



# Improvement of Experimental SEA model accuracy using Independent Component Analysis

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## ABSTRACT

Statistical Energy Analysis (SEA) is a suitable tool to predict vibration stationary responses. It is roughly categorized into analytical, FEM based, and experimental SEA due to the process of model construction. SEA is expected to be used for designing machines with optimal vibration transfer paths. However experimental SEA model accuracy especially in low frequency is highly dependent on experimental condition and target structure. It is required to develop model construction process for more convenient use of SEA. In this paper, preprocessing algorithm using independent component analysis (ICA) is proposed for improvement of model construction of SEA. Here, ICA is a signal processing method which is originally developed for bio-signal analysis and it is used for separation of complex mixture of several vibration sources. Feasibility of the proposed method is examined through an experiment with a test structure, composed of three flat steel plates.

Keywords: Statistical Energy Analysis, Independent Component Analysis, I-INCE Classification of Subjects Number(s): 74.9

## 1. INTRODUCTION

Statistical energy analysis, so called SEA, is a modelling procedure for the estimation of the vibrational response levels of built-up structures using energy flow relationship. SEA has been used for many years to predict the response of complex engineering systems especially to high-frequency excitations [1-3].

The advantages of SEA over the conventional approaches are that relatively few degrees of freedom are involved, it is easy to determine how the external power inputs act on the system, and the SEA parameters have physical meaning, which is useful when considering countermeasures.

The matrix of SEA parameters are also called SEA model and calculated based on excitation input and response on each subsystem. SEA can be roughly categorized into analytical SEA, SEA based on FEM and experimental SEA (1) according to the process of SEA model construction. The authors have proposed a process for solving structure-borne sound problems in machinery using experimental SEA and have carried out noise reduction for some kinds of mechanical product (3).

However, there are some problems while constructing procedure of experimental SEA model. Even though SEA assumes linearity of target mechanical system, complex vibration transfer occurs in actual experiments. To construct more accurate SEA model, unintentional vibration should be considered separately. Thus, development of separation method may enhance noise and vibration analysis using SEA.

On the other hand, in the field of bio-signal analysis, Independent Component Analysis (ICA) is used to decode multi-channel signals from independent sources (4-7). ICA is a signal processing method categorized as Blind Source Separation (BSS), a method that estimates input source signals from the output without a transfer process model. ICA restores source signals with an assumption that the original signals are generated independently, so the detection of source signals is achieved without identifying the transfer process. Principle Component Analysis (PCA) is also known as a technique similar to ICA which uses BSS.

In terms of mechanical engineering, a BSS method is useful because it enables the remote

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estimation of the vibration source and estimation can proceed without modeling the whole system, which is sometimes too complicated. From these backgrounds many research studies propose the application of ICA to mechanical systems (8-14). For example, Gelle et al. suggests (8) that the diagnosis of bearing error is possible by carrying out a Fourier transform as a pre-process to ICA. The disintegrated Fourier functions are treated as observed signals and ICA is carried out. Zuo et al. proposed a method in which wavelet transform was carried out before ICA (10). These methods achieved fault detection of a rotating system with fewer sensors. However, the frequency of the targets being diagnosed needs to be known first. Roan et al., (9), proposed the application of information maximization based on a blind source separation algorithm, where the detection of impulsive and random changes in the data is achieved using the learning curve of the updated parameter. Takada et al. proposed an application of ICA on the modal analysis of beams and plates (14). Furthermore, they suggested that it is possible to detect faults by observing modal shapes. These studies showed the effectiveness of BSS applied to mechanical systems. Samuel et al. summarized the various methods of fault detection of helicopter transmission systems (15).

The aim of this study is to develop a preprocessing method of SEA to construct more accurate SEA model. In this paper two are . Firstly, separation of unintentional vibration transfer with ICA is performed. Then, SEA model constructed with independent components are compared with basic SEA model.

## 2. METHODOLOGY

### 2.1 Statistical Energy Analysis

Statistical energy analysis, SEA is In an SEA model, the system is regarded as an assembly of subsystems. It is assumed that energy dissipation is proportional to the vibration energy in a subsystem; transfer energy is also taken to be proportional to the vibration energy between two subsystems. Considering the power balance leads to a set of equations. The SEA equation is expressed as follows, and most simple SEA system model is shown in figure 1.

$$\mathbf{P}(\omega) = \omega \mathbf{L}(\omega) \mathbf{E}(\omega), \tag{1}$$

where P is the external power input, E is the subsystem energy stored, and  $\omega$  is the band center angular frequency. L is the matrix of loss factors written as

$$\mathbf{L}(\omega) = \begin{bmatrix} \eta_1 + \eta_{12} + \eta_{13} + \dots & -\eta_{21} & \dots \\ -\eta_{21} & \eta_2 + \eta_{21} + \eta_{23} + \dots & \dots \\ \vdots & \vdots & \ddots \end{bmatrix}, \tag{2}$$

where  $\eta_i$  and  $\eta_{ij}$  are the internal loss factors of subsystem i and the coupling loss factors from subsystems i to j, respectively. The loss factors are dependent on the frequency.

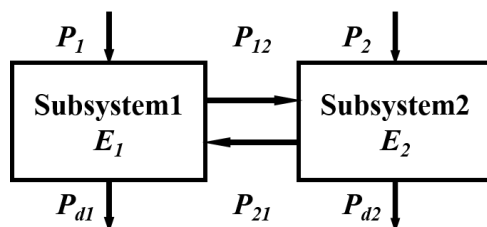


Figure 1 – An example model of SEA

In experimental SEA, the loss factors are evaluated from the data measured in the hammering test. They are given by

$$\eta_{ij} = \frac{E_{ji} / P_i}{\omega E_{ii} / P_i \cdot E_{jj} / P_j} \quad \text{and} \quad (3)$$

$$\eta_i = \frac{1 - \omega \sum_{j \neq i}^n (\eta_{ij} E_{ii} / P_i - \eta_{ji} E_{ji} / P_j)}{\omega E_{ii}} \quad , \quad (4)$$

where  $E_{ij}$  is the energy of subsystem  $i$  under power input  $P_j$  to subsystem  $j$  in the hammering test. The subsystem energy and the power inputs to the subsystem by the hammer are given by

$$P_j = \text{Im}[FA^*] / \omega \quad \text{and} \quad (5)$$

$$E_{ij} = \frac{m_i}{2} \cdot \sum_k^q (A_k / \omega)^2 / q \quad , \quad (6)$$

where  $F$  is the excitation force spectrum,  $A$  is the acceleration spectrum at the excitation,  $m_i$  is the mass of subsystem  $i$ , and  $A_k$  is the acceleration spectrum at point  $k$  on subsystem  $i$ .

### 2.2 Independent Component Analysis

A schematic diagram of ICA is shown in Fig. 1, and the relationship between the source signals  $s(t)$  and observed signals  $x(t)$  can be expressed by equations (7) and (8):

$$x(t) = A s(t) \quad , \quad (7)$$

$$s(t) = W x(t) \quad . \quad (8)$$

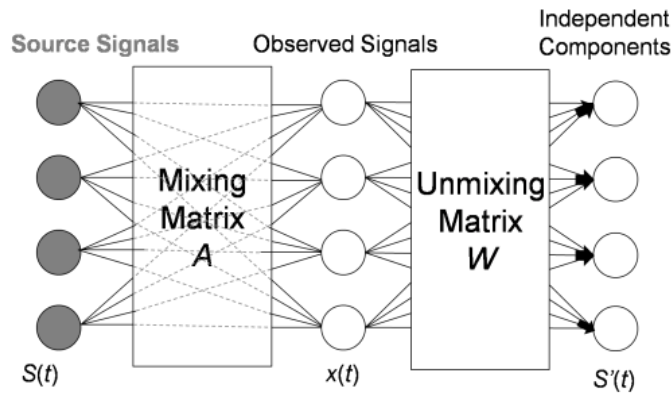


Figure 2- Scheme of independent component analysis

Here,  $s(t)$  and  $x(t)$  are multi-channel vectors, and the dimensions of  $s(t)$  are assumed to be equal to or less than the dimensions of  $x(t)$ . In addition, the mixed signals are the linear couplings of several source signals, so the transfer process can be expressed with a matrix  $A$  which consists of constant elements. When the transfer matrix  $A$  is known,  $W=A^{-1}$  and the source signals  $s(t)$  can be estimated from the observed signals  $x(t)$  using equation (8). However,  $A$  is unknown and difficult to evaluate in some systems. It means that some assumptions are needed to determine  $W$ . At this point, ICA assumes each of the source signals is generated by a different process, and they are independent in terms of how they are distributed. With this assumption, ICA calculates  $W$  to obtain mostly independent sets of  $s(t)$ . In this study, an algorithm called FastICA is used (). This algorithm evaluates independency by kurtosis, which is a 4th-order statistic:

$$kurt(x) = E(x^4) - 3[E(x^2)]^2. \quad (9)$$

FastICA first carries out decorrelation, then maximizes kurtosis by iteration and finds  $\mathbf{W}$ . Independent components are obtained when  $\mathbf{W}$  with a maximum kurtosis is calculated. More specifically, the calculation of FastICA focuses on one of row  $\mathbf{W}_i$  ( $i=1, 2, \dots, n$ ) in  $\mathbf{W}$ , and the direction that increases kurtosis most rapidly is calculated. Here  $y_i = \mathbf{W}_i \mathbf{x}(t)$  and its kurtosis is calculated with the observed  $\mathbf{x}(t)$ , and we find the direction that most rapidly increases the value of kurtosis. By iteration of this process,  $\mathbf{W}_i$  converges. One of the independent components is calculated with  $\mathbf{W}_i$  from  $y_i = \mathbf{W}_i \mathbf{x}(t)$ . The same process is performed on the other rows and matrix  $\mathbf{W}$  finally converges.

### 2.3 SEA Model Construction with Independent Components Signals

Scheme of analysis flow is shown in figure 3. Left hand side is a common experimental SEA model construction process, and right hand side is proposing method; process ICA before model construction. Commonly, impulsive hammering test are performed to construct SEA model. On each subsystem, accelerations at several points are measured and average vibration response are considered as representative value, or statistical value.

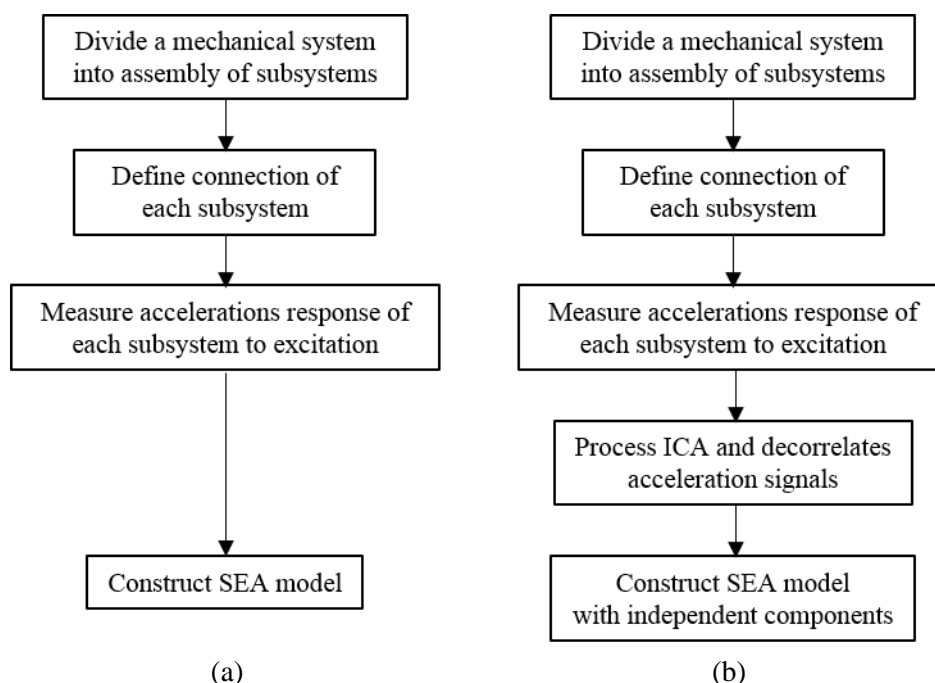


Figure 3 - Flow of common experimental SEA (a) and proposing method using ICA (b)

## 3. SEPARATION OF UNINTENTIONAL VIBRATION

### 3.1 Apparatus

Before discuss about SEA model accuracy, separation of unintentional vibration using ICA is performed. Figure 4 shows the test structure used in this study. This system is composed of three rectangular flat steel plates of 2 mm thickness. Plates 1, 2, and 3 are considered as subsystems 1, 2, and 3, respectively. The structure was freely suspended by strings at 2 points along each edge of plates 1 and 3. Four accelerometers are set on subsystem 1 and 2, and 2 accelerometers are set on subsystem 3. Each sensor measures antiplane vibration. Simultaneously, impulsive force applied on each subsystem by a hammer is measured. The measured responses are normalized by the maximum value of the excitation force for comparability.

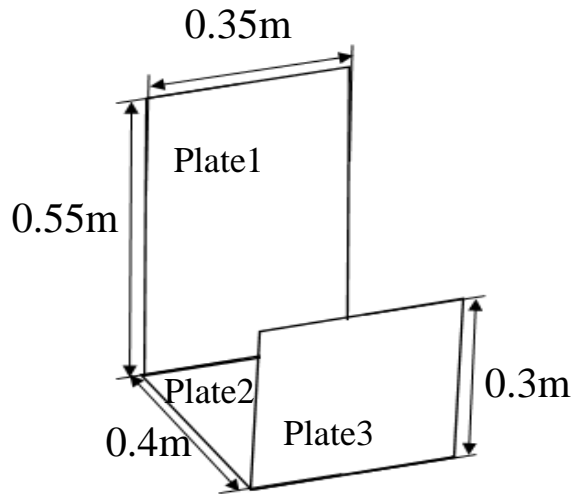


Figure 4 - The test system composed of three rectangular flat steel plates of 2 mm thickness

### 3.2 Measurement of vibration transfer process

Impulsive force is applied at the center of subsystem 1 and the transient responses of each subsystem are recorded. Average of accelerations in each subsystem are calculated. Then, short time FFT is processed to check vibration response at each subsystem by frequency. An example of measured acceleration is shown in Figure 5; (a), (b), (c), and (d) represent input force on subsystem 1, acceleration of subsystem 1, acceleration of subsystem 2, and acceleration of subsystem 3 respectively. Short time FFT data is shown in figure 6. At some frequency bands, peak acceleration of subsystem 3 is earlier than subsystem 2. This phenomenon is inconvenient for SEA model construction.

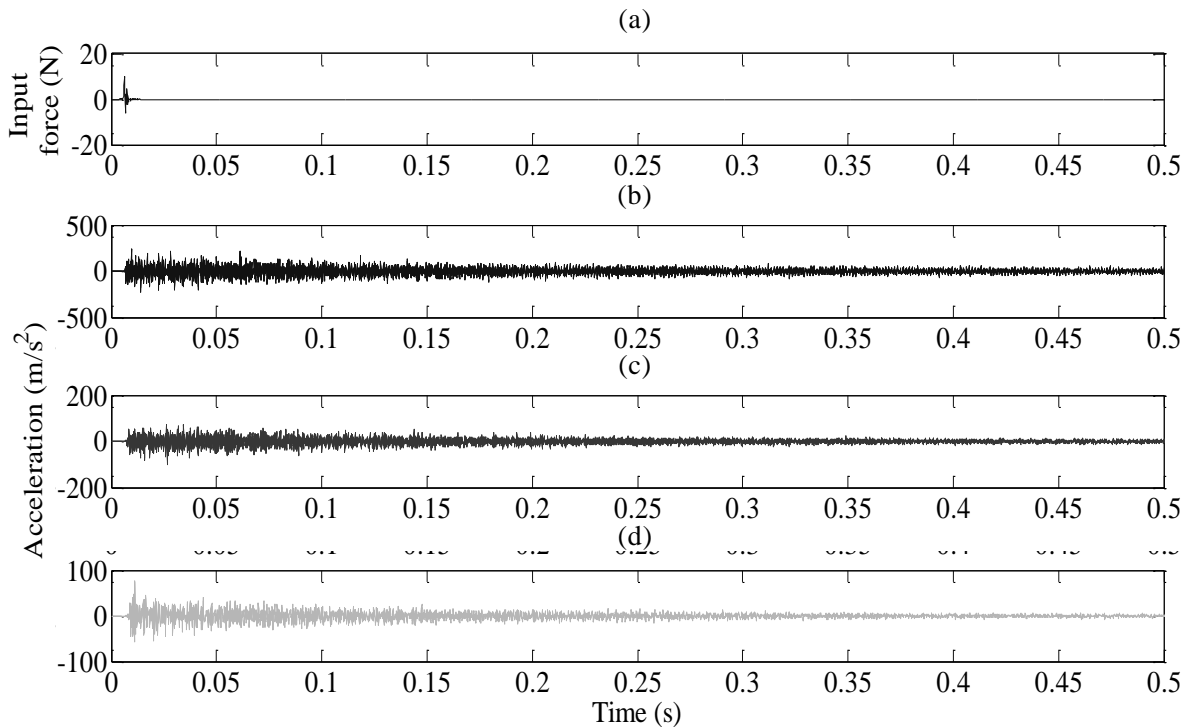


Figure 5 – Impulsive force and measured acceleration on each subsystem; (a), (b), (c), and (d) represent input force on subsystem 1, acceleration of subsystem 1, acceleration of subsystem 2, and acceleration of subsystem 3 respectively.

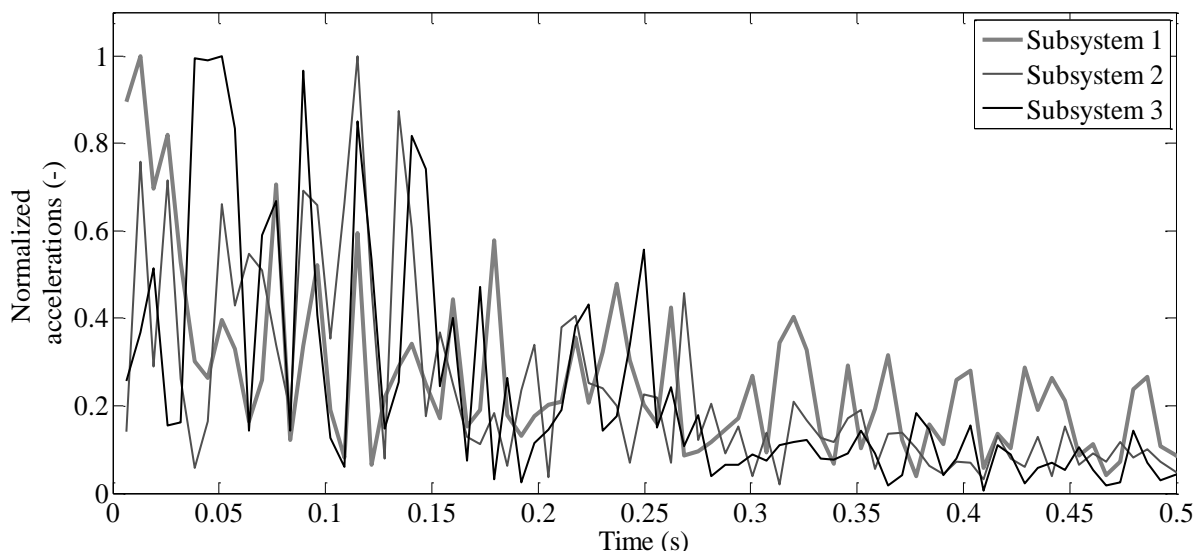


Figure 6 – Short time FFT data of average acceleration on each subsystem (2400 Hz); Gray bold line, gray thin line, black thin line represent normalized acceleration of subsystem 1, 2, and 3 respectively.

### 3.3 Signals separated by ICA

Applied impulsive force and measured accelerations are processed into ICA. Accelerations measured at subsystem 3 is shown in figure 7; (a) represents input force on subsystem 1, (b) and (c) represent measured accelerations on subsystem 3. These signals are processed with ICA and decorrelated. In the same way, accelerations of subsystem 1 and 2 are also decorrelated.

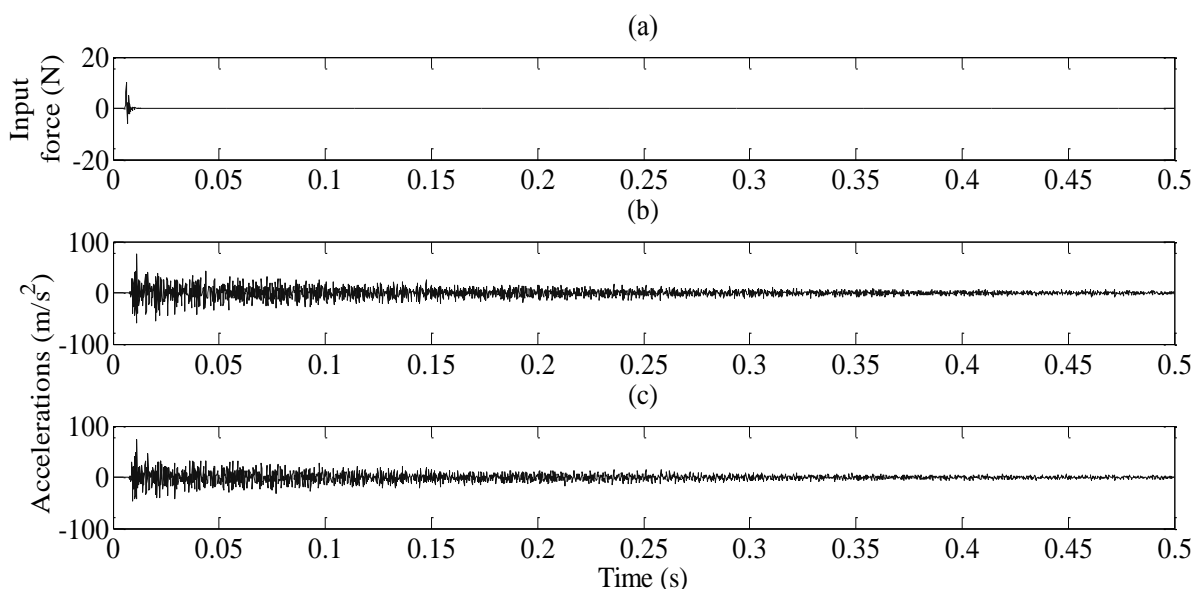


Figure 7 – Input force (a) and measured accelerations on subsystem 3 (b, c).

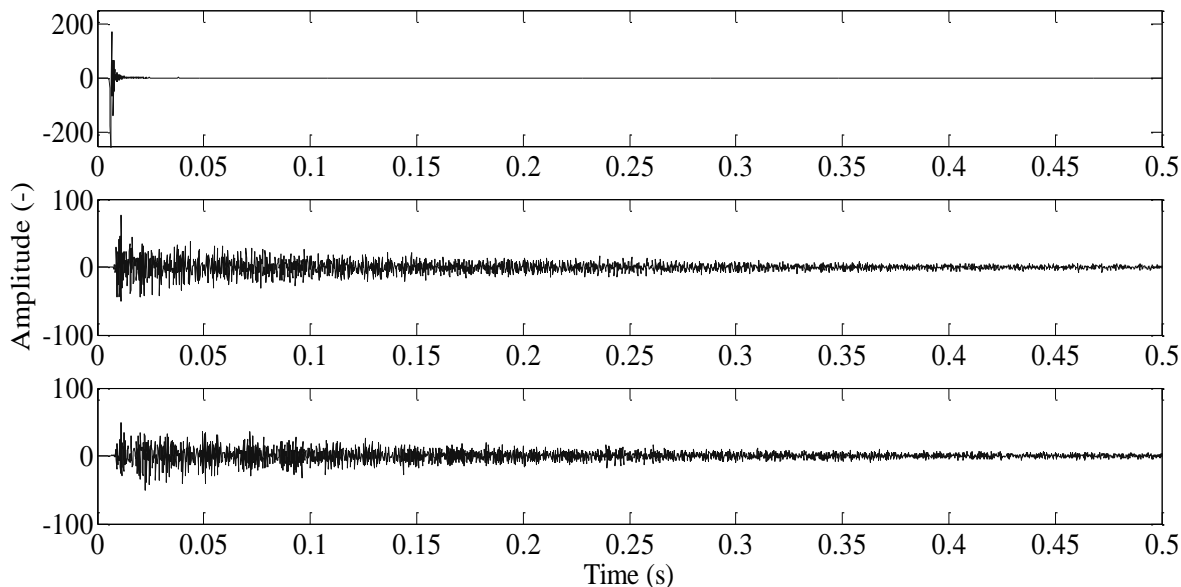


Figure 8 –independent components decorrelated from subsystem 3

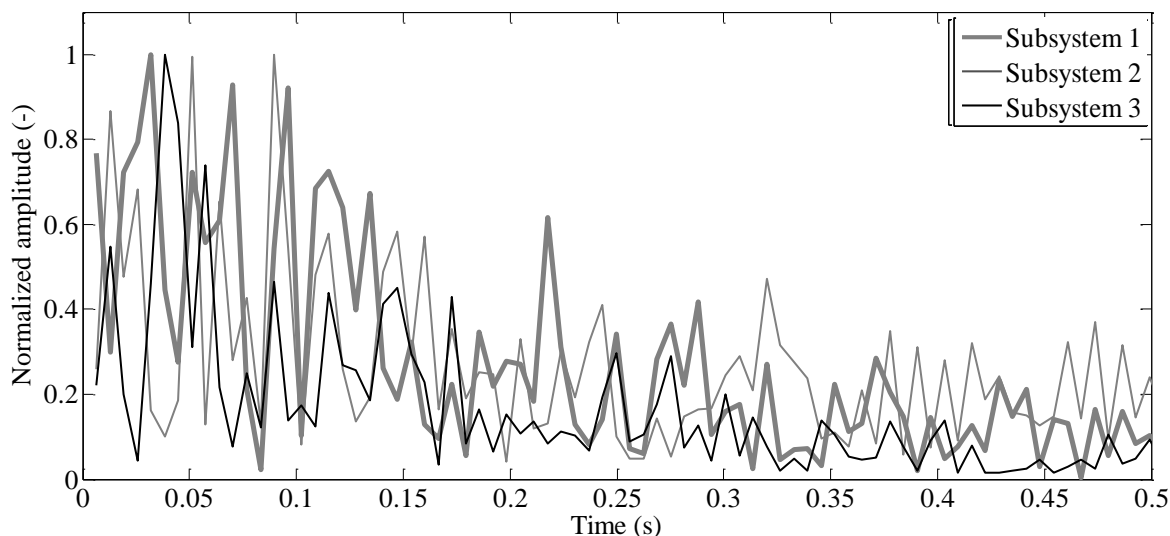


Figure 9 – Short time FFT data of average acceleration on each subsystem (2400 Hz); Gray bold line, gray thin line, black thin line represent normalized acceleration of subsystem 1, 2, and 3 respectively.

#### 4. SEA MODEL CONSTRUCTION

##### 4.1 Overview and apparatus

In this section, SEA model accuracy constructed with independent components signals are discussed. The same plate mentioned in section 3 is experimented. Instead of raw accelerations signals, independent components are put in to SEA model calculation process.

##### 4.2 Result and Discussion

Predicted energy calculated with normal SEA and ICA-SEA are shown in figure 10; Bold lines represent measured energy as reference, dash lines represent predicted energy with normal SEA, and thin solid lines represent predicted energy with ICA-SEA. Gray, green, and red lines represent subsystem 1, 2, and 3 respectively.

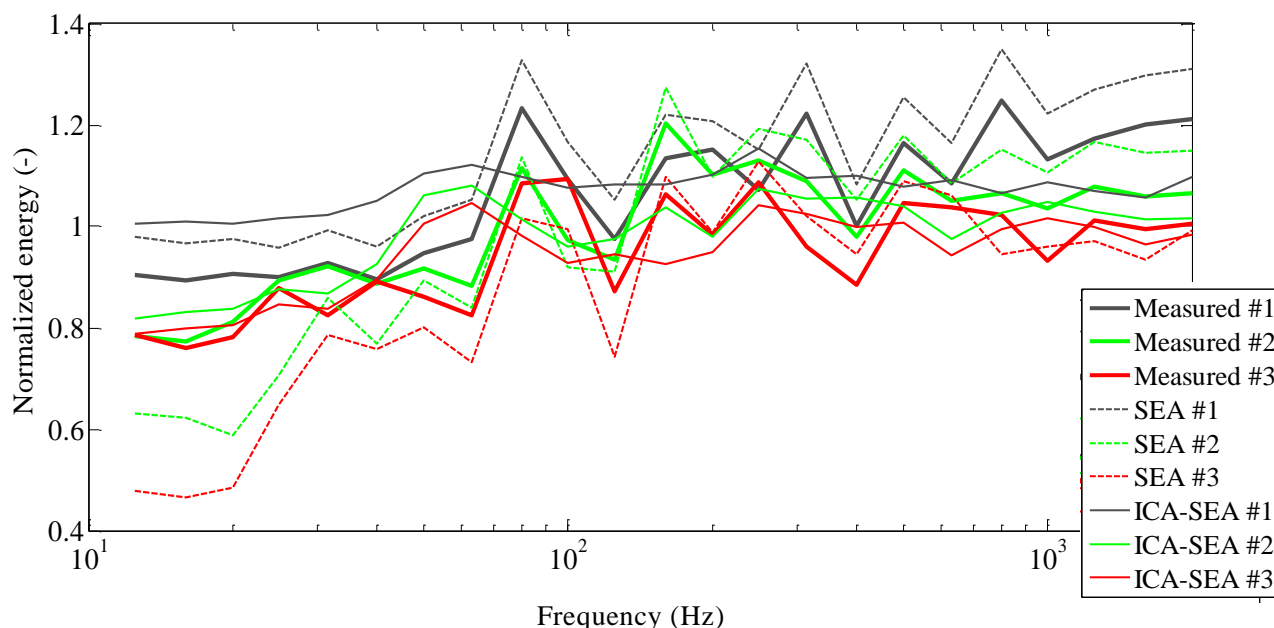


Figure 10 – Comparison of measured energy and predicted energies with SEA, and ICA-SEA

The result suggests that ICA-SEA can predict subsystem energy more accurately than normal SEA especially in low frequency, where normal SEA are not suitable. Further, energy of remote subsystem 3 is well estimated with ICA-SEA. It suggests that separating unintentional vibration before constructing SEA model is effective.

## 5. CONCLUSION

In this paper, preprocessing algorithm using independent component analysis (ICA) is proposed for improvement of model construction of SEA. Through an experiment target on 3 subsystems' structure, bended steel plate, feasibility of separation of inconvenient vibration using ICA, and ICA based SEA is shown. First, initial shock input on remote subsystem 3 is separated with ICA to remain only linear vibration transfer. Then, performance of energy prediction using ICA-SEA is compared with normal SEA, and advantage of new method is shown.

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