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Impulsive Response Analysis

Using Transient Energy Distribution Analysis

Shohei CHIDA[1]; Hiroki NAKAMURA[2]; Toru YAMAZAKI[3]

[1] Graduate School of Kanagawa University, Japan

[2,3] Department of Mechanical Engineering, Kanagawa University, Japan

ABSTRACT

Statistical Energy Analysis (SEA) and Energy Distribution Analysis (EDA) are suitable for predicting responses to stationary vibrations. These methods consider a structure of interest as an assembly of subsystems and analyze energy flows between the subsystems. In particular, EDA can analyze vibrations over the whole frequency range, while SEA is basically used only for high frequencies. However, these methods do not consider transient dynamic characteristics, so they are not applicable to the analysis of unsteady vibrations such as impulsive vibrations. This paper proposes transient EDA (TEDA) as is extension of EDA to impulsive responses. The proposed method is expected to be a suitable tool for impulsive analysis of huge structures like ships. The feasibility of the proposed method is examined through an experiment with a test structure composed of three flat steel plates. Also, the proposed method is compared with TSEA. We find that TEDA can predict the vibration energy of each subsystem more accurately than TSEA. Measurements of the energy response of a structure composed of three flat steel plates, including a coupling at right angles, showed specific energy transfer characteristics: the maximum energy of the end plate that is far from the input plate was larger than that of the second plate that is between the input and the end plates; and energy peak rise time of the end plate was faster than that of the second plate.

Keywords: SEA, Transient, Shock I-INCE Classification of Subjects Numbers: 75.2, 41.3

1. INTRODUCTION

Complex mechanical systems like vehicles and buildings often experience external excitation. In particular, large impact forces may cause damage around the impact point and may also cause trouble for precision electric devices and fragile items located at a distance by the propagation of the transient vibration via various transfer paths from the impact point. Finite Element Method (FEM) and Transient Statistical Energy Analysis (TSEA) are often used to predict the impulsive responses of structures. In large structures especially, FEM is not suitable because it requires a large number of FEM meshes and is time consuming. TSEA is, therefore, a more suitable method to analyze the problem (1,2).

SEA has been used for many years to predict the response of complex engineering systems to high-frequency excitations. SEA considers a structure of interest as an assembly of subsystems and analyzes energy flows between the subsystems. SEA can be roughly categorized into analytical SEA, SEA based on FEM and experimental SEA (1). The authors have proposed a process for solving structure-borne sound problems in machinery using experimental SEA and have carried out noise reduction for some kinds of mechanical product (3). However SEA is a method for steady state problems. For analyzing the unsteady state problem, TSEA is proposed as an extension of SEA. TSEA enabled us to predict the impulsive energy response analysis. Pinnington et al. and the authors have discussed the formulation of TSEA by using a system composed of two subsystems (4,5). Musser et al.

[1] mistymountainhop76@gmail.com.com
[2] hiroki-nak@kanagawa-u.ac.jp,
[3] toru@kanagawa-u.ac.jp

have utilized TSEA for sound quality design in automotive door closure (6). The authors have also proposed a method of measuring transient energy responses for verifying TSEA prediction results (7). However, TSEA is inherently limited to predict energy responses in higher frequency ranges as is SEA.

On the other hand, the idea of Energy Distribution Analysis (EDA) has been proposed for building a proper SEA model using FEM (8). EDA also focuses on the energy flow between subsystems (9). It can be said that EDA is SEA that considers connections between all subsystems even though the subsystems are not physically coupled. This means that EDA considers the influence of global modes, so there are no restrictions on the frequency range to be analyzed. In addition, the EDA model can be converted to the SEA parameters, internal loss factors, and coupling loss factors, so EDA can easily be used for the design of structural modifications and countermeasures. EDA is, of course, suitable for steady state problems and there is no report concerning EDA for transient energy flow.

In this paper, EDA is extended to Transient Energy Distribution Analysis (TEDA) to analyze transient energy flow for predicting the impulsive energy responses. The results from TEDA are compared with those from TSEA on a simple structure with three flat plate subsystems. It is shown that TEDA can predict the transient energy responses with high accuracy in low frequency ranges, where TSEA results are not in good agreement with the measured values.

2. Formulations of SEA, EDA, and TSEA

In this section the basic equations of SEA, EDA, and TSEA are summarized and a TEDA equation will be derived in the next section.

2.1 SEA

In an SEA model, the system is regarded as an assembly of subsystems. It is assumed that energy dissipation is proportional to the vibration energy in a subsystem; transfer energy is also taken to be proportional to the vibration energy between two subsystems. Considering the power balance leads to a set of equations. The SEA equation is (2)

$$\mathbf{P}(\omega) = \omega \mathbf{L}(\omega)\mathbf{E}(\omega), \tag{1}$$

where **P** is the external power input, **E** is the subsystem energy stored, and ω is the band center angular frequency. **L** is the matrix of loss factors written as

$$\mathbf{L}(\omega) = \begin{bmatrix} \eta_1 + \eta_{12} + \eta_{13} + \cdots & -\eta_{21} & \cdots \\ -\eta_{21} & \eta_2 + \eta_{21} + \eta_{23} + \cdots & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}, \tag{2}$$

where η_i and η_{ij} are the internal loss factors of subsystem i and the coupling loss factors from subsystems i to j, respectively. The loss factors are dependent on the frequency.

In experimental SEA, the loss factors are evaluated from the data measured in the hammering test. They are given by

$$\eta_{ij} = \frac{E_{ji} / P_i}{\omega E_{ii} / P_i \cdot E_{jj} / P_j} \quad \text{and} \tag{3}$$

$$\eta_i = \frac{1 - \omega \sum_{j \neq i}^{n} (\eta_{ij} E_{ii} / P_i - \eta_{ji} E_{ji} / P_i)}{\omega E_{ii}}, \tag{4}$$

where E_{ij} is the energy of subsystem i under power input P_j to subsystem j in the hammering test. The subsystem energy and the power inputs to the subsystem by the hammer are given by

$$P_j = \mathrm{Im}\left[FA^*\right]\Big/\omega \quad \text{and} \tag{5}$$

$$E_{ij} = \frac{m_i}{2}\cdot\sum_k^q \left(A_k/\omega\right)^2 \Big/ q, \tag{6}$$

where F is the excitation force spectrum, A is the acceleration spectrum at the excitation, m_i is the mass of subsystem i, and A_k is the acceleration spectrum at point k on subsystem i.

2.2 EDA

In an EDA model, the energies **E** and time-averaged input powers **P** averaged over some frequency band are related by

$$\mathbf{E}(\omega) = \mathbf{A}(\omega)\mathbf{P}(\omega), \tag{7}$$

where **A** is a matrix of energy influence coefficients (EIC) in the relevant frequency band. The element A_{ij} shown in the next equation gives the (time and frequency averaged) energy in subsystem i per unit (time and frequency averaged) power input to subsystem j.

$$\mathbf{A}\left(\omega\right) = \begin{bmatrix} A_{11} & A_{12} & \cdots \\ A_{21} & A_{22} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \tag{8}$$

The EDA model is valid to for all kinds of linear structures. In most structures it is not easy to estimate the EIC, **A**, theoretically, so it is often evaluated by from measurement data or numerical data by using FEM and other methods. Generally **A** becomes a full matrix and is not usually symmetric.

The internal and coupling loss factors in an SEA model can be estimated from an ED model. If an ED model is formed, then equation (7) can be inverted to give

$$\mathbf{P}(\omega) = \mathbf{A}^{-1}(\omega)\mathbf{E}(\omega), \tag{9}$$

where

$$\mathbf{A}^{-1}(\omega) = \begin{bmatrix} a_{11} & a_{12} & \cdots \\ a_{21} & a_{22} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}. \tag{10}$$

By comparing equations (2) and (10), the relation between the EIC and the SEA loss factors can be derived:

$$a_{ii} = \omega\left(\eta_i + \sum_{k\neq i}^{n}\eta_{ik}\right) \quad \text{and.} \tag{11}$$

$$a_{ij} = -\omega\eta_{ji}. \tag{12}$$

These relations allow us to regard an EDA model as an SEA model; then it is easy to understand the problem by using only two design parameters, the damping of subsystems related to internal loss factors and the transmissibility between subsystems related to coupling loss factors.

An EDA model can be built by manipulations based on equations (7), (5), and (6) to estimate the power inputs and the subsystem energies under the hammer test.

2.3 TSEA

SEA is a tool for predicting the steady-state responses of structures. For transient phenomena, we can use Transient SEA. The equation of TSEA is given by

$$\mathbf{P}(\omega,t)=\frac{d}{dt}\mathbf{E}(\omega,t)+\omega\mathbf{L}(\omega)\mathbf{E}(\omega,t)\,, \tag{13}$$

where the input powers **P** and the subsystem energies **E** are time dependent in the relevant frequency band ω. There are different ideas about the use of the loss factor matrix **L**. One idea is to use the loss factors in equation (2) of the SEA model, which are time invariant. Another idea is to use time variant loss factors. In this study we simply use the time-invariant SEA loss factors of **L** in the equation (13).

2.4 Parameter study of TSEA

Here, the change of the predicted subsystem energies due to the loss factors is discussed. The test system is composed of three subsystems connected in a chain, as shown in Figure 1. Consider all internal and coupling loss factors are initially set to be 0.1 and 1 W is input to the subsystem 1 at the center angular frequency of ω=20 rad/s as the nominal case. Each loss factor is individually varied by a factor of 10 times and then the changes of the transient subsystem energies are recorded.

Figure 2 shows a comparison of the transient energy responses of subsystems 2 and 3 due to the internal and coupling loss factors. The maximum response values and the rise time vary with each loss factor change. It should be noted that the values of subsystem 3 are always less than those of

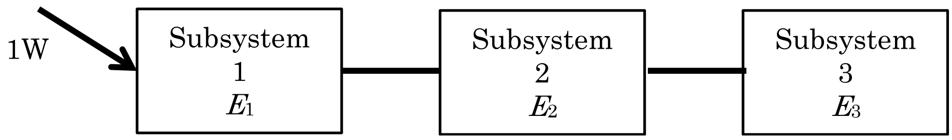
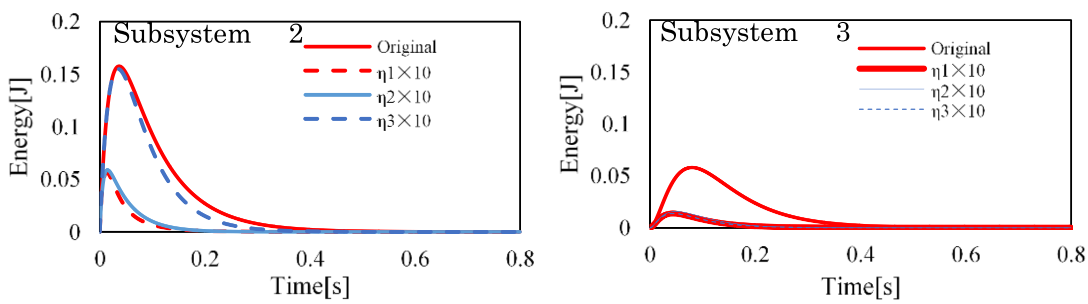
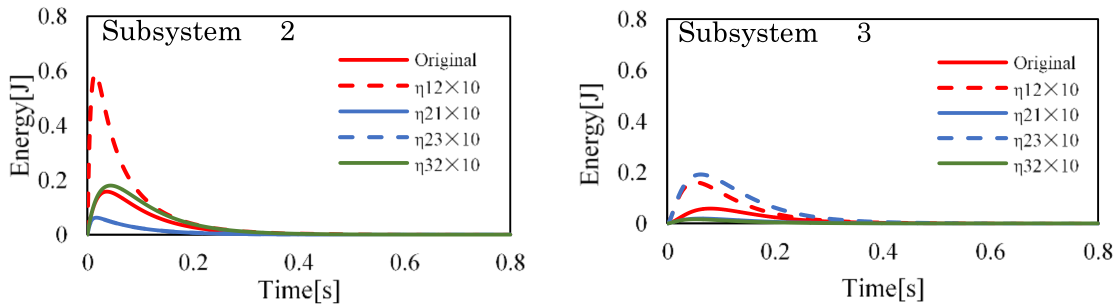


Figure 1 – Test system composed of three subsystems connected in a chain for parameter study



(a) Changes in internal loss factors



(b) Changes in coupling loss factors

Figure 2 –TSEA parameter study results for subsystem energy responses with different loss factors

subsystem 2 and the rise times on subsystem 2 are always shorter than those on subsystem 3. These facts are a normal consequence of the subsystem locations.

3. Proposal for TEDA

Like SEA, EDA is a tool for predicting steady-state energy responses. It seems that the extension of EDA to analyze transient responses can be carried out just as it was for TSEA. We propose the new transient method, TEDA, in this section and a comparison between TSEA and TEDA is investigated in the next section.

3.1 TEDA

EDA is also a tool for predicting the steady-state responses on of structures. For the case of transient phenomena, we propose TEDA in a form similar to that for TSEA, as shown in the equation (13). The equation of TEDA is therefore given by

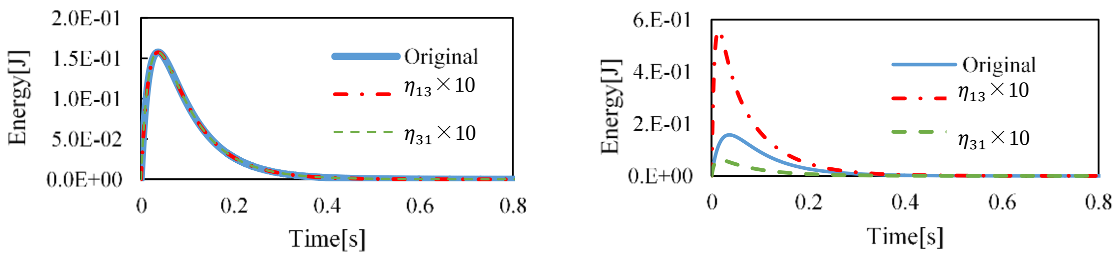
$$\mathbf{P}(\omega,t)=\frac{d}{dt}\mathbf{E}(\omega,t)+\mathbf{A}^{-1}(\omega)\mathbf{E}(\omega,t)\,. \tag{14}$$

The inverse of the EIC is also time-invariant. Thus, if an EDA model is formed, a TEDA model is easily obtained.

3.2 Parameter study of TEDA

We carried out a TEDA parameter study similar to the TSEA parameter study described in section 2.4. In this study, the system shown in Figure 1 and the nominal values are same as those described in section 2.4. The fluctuation of the subsystem energy responses due to changes in each loss factor is investigated.

Figure 3 shows the energy response results for subsystems 2 and 3 due to changes of the indirect coupling loss factors, η_{13} and η_{31}. It is should be noted that the responses of subsystem 3 are larger than those of subsystem 2 and the rise time on subsystem 3 is shorter than that on subsystem 2. This is because subsystem 3 receives energy input directly from subsystem 1. Of course, the indirect coupling loss factors have a large effect under in this study as the values of the indirect coupling loss factors are as large as those of the direct coupling loss factors.



(a) Subsystem 2 (b) Subsystem 3

Figure 3 –TEDA parameter study results for the subsystem energy responses with different indirect coupling loss factors

3.3 Setting of the input power

The maximum value of the response, its rise time, and the decay behavior are important in instantaneous-response analysis. In TEDA and TSEA predictions, the power input is an important factor in the response of each subsystem. There are many types of power input to be considered. The integration of the power input is equal to those of the energy responses (7). So we use the triangle signal for the impulsive input. Also the solution of equations (13) and (14) uses the initial energy rather than the set of power inputs. The initial subsystem energy for a given input is determined as follows.

(a) The excitation force and the acceleration near the excitation point are measured to evaluate the power input. We use a digital filter to extract the data in each frequency band (10).

(b) The input powers are evaluated by the product of the force and the acceleration found in the previous step.
(c) The initial energy E_0 input to a subsystem is determined from the time integration over an interval of 0.05s of the input powers evaluated in step (b).

3.4 Procedure for verification of TEDA predictions with the measured results

The results of TSEA and TEDA predictions are evaluated by comparison with the direct measured transient energy response on each subsystem at each frequency band. TSEA and TEDA results contain no information concerning the spatial distributions on the subsystems. So we have to process the direct measured results to compare with the predictions. We have already proposed a method to estimate the directly measured energy averaged response on a subsystem under the hammer test, which evaluates the averaged response over time from the data at several measurement points with an average location that is the same as the center of gravity of the subsystem. Then the measured results for comparing with the TSEA and TEDA predictions are determined as the following steps.

(a) The measurement points on a subsystem are selected to estimate the subsystem energy response. The points should be selected to match their spatially averaged location and the center of gravity of the subsystem.
(b) The accelerations and input force are measured and separated into frequency bands using digital filter processing (10).
(c) The acceleration data are converted to velocity data by the Simpson method.
(d) The subsystem energy in a frequency band is calculated from the squared normalized velocity and the mass of the subsystem.

4. Impulsive Response Analysis by TEDA

Here TEDA is applied to a simple structure with three subsystems for a comparison with TSEA.

4.1 Test structure

Figure 4 shows the test structure used in this study. This system is composed of three rectangular flat steel plates of 2 mm thickness. Plates 1, 2, and 3 are considered as subsystems 1, 2, and 3, respectively. The structure was freely suspended by strings at 2 points along each edge of plates 1 and 3.The impulsive force produced by the impact hammer is applied at the center of subsystem 1 and the transient responses of each subsystem are recorded.

The transient responses are measured by three accelerometers on each subsystem. The measured responses are normalized by the maximum value of the excitation force for comparability.

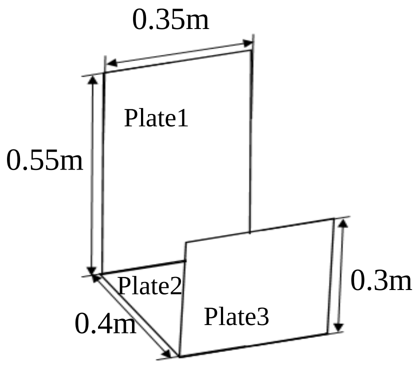
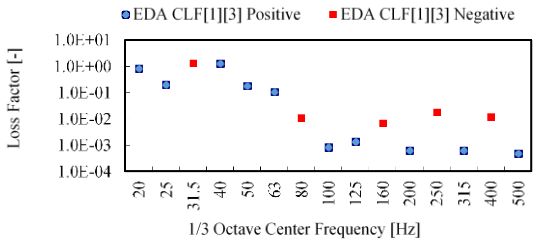


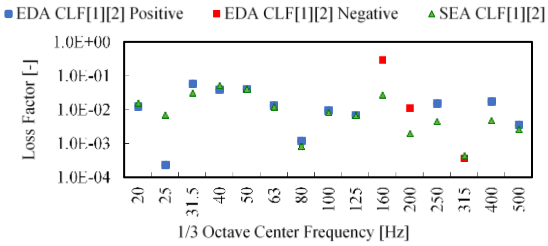
Figure 4 - The test system composed of three rectangular flat steel plates of 2 mm thickness

4.2 EDA and SEA models

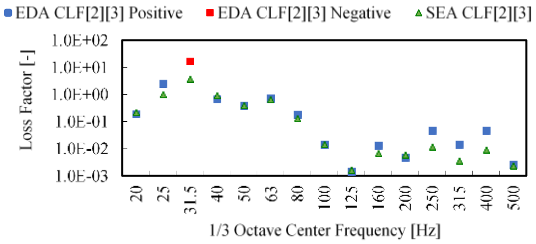
The EDA and SEA models are constructed according to the methods described in sections 2.1 and 2.2. Figure 5 shows the estimated loss factors. In the EDA model the indirect coupling loss factors (η_{13}, η_{31}) are also considered, as shown in Figure 5(a). The values are large in lower frequency ranges and decrease as the frequency increases. Some of the values are negative. Figure 5(b) shows the direct coupling loss factors (η_{12}, η_{21}, η_{23}, η_{32}) evaluated in both models. These factors are similar in almost all frequency bands, but in some bands the EDA factors are negative and different from the SEA factors.



(a) Indirect coupling loss factors, η_{13}

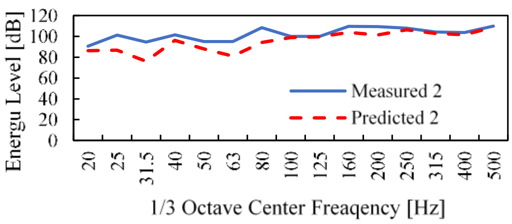


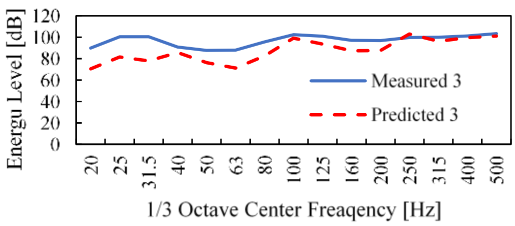
(b) Direct coupling loss factors, η_{12}



(c) Direct coupling loss factors, η_{23}

Figure 5 - Comparison of coupling loss factors in the SEA and EDA models.





(a) Subsystem 2 (plate 2)   (b) Subsystem 3 (plate 3)

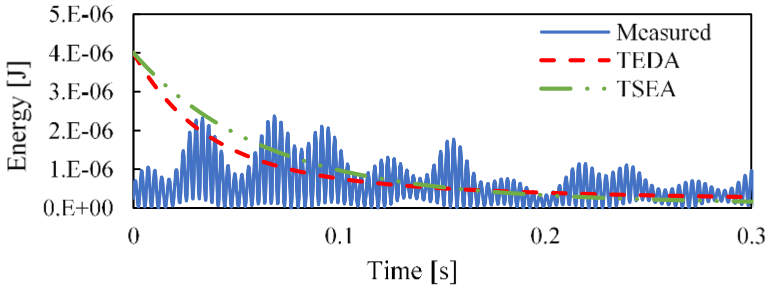
Figure 6 –SEA model accuracy. A comparison of the SEA prediction of the subsystem energy in the frequency domain with measurements.

In order to verify both models in the frequency domain, the predicted energies are compared with the measured results, as shown in Figure 6. The prediction by the EDA model is, of course, in agreement with the measured results because of the use of equation (7). Also the SEA predicted energy of the input subsystem 1 is, of course, in agreement with the measured energy, so the comparison is not shown. It is confirmed that the SEA model has some errors in the energy prediction, especially in low frequency ranges. The measured and predicted values become closer in higher frequency bands.

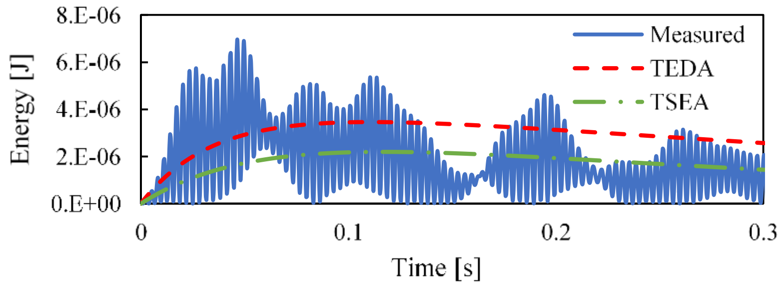
4.3 Comparison of TEDA with TSEA

Figure 7 shows a comparison of the transient subsystem energy responses predicted by TEDA and TSEA in the 160 Hz band on subsystems 2 and 3. It is shown that the predictions of TEDA and TSEA both give the mean time responses. The initial responses of input subsystem 1 are not in agreement because equations (13) and (14) are solved by using the initial energy in the TEDA and TSEA calculation while the hammering power input has some time interval. The predictions from TEDA are in better agreement with the measured results than those from TSEA in terms of the maximum response, its rise time, and the time decay.

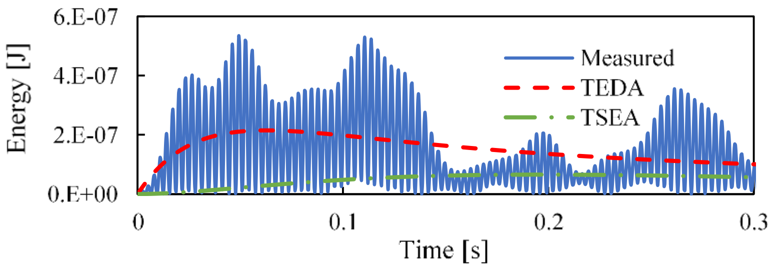
Figures 8 and 9 compare the measured and predicted results for the maximum values and the rise time in each frequency band. The maximum results shown in Figure 8 are in good agreement through the whole frequency band, but the results from TSEA have some differences in the lower frequency ranges. In Figure 9 it can be seen that the rise time results from TEDA are closer to the measured results.



(a) Subsystem 1 (Input plate 1)



(b) Subsystem 2 (plate 2)



(c) Subsystem 3 (plate 3)

Figure 7 - Comparisons of transient subsystem energy responses on the receiver plate in the 160 Hz band

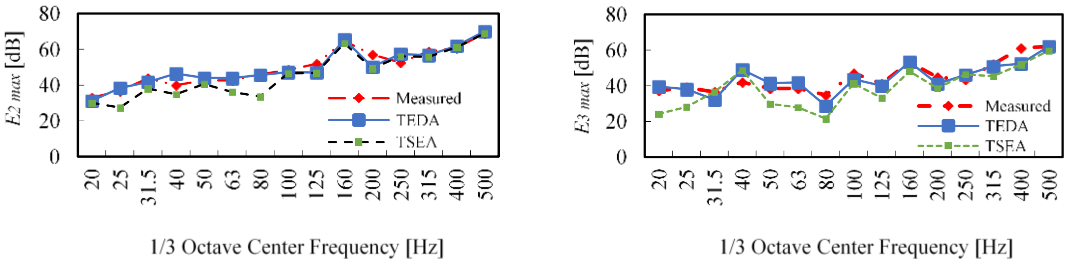
Overall, TEDA gives better predictions than TSEA in low frequency bands in terms of the maximum responses and rise times.

4.4 Discussion of results

In this section we discuss the reasons why the TEDA predictions are more accurate than those of TSEA. First, the direct transient energy responses are discussed.

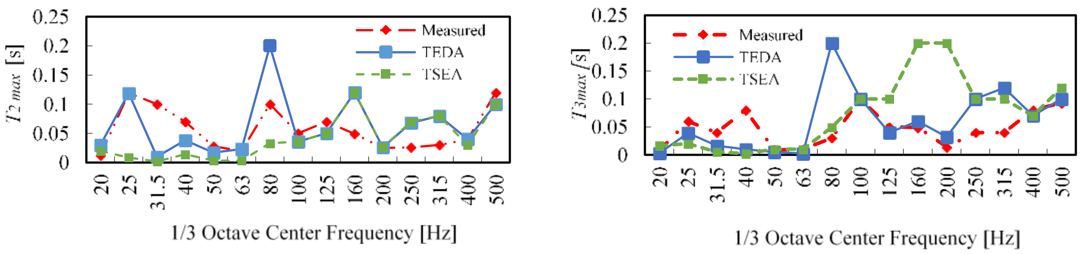
Figure 10 shows a comparison of the responses on subsystems 2 and 3. The response of subsystem 2 which is near the input subsystem is smaller and slower than the response of subsystem 3 which is far from the input subsystem. This indicates that the internal force is transmitted to subsystem 3 by the hammering on subsystem 1. The internal force would be transferred not as vibratory but as rigid motion. This phenomenon is called the “bridge effect” in this paper.

As shown in Figure 7, the response of subsystem 2 is larger than that of subsystem 3 and the bridge effect is not observed in the 160 Hz band frequency. In such frequency bands, TSEA is also useful to predict transient responses, as shown in Figures 8 and 9. On the other hand, especially in the lower frequency bands (e.g., 20 and 40 Hz), the responses of subsystem 2 are smaller than those of subsystem 3, as shown in Figure 8. In such bands, TSEA does not work well but TEDA does. As discussed in section 3.2, TEDA can include the bridge effect by employing indirect coupling loss factors. As a result, the transient energy transfer is not sequential and the response of a subsystem far from the excitation location is earlier and larger, especially in low frequency range. Thus, TSEA is not a suitable tool for energy prediction in a structure in which the bridge effect is important, whereas TEDA is suitable because it can take into account the bridge effect by employing indirect coupling loss factors.



(a) Subsystem 2 (plate 2) (b) Subsystem 3 (plate 3)

Figure 8 - Comparison of the maximum transient energy responses for measurements and predictions by TEDA and TSEA



(a) Subsystem 2 (plate 2) (b) Subsystem 3 (plate 3)

Figure 9 - Comparison of rise time responses for measurements and predictions by TEDA and TSEA

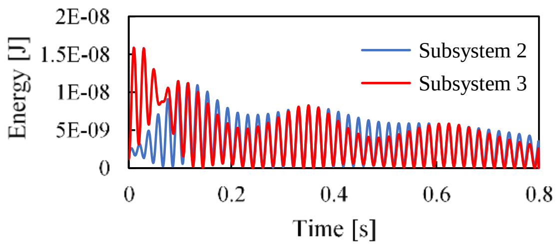


Figure 10 – Examples of transient subsystem energy responses in the 25 Hz band on subsystems 2 and 3

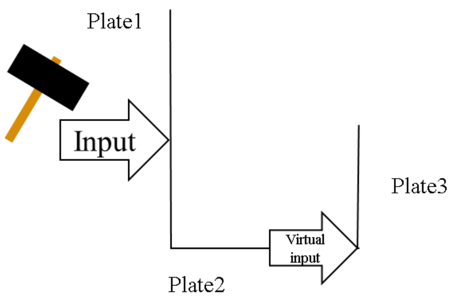


Figure 11 - Hypothesis about the input behavior of the bridge structure

5. CONCLUSIONS

In this paper, the concept of TEDA for predicting the transient energy responses was introduced. The basic equation of TEDA is same as that of TSEA. However, the loss factor matrix is replaced the energy influence coefficient of EDA. Parameter studies of TSEA and TEDA were carried out numerically. The employment of indirect coupling loss factors gives rise to the bridge effect, which refers to force transmission from the input subsystem to a distant subsystem. TEDA works well in structures with the bridge effect, such as the three-plate system employed in this work. When comparing the TEDA and TSEA predictions with directly measured results, the prediction by TEDA is in good agreement over all frequency bands, while the TSEA prediction is accurate only in higher frequency bands.

In future work, TEDA will be applied to actual complex structures such as violins and automotive bodies.

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