac{1}{c}\sin\theta\right)\frac{\partial}{\partial \theta}\frac{1}{r}\left(\cos\theta - \frac{1}{c}\sin\theta\right)\frac{\partial}{\partial \theta}\\ &= 0.\end{aligned} \tag{22}$$

The expansion derives the following.

$$\left(\sin^2\theta-\frac{1}{c^2}\cos^2\theta\right)\frac{\partial^2\mu}{\partial r^2}+\frac{1}{r}\left(\cos^2\theta-\frac{1}{c^2}\sin^2\theta\right)\frac{\partial\mu}{\partial r}+\frac{2}{r}\left(\cos\theta\sin\theta+\frac{1}{c^2}\cos\theta\sin\theta\right)\frac{\partial\mu}{\partial r}\frac{\partial\mu}{\partial\theta}$$
$$-\frac{2}{c^2}\left(\cos\theta\sin\theta+\frac{1}{c^2}\cos\theta\sin\theta\right)\frac{\partial}{\partial\theta}+\frac{1}{r^2}\left(\cos^2\theta-\frac{1}{c^2}\sin^2\theta\right)\frac{\partial^2}{\partial\theta^2}=0. \tag{23}$$

Applying the double-angle rule, we obtain

$$\frac{1}{2}\left(c_n-c_p\cos 2\theta\right)\frac{\partial^2\mu}{\partial r^2}+\frac{1}{2r}\left(c_n+c_p\cos 2\theta\right)\frac{\partial\mu}{\partial r}+\frac{2}{r}c_p\sin 2\theta\frac{\partial\mu}{\partial r}\frac{\partial\mu}{\partial\theta}$$
$$-\frac{1}{r^2}c_p\sin 2\theta\frac{\partial\mu}{\partial\theta}+\frac{1}{2r^2}\left(c_n+c\cos 2\theta\right)\frac{\partial^2\mu}{\partial\theta^2}=0, \tag{24}$$

where

$$c_p = 1+\frac{1}{c^2}, \tag{25}$$
$$c_n = 1-\frac{1}{c^2}. \tag{26}$$

The impulse responses in space-time can be iteratively calculated along with the distance r, and that each iteration requires a calculation of the difference for the argument θ domain in Eq. (24). The recurrence formula can be defined by applying DFT of the argument domain θ to Eq. (24). Although the DFT requires periodic extension, the argument domain is originally periodic from $-\pi$ to π. As the result, the spatially wrapped propagation can be avoided even applying DFT to compute. By applying the follows,

$$U\left(r,\omega_{\boldsymbol{\theta}}\right) = \mathscr{F}\left[\mu\left(r,\boldsymbol{\theta}\right)\right], \tag{27}$$
$$-2\pi j\omega_{\boldsymbol{\theta}}U\left(r,\boldsymbol{\theta}\right) = \mathscr{F}\left[\frac{\partial}{\partial\boldsymbol{\theta}}\mu\left(r,\boldsymbol{\theta}\right)\right], \tag{28}$$
$$4\pi^2 j\omega_{\boldsymbol{\theta}}^2U\left(r,\boldsymbol{\theta}\right) = \mathscr{F}\left[\frac{\partial^2}{\partial\boldsymbol{\theta}^2}\mu\left(r,\boldsymbol{\theta}\right)\right], \tag{29}$$

an equation is derived as the recurrence formula for the distance domain. The recurrence formula requires only argument θ for that calculation.

However, in Eq. (24), the 4th item is multiplying of $\sin 2\theta$ and $\partial\mu/\partial\theta$, The multiplication is converted as the convolution in angular frequency domain ω_θ. The trigonometric functions are converted in angular frequency domain as

$$\frac{1}{2}\left[\delta\left(\omega_\theta-2\Delta\omega_\theta\right)+\delta\left(\omega_\theta+2\Delta\omega_\theta\right)\right] = \mathscr{F}\left[\cos 2\theta\right], \tag{30}$$
$$\frac{j}{2}\left[\delta\left(\omega_\theta-2\Delta\omega_\theta\right)-\delta\left(\omega_\theta+2\Delta\omega_\theta\right)\right] = \mathscr{F}\left[\sin 2\theta\right], \tag{31}$$

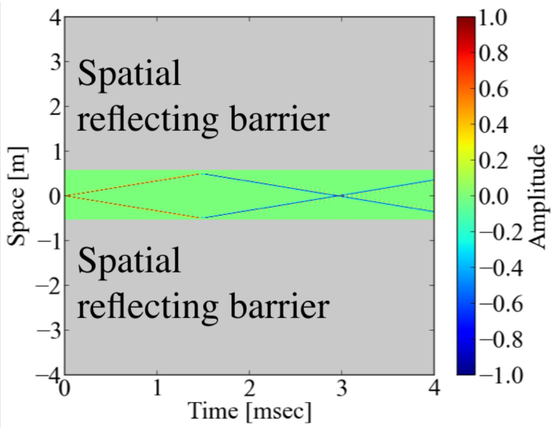
where $\delta\left(\cdot\right)$ is the delta function.

The finite difference is approximately utilized for $\partial^2/\partial r^2$ for that recurrence formula.

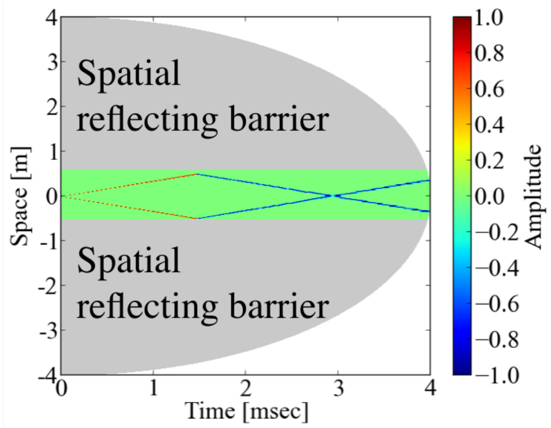
$$\frac{1}{2\Delta r}\left[U\left(r+\Delta r,\omega_\theta\right)-U\left(r-\Delta r,\omega_\theta\right)\right] = U_{\mathrm{dr}}\left(r,\omega_\theta\right)\left(\simeq\frac{\partial}{\partial r}U\left(r,\omega_\theta\right)\right), \tag{32}$$
$$\frac{1}{\Delta r^2}\left[U\left(r+\Delta r,\omega_\theta\right)+U\left(r-\Delta r,\omega_\theta\right)-2U\left(r,\omega_\theta\right)\right] = U_{\mathrm{ar}}\left(r,\omega_\theta\right)\left(\simeq\frac{\partial^2}{\partial r^2}U\left(r,\omega_\theta\right)\right), \tag{33}$$

Eq. (24) can be transformed into angular frequency domain ω_θ. By applying Eqs. (28)-(33), the recurrence formula is derived as

$$\mathrm{C}_1\otimes\frac{1}{\Delta r^2}\left[U\left(r+\Delta r,\omega_\theta\right)+U\left(r-\Delta r,\omega_\theta\right)-2U\left(r,\omega_\theta\right)\right]+\mathrm{C}_2\otimes\frac{1}{2\Delta r}\left[U\left(r+\Delta r,\omega_\theta\right)-U\left(r-\Delta r,\omega_\theta\right)\right]$$
$$+\mathrm{C}_3\otimes\pi j\omega_\theta\frac{1}{2\Delta r}\left[U\left(r+\Delta r,\omega_\theta\right)-U\left(r-\Delta r,\omega_\theta\right)\right]+\mathrm{C}_4\otimes\pi j\omega_\theta U\left(r,\omega_\theta\right)+\mathrm{C}_5\otimes-\pi^2\omega_\theta^2U\left(r,\omega_\theta\right)=0, \tag{34}$$



(a) In cartesian coordinate with the conventional method



(b) In polar coordinate with the proposed method

Figure 3 – Simulated space-time impulse responses with spatial barriers at (0.5 m,0.5 m)

where $\mathrm{C}_1 - \mathrm{C}_5$ are coefficients in the recurrence formula, which satisfy the Eqs. (35)-(39).

$$\mathrm{C}_1 = \frac{1}{2}[c_p\delta(\omega_\theta - 2\Delta\omega_\theta) + 2c_n\delta(\omega_\theta) - c_p\delta(\omega_\theta + 2\Delta\omega_\theta)], \tag{35}$$

$$\mathrm{C}_2 = \frac{1}{2r}[c_p\delta(\omega_\theta - 2\Delta\omega_\theta) + 2\delta(\omega_\theta) + c_p\delta(\omega_\theta + 2\Delta\omega_\theta)], \tag{36}$$

$$\mathrm{C}_3 = \frac{jc_p}{r}[\delta(\omega_\theta - 2\Delta\omega_\theta) - \delta(\omega_\theta + 2\Delta\omega_\theta)], \tag{37}$$

$$\mathrm{C}_4 = \frac{jc_p}{r^2}[\delta(\omega_\theta - 2\Delta\omega_\theta) + \delta(\omega_\theta + 2\Delta\omega_\theta)], \tag{38}$$

$$\mathrm{C}_5 = \frac{1}{r^2}[\frac{c_p}{2}\delta(\omega_\theta - 2\Delta\omega_\theta) + c_n\delta(\omega_\theta) + \frac{c_p}{2}\delta(\omega_\theta + 2\Delta\omega_\theta)], \tag{39}$$

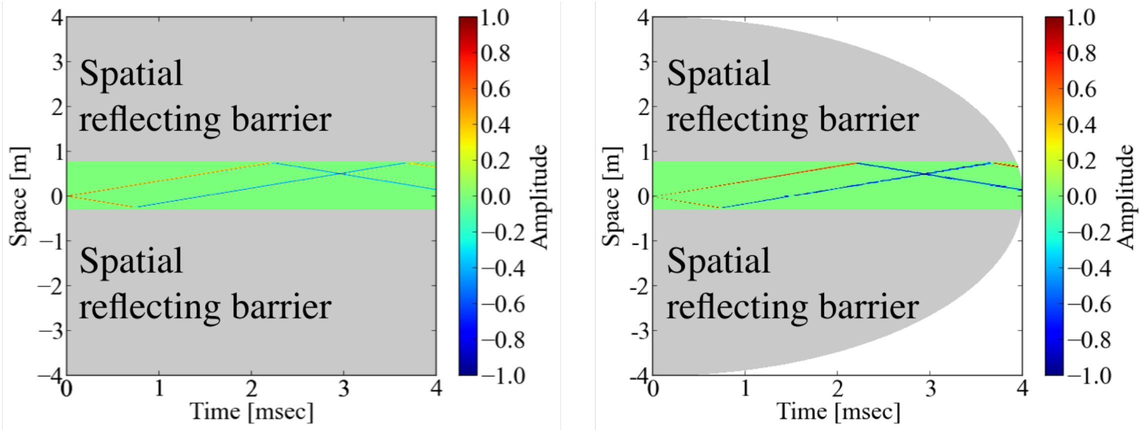
where $\otimes$ is the convolution operator. By applying Eq. (34) to simulate the sound propagation, the sound pressure distribution in space-time can be solved. In the solution, only angular frequency domain is required for the calculation, and the calculation can reduce the numerical dispersion without any approximation. In addition, the argument domain in polar coordinate is originally wrapped from $-\pi$ to π, which is expected to avoid spatially wrapped propagation due to DFT.

4. NUMERICAL EXPERIMENTS

We evaluated the performance of the proposed method by numerical experiments. We defined a closed space with perfectly reflecting walls, The distances between origin and walls were (0.5 m, 0.5 m) and (0.25 m,0.75 m). and simulate the propagation of delta function $\delta(r)$. We obtained the cartesian-temporal impulse responses h_η at $\eta = 0.0$ m, 0.2 m and 0.4 m from origin. As the conventional method, we applied FDTD(2,2) method which consists of 2nd order finite differences for space and time. In addition, we applied linear interpolation to polar function μ to project to cartesian function u. The other common conditions are displayed in Tbl. 1.

Table 1 – Conditions for computer simulation

Sonic velocity	340 m/s
Temporal resolution	5.0×10^5 samples/s
Spatial resolution	5.0×10^3 samples/m
Angular resolution	1024 samples/2π
Iteration steps	2000



(a) In cartesian coordinate with the conventional method

(b) In polar coordinate with the proposed method

Figure 4 – Simulation space-time impulse responses with spatial barriers at (0.25 m,0.75 m)

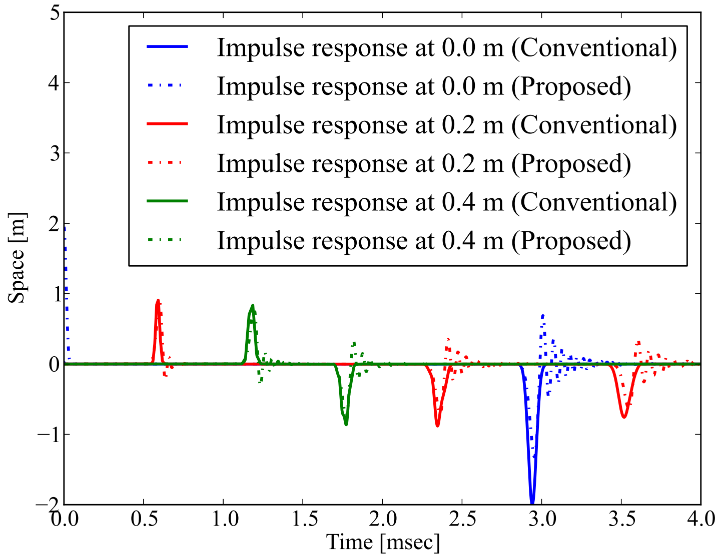


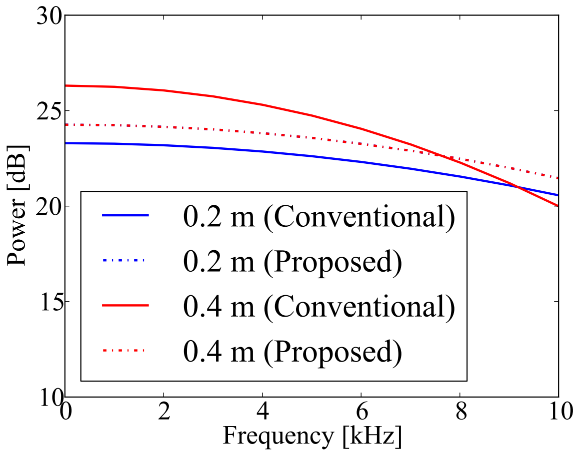
Figure 5 – Temporal impulse responses at several spatial positions obtained with the conventional and th proposed methods

4.1 Experimental results and discussions

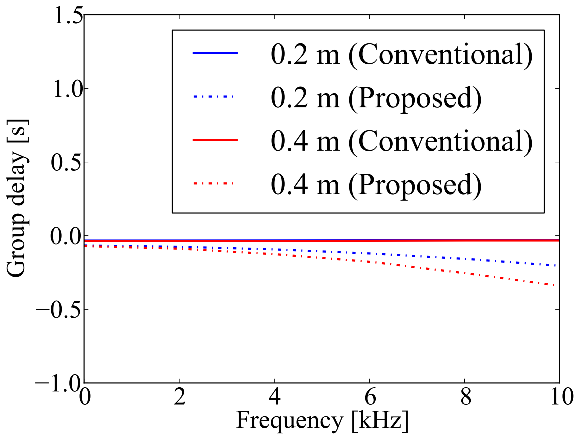
Space-time impulse responses simulated by the conventional and proposed methods are displayed in Figs. 3, 4. They are impulse responses in Cartesian represented space-time, whose horizontal axes are time and vertical axes are space. Figures. 3(b) and 4(b) are impulse responses in polar represented space-time, which was computed from origin $r = 0$ for the iteration 2000. Figure 3(b) and 4(b) display impulse responses for future direction although the proposed method solves impulse responses for both the past and future directions. The power spectra and group delays of 1st reflection are displayed in Figs. 6. Figures 3 and 4 indicated that the proposed method has achieved to simulate wave propagation same as conventional method without spatially-wrapping. Figure 5 however indicated the waveforms of the proposed method were different from ones of the conventional methods. According to Fig. 6, these results indicated the power gain was distorted like low-pass-filter although the distortion of group delay was fixed.

These results indicate that the distortion in cartesian coordinate was caused by coordinate conversion. The interval of lattice point broaden in proportion as the distance from the origin increases. That distorts the numerical results in cartesian coordinate.

According to all results, the proposed method has achieved numerical-dispersion-free simulation for acoustics in polar represented space-time. However, coordinate conversion is required to compose cartesian waveform such as impulse responses, which causes lowpass-filter-like distortion due to the interval of lattice points. To improve the proposed method, accurate coordinate conversion is required.



(a) Power



(b) Group delay

Figure 6 – Spectrum of 1st reflected waveforms at Fig. 3

5. CONCLUSIONS

In this study, the DFT-based FDTD method was used to reduce numerical dispersion for acoustic simulation. The DFT-based computation caused spatial periodicity in sound propagation due to the periodic extension. Therefore, an approach is proposed to avoid the periodic extension for the DFT-based computation in this paper. With the approach, sound pressure distribution in cartesian coordinate was converted into one in polar-form space-time, and the DFT computation was conducted along with the argument domain. This approach has achieved to avoid the wrapped propagation. However, it has been indicated that the coordinate conversion distorted the numerical result. Thus, the accurate coordinate conversion is the most important issue for future work, and we intent to study the method for coordinate conversion.

ACKNOWLEDGEMENTS

This research was partly supported by a grant-in-aid for scientific research, funded by MEXT, Japan.

REFERENCES

1. Thomas D. Rossing, “Springer Handbook of Acoustics, ” Springer, 2007.

2. P. H. Aoyagi, J.-F. Lee, and R. Mittra, “A hybrid Yee algorithm/scalar-wave equation approach, ” IEEE Trans. Micro. Theory, Vol. 41, No. 9, pp.1593-1600, 1993.

3. R. Courant, K. O. Fredrichs, and H. Lewy, “Uber die Differenzengleichungen der Mathematischen Physik,” Math. Ann, Vol.100, pp.32-74, 1928.

4. Alan V. Oppenheim and Ronald W. Schafer, “Discrete-Time Signal Processing, ” Prentice Hall, 2009.

5. Q. H. Lin and J.-Q. He, “An Efficient PSTD Algorithm in Cylindrical Coordinates, ” IEEE Trans. Ant. Propagat., Vol. AP-1, pp.558-561, 1998.

6. H. Minkowski, “Raum und Zeit, ” Phys. Z., vol. 10, pp. 104-117, 1909.