Transient response of complex stiffness system using a green function from the Hilbert Transform and the steady space technic.

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ABSTRACT

Transient response of complex stiffness system using a green function from the Hilbert Transform and the Steady Space Technic. The characteristics of damped structures which are designed to reduce the strength of vibrations and shock are expressed in complex stiffness. It is not easy to solve second-order differential equations which have complex stiffness because of the governing equation's singular points that cause time solution divergence. To solve this problem, free vibrations of these systems was obtained theoretically by the Hilbert Transform and the Steady Space Technic in which singular points are avoided and provides green function of the convolution integral. The result that are calculated by the numerical integration process for transient responses show accurate amplitude and phase differences. Therefore, it is suggested that this method provides an accurate way to estimate the maximum amplitude of time responses.

Keywords: Complex stiffness, Transient, Hilbert transform I-INCE Classification of Subjects Number(s): 75.9

1. INTRODUCTION

Viscoelastic material, such as rubber is widely used in industrial field to reduce the strength of vibrations and shock. For example rubber mounts are used for vibration isolator and sandwich structures which are laminated by viscoelastic and elastic material absorb the vibration energy through the viscoelastic layer. These damped structures are easy to make also have good performance. But in case of analysis, it is not same with elastic structure analysis because of viscoelastic material's damping ability.

There are various mathematical models to express the characteristics viscoelastic material. From among these, the hysteresis damping is commonly used and easy to apply as a complex stiffness. In frequency range, a complex number express the phase difference which causes the energy dissipation and so complex number's character well agree with hysteresis damping. Therefore complex number widely used in frequency analysis.

To get a time solution, equation of motion in frequency range would be changed in to time range. Inaudi (1, 2) used the Hilbert Transform and Inverse Fourier transform to make equation of motion in time domain and applied the time inverse method to get time response of complex stiffness system also suggested the iteration method. M. Salehi (3) used Inverse Fourier Transform for solving time response of sandwich structure. Bae (4) suggested numerical method to solve the initial condition problem for the five-layered viscoelastic sandwich beam.

But these methods base on discrete Fourier transform and so signal's periodicity strongly affects the time response. It means that force signals and time responses are periodic signal which can make interference effect. And initial condition problem cannot be solved by these methods.

In this study, continuing the previous study, the way to obtain the 1-DOF transient response of complex stiffness which has the initial condition problem was suggested theoretically with equation of

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motion in complex stiffness system. First, free vibrations of these systems were obtained theoretically by the Hilbert Transform and the Steady Space Technic in which provides green function of the convolution integral. Second, the response at the arbitrary force of this system was obtained by green function and convolution integral. At last, from this method identification method also suggested.

2. HIBERT TRANSFORM AND COMPLEX STIFFNESS SYSTEM

2.1 Hilbert transform

The Hilbert transform is a form of integral transform defined by Cauchy's principle value and give us the $\pm\pi/2$ phase shift operator and also conserve the energy of signal. (5) The Hilbert transform of function f(x) is given by

$$\hat{f}(t) = (1/\pi)P\int_{-\infty}^{\infty} f(\tau)\,/(\,t-\tau\,)d\tau \quad (1)$$

Here, Capital letter P means Cauchy's principle value and its definition is given by

$$P\int_{\alpha}^{\beta} p(y)dy = \lim_{\epsilon\to 0^+}(\int_{\alpha}^{y_0-\epsilon} p(y)dy + \int_{y_0+\epsilon}^{\beta} p(y)dy) \quad (2)$$

Cauchy's principle value represents the preceding limiting process also balancing (or canceling) process because symmetric or even interval provides cancelation of integration area when p(x) has singular point at at $z = y_0$. (6)

Hilbert transform phase shift operator is explained by Fourier transform. The Hilbert transform Eq.(1) can be expressed as convolution integral as follow

$$\hat{f}(t) = f(t) * (1\,/\,\pi t\,) \quad (3)$$

and the Fourier transform of Eq.(3) becomes

$$\hat{F}(\omega) = FFT[\hat{f}(t)] = -j\,sgn(\omega)F(\omega) \quad (4)$$

where,

$$sgn(\omega) = -1\ (\text{for}\ \omega < 0),\qquad 0\ (\text{for}\ \omega = 0),\qquad 1\ (\text{for}\ \omega > 0)$$

In frequency domain, the Hilbert transform multiply $\mp j$ and $F(\omega)$ according to plus and minus frequency sign. And here, imaginary number j is phase shift operator and same with $e^{j\pi/2}$ in phase plane. Hilbert transform can be easy in frequency domain by only multiplication without any integration. Therefore, the process to get Eq. (1) consisted of Fourier transform and multiplication and inverse Fourier transform. And this process can be easier if we use the analytic signal as Eq. (5)

$$f_a(t) = f(t) + j\hat{f}(t) \quad (5)$$

And the Fourier transform of Eq. (5) using Eq. (4) is one-side-spectrum or engineering spectrum which negative frequency component are zero and analytic signal $f_a(t)$ is obtained by inverse Fourier transform of one-side-spectrum.

$$f_a(t) = \int_{0}^{\infty} 2F(\omega)e^{j\omega t}d\omega \quad (6)$$

But actually, we use the discrete Fourier transform which mean Fourier series and periodicity is one of the characteristic of Fourier series.

2.2 Time Domain Equation of motion in Complex Stiffness System

The transient response needs the time domain equation of motion. However the complex stiffness system typically defined in Frequency domain. Accordingly we used the inverse Fourier transform and Hilbert transform to change the domain.

Frequency domain 1-DOF equation of motion is given

$$-\omega^2 mX(\omega) + (k + j\,k\eta\,sgn(\omega))X(\omega) = F(\omega) \quad (7)$$

using Eq. (4) We have

$$-\omega^2 mX(\omega) + kX(\omega) - k\eta\hat{X}(\omega) = F(\omega) \quad (8)$$

and Hilbert transform of Eq. (8) as follow

$$-\omega^2 m\hat{X}(\omega) + k\hat{X}(\omega) + k\eta X(\omega) = \hat{F}(\omega) \quad (9)$$

Multiplying j and Eq. (9), adding to Eq. (8), and applying Eq.(5), we obtain the equation of motion as a form Fourier transform of analytic signal.

$$-\omega^2 mX_a(\omega) + kX_a(\omega) + jk\eta X_a(\omega) = F_a(\omega) \quad (10)$$

Therefore, inverse Fourier transform of Eq. (10) is time domain equation of motion of complex stiffness system and given by

$$m\ddot{x}_a(t) + k(1 + j\eta)x_a(t) = f_a(t) \quad (11)$$

3. TIME ANALYSIS OF COMPLEX STIFFNESS SYSTEM

3.1 Initial Condition Problem of Complex Stiffness System

Referenced method (Inaudi, Ifft) start from the analytic signal of force $f_a(t)$ and this force signal summed as periodic wave because of characteristic of Fourier series. So there is no room for initial condition problem.

Therefore, to find the initial condition problem, we avoid using the discrete Hilbert transform and assumed that $x_a(t)$ is kind of analytic functions u(t)+j*v(t) containing analytic signals. This assumption means that the result of real function u(t), v(t) are related by phase-shift and $x_a(t)$ does not separately consists of x(t) and discrete Hilbert transform of x(t).

Substituting u(t)+j*v(t) with $x_a(t)$, Eq. (11) is separated into real part and imaginary part.

$$m\ddot{u}(t) + ku(t) - k\eta v(t) = 0 \quad (12)$$

$$m\ddot{v}(t) + k\eta u(t) + kv(t) = 0 \quad (13)$$

Eq. (12) is the real part of Eq. (11) and Eq. (13) is imaginary part. Since Eq. (12,13) is related each other, both equation should be solved altogether. So express Eq. (12,13) into Steady space technic.

$$\frac{d}{dt}\begin{Bmatrix} u \\ \dot{u} \\ v \\ \dot{v} \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_n{}^2 & 0 & \eta\omega_n{}^2 & 0 \\ 0 & 0 & 0 & 1 \\ -\eta\omega_n{}^2 & 0 & -\omega_n{}^2 & 0 \end{bmatrix}\begin{Bmatrix} u \\ \dot{u} \\ v \\ \dot{v} \end{Bmatrix} \quad (14)$$

Changing Eq. (14) as a simple form,

$$\{\dot{x}\} = [L]\{x\} \quad (15)$$

Here we can get modal coordinates and Eigen value though the Eigen value problem to find the free vibration response. Modal coordinates and Eigen value are given by

$$\{x\} = [\,V\,]\{p\} \quad (16)$$

$$\{\dot{p}\} = [\,V\,]^{-1}[\,L\,][\,V\,]\{p\} = [\,\lambda\,]\{p\} \quad (17)$$

Where, Modal coordinates and Eigen values are given by

$$[\,\lambda\,] = \begin{bmatrix} z & 0 & 0 & 0 \\ 0 & \bar{z} & 0 & 0 \\ 0 & 0 & -\bar{z} & 0 \\ 0 & 0 & 0 & -z \end{bmatrix}$$

$$[\,V\,] = \begin{bmatrix} -(\alpha_I + j\alpha_R)/s & (-\alpha_I + j\alpha_R)/s & (\alpha_I - j\alpha_R)/s & (\alpha_I + j\alpha_R)/s \\ -j & j & j & -j \\ (\alpha_R - j\alpha_I)/s & (\alpha_R + j\alpha_I)/s & -(\alpha_R + j\alpha_I)/s & (-\alpha_R + j\alpha_I)/s \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$s = \alpha_R^2 + \alpha_I^2, r = \sqrt{2 + 2/s}$$

Here, Eigen values of [λ] are located in first quadrant and the other Eigen values symmetrically located in each quadrant.

The solution of Eq. (17) is form of the exponential formula e^{jzt}. So modal coordinate solution turn into normal coordinate by using Eq. (16)

$$\{x\} = C_1\{\vec{v}_1\}e^{zt} + \bar{C}_1\{\bar{\vec{v}}_1\}e^{\bar{z}t} + C_2\{\vec{v}_3\}e^{-\bar{z}t} + \bar{C}_2\{\bar{\vec{v}}_3\}e^{-zt} \quad (18)$$

Where, $C_1 = \sigma_1 + \nu_1$, $C_2 = \sigma_2 + \nu_2$

Here, the $v_i$means column ith vector of modal coordinates [V]. v_2 is complex conjugate of v_1 also column vectors v_3, v_4 have same relation. Followed by these relations that cause cancelation of imaginary part, the components of {x} only remain real value. Free vibration response {x} is given by

$$\{x\} = \begin{matrix} e^{\alpha_R t}[(A\cos(\alpha_I t) + B\sin(\alpha_I t))\mathrm{Re}\{\vec{v}_1\} + (B\cos(\alpha_I t) - A\sin(\alpha_I t))\mathrm{Im}\{\vec{v}_1\}] \\ e^{-\alpha_R t}[(C\cos(\alpha_I t) + D\sin(\alpha_I t))\mathrm{Re}\{\vec{v}_3\} + (D\cos(\alpha_I t) - C\sin(\alpha_I t))\mathrm{Im}\{\vec{v}_3\}] \end{matrix} \quad (19)$$

Where, $A=2\sigma_1$, $B=2\nu_1$, $C=2\sigma_2$, $D=2\nu_2$

Then, we check the boundary condition and initial condition. In a spring mass system, initial condition mean amount of total energy which system have it first time. This total energy is diffused by hysteresis damping in complex stiffness system and it mean that through the infinite time total energy naturally goes to zero. Implying this assumption, unknown quantity A, B are zero.

Applying t=0 to Eq. (19) and, matrix form of initial condition problem are

$$\{x(0)\} = \begin{Bmatrix} u_0 \\ \dot{u}_0 \\ v_0 \\ \dot{v}_0 \end{Bmatrix} = r^{-1} \begin{bmatrix} \alpha_I s^{-1} & -\alpha_R s^{-1} \\ 0 & 1 \\ -\alpha_R s^{-1} & -\alpha_I s^{-1} \\ 1 & 0 \end{bmatrix} \begin{Bmatrix} C \\ D \end{Bmatrix} \quad (20)$$

Solving upper part matrix (2×2) from Eq. (20) with real part of initial condition u_0 and $\dot{u}_0$, unknown constant C and D is given by

$$C = r(su_0 + \alpha_R \dot{u}_0)/\alpha_I \,, D = r\dot{u}_0 \quad (21)$$

Applying C and D to lower part matrix (2×2), we have imaginary initial condition v_0 and $\dot{v}_0$

$$v_0 = -(\dot{u}_0 + u_0\alpha_R)/\alpha_I \,, \dot{v}_0 = (\dot{u}_0\alpha_R + u_0 s)/\alpha_I \quad (22)$$

Substituting Eq. (21) in Eq. (19) and arranging equation by Eq. (22) We have simple form of free vibration response of complex stiffness system, where mass is 1.

$$u(t) = e^{-\alpha_R t}\{u_0\cos(\alpha_I t) - v_0\sin(\alpha_I t)\} \quad (23)$$

$$\dot{u}(t) = e^{-\alpha_R t}\{\dot{u}_0\cos(\alpha_I t) - \dot{v}_0\sin(\alpha_I t)\} \quad (24)$$

$$v(t) = e^{-\alpha_R t}\{u_0\sin(\alpha_I t) + v_0\cos(\alpha_I t)\} \quad (25)$$

$$\dot{v}(t) = e^{-\alpha_R t}\{\dot{u}_0\sin(\alpha_I t) + \dot{v}_0\cos(\alpha_I t)\} \quad (26)$$

These forms of equations satisfy the assumption that $x_a(t)$ is related with Hilbert transform following by Bedrosian identity, product of lowpass and highpass signals with nonoverlapping spectra and Hilbert transform of this signal is defined by a product of the lowpass signal and Hilbert transform of the highpass signal. (7)

3.2 Time Response of complex stiffness system at arbitrary force

A convolution integral is basically used for getting transient response. As a green function, unit impulse response is made of free vibration response substituting initial condition $u_0 = 0$, $\dot{u}_0 = 1$.

$$\{x(t)\} = (1/\pi)P\int_0^t f(\tau)\{g(t-\tau)\}d\tau \quad (27)$$

Where, {g(t)} is unit impulse response of complex stiffness system.

Also this numerical convolution integral can be solved variously depend on how to approximate the force signal and we try to approximate in three way

3.3 Superposition method

The impulses ($F_i dt$) during the unit time (dt) is represented in Figure 1. And responses due to the impulses are produced continually following by arbitrary force signal $f(t_i)$. Therefore superposition of unit impulse response is transient response of $f(t_i)$. Formulation of superposition method is given by

$$\{x(t)\} = \sum_{n=1}^{N} us(t-ndt)(F_i dt\{x_u(t-ndt)\})/m \quad (28)$$

Where, us(t) is unit step function and $x_u(t)$ is unite impulse response which meet the initial condition $u_0 = 0$, $\dot{u}_0 = 1$. And Eq. (27) goes to form of convolution integral when $N\to\infty$ and $dt\to 0$

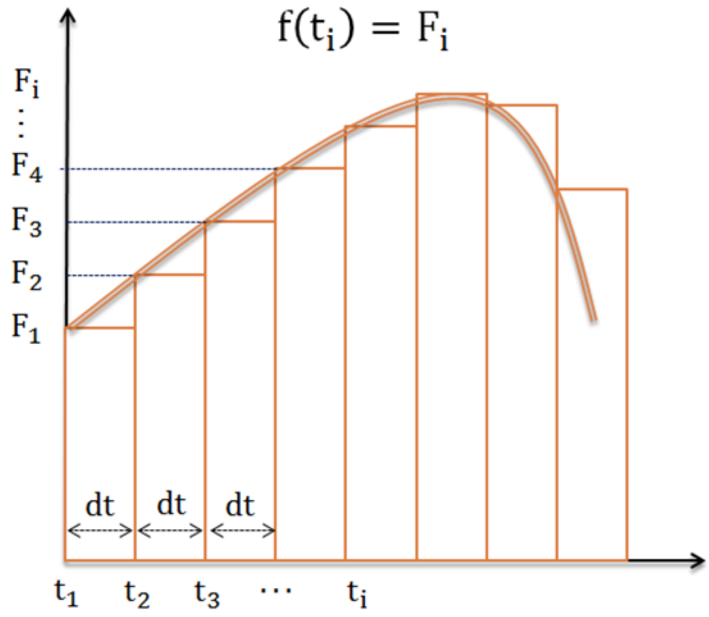

Figure 1 – Zero order discrete arbitrary force signal $f(t_i)$

3.4 Superposition method

This method used same force approximation which the magnitude of unit impulses is constant during small time dt. But we used free vibration response and analytic convolution integral (from 0 to dt) to predict the next time step value. The solution of 1-DOF differential equation is summation of particular solution and homogeneous solution so that the next time step value can be obtained by adding free vibration response to analytic convolution integral. For the convenience, defining the exponential sine and cosine integration

$$ESI = \int_0^{dt} e^{-\alpha_R(t-\tau)}\sin(\alpha_I(t-\tau))d\tau, ECI = \int_0^{dt} e^{-\alpha_R(t-\tau)}\cos(\alpha_I(t-\tau))d\tau \quad (29)$$

Using Eq. (29) zero order transient response due to $f(t_i)$ is given by

$$u_{i+1} = F_{i+1}ESI/(\alpha_I m) + e^{-\alpha_R t}\{u_i\cos(\alpha_I dt) - v_i\sin(\alpha_I dt)\} \quad (30)$$

$$\dot{u}_{i+1} = (F_{i+1}/m)(ECI - \alpha_R ESI/\alpha_I) + e^{-\alpha_R t}\{\dot{u}_0\cos(\alpha_I dt) - \dot{v}_0\sin(\alpha_I dt)\} \quad (31)$$

From Eq. (22), (30) and (31), predicting the next time step value is possible by previous time step value when we know the system's Eigen value. And also v_{i+1} and $\dot{v}_{i+1}$ can be calculate in the similar way.

$$v_{i+1} = F_{i+1}ECI/(\alpha_I m) + e^{-\alpha_R t}\{u_0\sin(\alpha_I t) + v_0\cos(\alpha_I t)\} \quad (32)$$

$$\dot{v}_{i+1} = (F_{i+1}/m)(ESI - \alpha_R ECI/\alpha_I) + e^{-\alpha_R t}\{\dot{u}_0\sin(\alpha_I t) + \dot{v}_0\cos(\alpha_I t)\} \quad (33)$$

3.5 Superposition method

Referring to Fig.2 in the first order approximation, force signal assumed as a linear function. And difference between Fig. 1 and Fig. 2 representing the force signal is small triangles which means linear component. So adding a linear component of integration to the Eq. (30,31) is the way to get time solution. In common with zero order approximation, let's define linear component of integration

$$LESI(t) = \int_0^{t} \tau e^{-\alpha_R(t-\tau)}\sin(\alpha_I(t-\tau))d\tau, LECI(t) = \int_0^{t} \tau e^{-\alpha_R(t-\tau)}\cos(\alpha_I(t-\tau))d\tau \quad (34)$$

Using Eq. (34) first order transient response due to $f(t_i)$ is given by

$$u_{i+1} = \Delta F_i LESI(dt)/(\alpha_I m dt) + F_i ESI(dt)/(\alpha_I m) + e^{-\alpha_R t}\{u_i\cos(\alpha_I dt) - v_i\sin(\alpha_I dt)\} \quad (35)$$

$$\dot{u}_{i+1} = \Delta F_{i+1}(LECI(dt) - \alpha_R LESI(dt)/\alpha_I)/(m dt) + F_{i+1}(ECI(dt) - \alpha_R ESI(dt)/\alpha_I)/m + e^{-\alpha_R t}\{\dot{u}_0\cos(\alpha_I dt) - \dot{v}_0\sin(\alpha_I dt)\} \quad (36)$$

Again, v_{i+1} , $\dot{v}_{i+1}$ can be calculate in the similar way.

$$v_{i+1} = \Delta F_i LECI(dt)/(\alpha_I m dt) + F_i ECI(dt)/(\alpha_I m) + e^{-\alpha_R t} e^{-\alpha_R t}\{u_0 \sin(\alpha_I t) + v_0 \cos(\alpha_I t)\} \quad (37)$$

$$\dot{v}_{i+1} = \Delta F_{i+1}(LESI(dt) - \alpha_R LECI(dt)/\alpha_I)/(mdt) + F_{i+1}(ESI(dt) - \alpha_R ECI(dt)/\alpha_I)/m + e^{-\alpha_R t}\{\dot{u}_0 \sin(\alpha_I dt) + \dot{v}_0 \cos(\alpha_I dt)\} \quad (38)$$

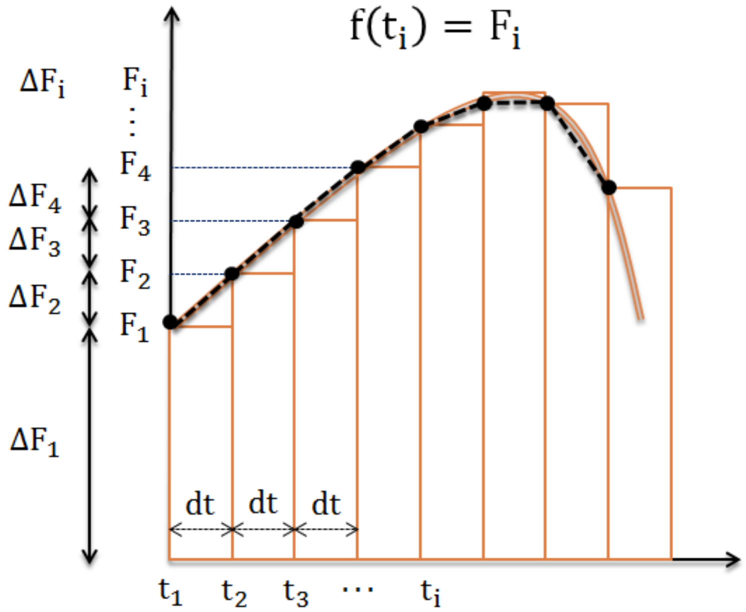

Figure 2 – Discrete arbitrary force signal $f(t_i)$

4. NUMERICAL EXAMPLE

4.1 Initial condition problem

In order to illustrate the Bedrosian identity described in section 3.1, we make the figure of given free vibration. Mass, stiffness and loss factor of system are given by $m = 1kg$, $k = (100\pi)^2 N/m^2$, $\eta = 0.2$ and governing equation is

$$\ddot{x}_a(t) + (100\pi)^2 N/m^2 (1 + 0.2j) x_a(t) = 0 \quad (39)$$

With u(0)=0, $\dot{u}$(0)=1, form Eq.(23) to Eq. (26), time solutions of $u(t)$, $\dot{u}(t)$, $v(t)$, $\dot{v}(t)$ are represented in Fig.3 and it is the analytic impulse responses of complex stiffness system. In each graph in Fig. 4, $u(t)$ and $\dot{u}(t)$ have $\pi/2$ phase difference with $v(t)$ and $\dot{v}(t)$ following by Bedrosian identity.

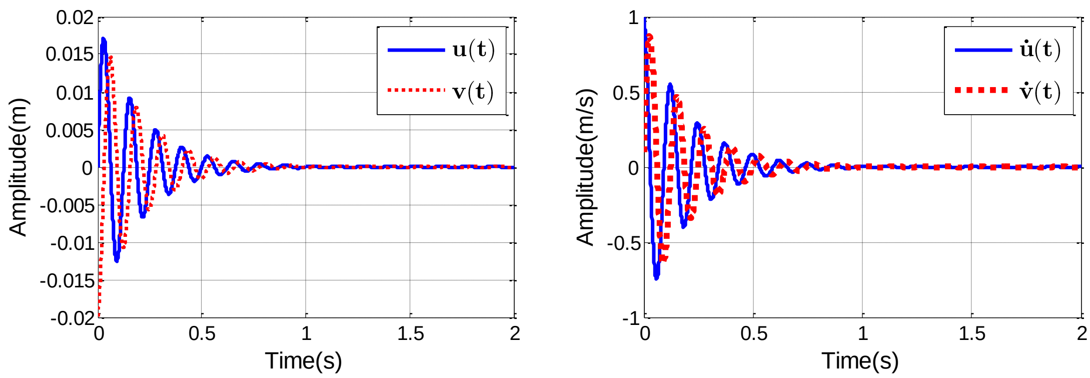

Figure 3 – Free vibration response $u(t)$, $\dot{u}(t)$, $v(t)$, $\dot{v}(t)$ of complex stiffness system

4.2 Transient response of triangular force

With the same governing Eq. (39), in order to compare the approximation method, the transient response of triangular force is calculated. Let the triangular force signal are

$$\mathrm{Tri}(t) = 20t \ \ [\ t < t_0\ ], \qquad -20(t - t_0)\ \ [\ t_0 \le t < 2t_0\ ], \qquad 0\ (t \ge 2t_0) \tag{40}$$

Triangular force signal in Fig. 3 is linear and discontinues at two points $t = t_0, 2t_0$ and slop of triangle is 20, $t_0 = 0.25s$.

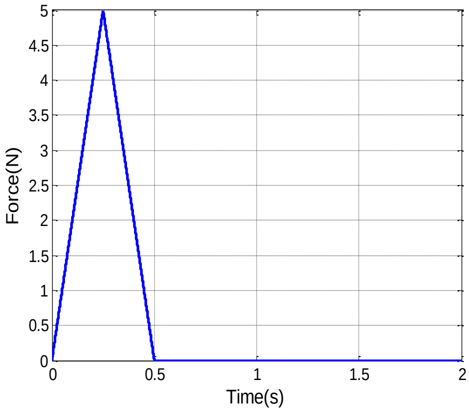

Figure 4 – triangular force signal

Convolution integral of force signal in Fig. 4 can be calculated as the same way to get Eq. (35) and (36) by separating an interval of integration. With zero initial condition, integration of triangular force from 0 to t_0 are given by

$$u(t) = -20\mathrm{LESI}(t)/\alpha_I\,, \ \ \dot{u}_{i+1} = 20(\,\mathrm{LECI}(t) - \alpha_R\mathrm{LESI}(t)/\alpha_I) \tag{41}$$

The response from t_0 to $2t_0$ is represented as a superposition of free vibration and forced vibration. It follows as

$$u(t + t_0) = -20\mathrm{LESI}(t)/\alpha_I + 20t_0\mathrm{ESI}(t)/\alpha_I + e^{-\alpha_R t}\{u_{t_0}\cos(\alpha_I t) - v_{t_0}\sin(\alpha_I t)\} \tag{42}$$

$$\dot{u}(t + t_0) = -20(\,\mathrm{LECI}(t) - \alpha_R\mathrm{LESI}(t)/\alpha_I) + 20t_0(\,\mathrm{ECI}(t) - \alpha_R\mathrm{ESI}(t)/\alpha_I) + e^{-\alpha_R t}\{\dot{u}_{t_0}\cos(\alpha_I t) - \dot{v}_{t_0}\sin(\alpha_I t)\} \tag{43}$$

The response after $2t_0$ is given by only form of free vibration

$$u(t + 2t_0) = e^{-\alpha_R t}\{u_{2t_0}\cos(\alpha_I t) - v_{2t_0}\sin(\alpha_I t)\} \tag{44}$$

$$\dot{u}(t + 2t_0) = e^{-\alpha_R t}\{\dot{u}_{2t_0}\cos(\alpha_I t) - \dot{v}_{2t_0}\sin(\alpha_I t)\} \tag{45}$$

4.3 Transient response of triangular force

In Fig.5 we compare the each approximation method according to the time interval dt. The time interval is set for dt=0.25s which is the roughest interval expressing the triangular force. Except for First order approximation, all other approximations show the inaccurate prediction and zero order approximation bring about time delay.

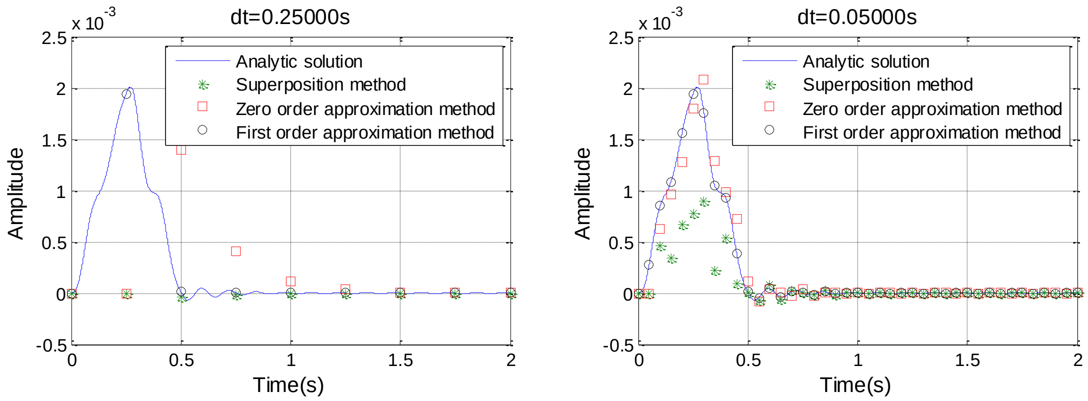

Figure 5 – Transient response of complex stiffness 1-dof system dt=0.25s , dt=0.05s

In the next figure we apply the time interval for $dt = 0.05s$ and zero order approximation again generates time delay because of using forward difference. Also superposition method show wrong prediction on account of long time interval dt which does not show the natural frequency of free vibration response. But when the time interval became short and short in Fig.5 three methods give us almost same results.

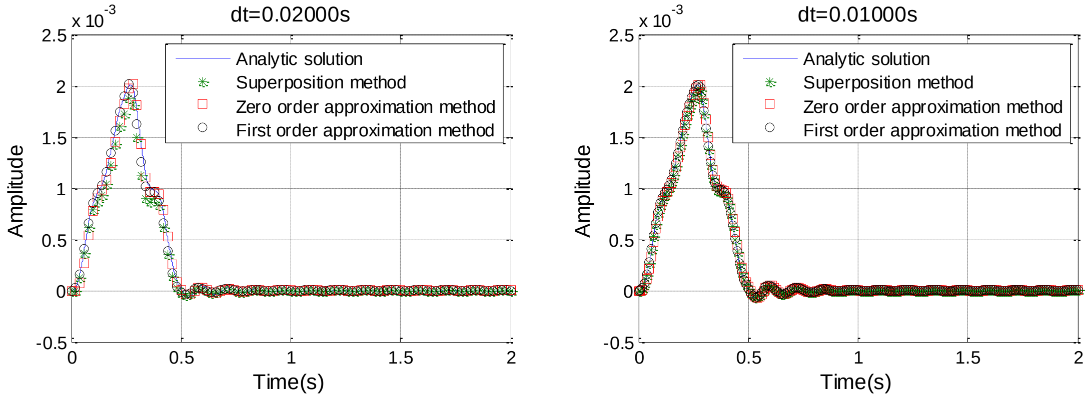

Figure 6 – Transient response of complex stiffness 1-dof system dt=0.02s , dt=0.01s

Referring these results in Fig 6, Fig 7, arbitrary excitation force time interval in order to obtain a response, should be chosen as small as the shortest time interval of the triangular wave interval for first order approximation. Otherwise, response of this system will have a time delay or ignore the short-step force signal.

4.4 Transient response of triangular force

Transient response with initial condition problem is solved by first order approximation method. Characteristic of system are given $m = 1kg$, $k = (100\pi)^2$, $\eta = 0.2$ and force shape is half sine wave. The initial condition of system is u(0)=5e-5m, $\dot{u}(0)$=3e-2m/s

In Fig.7 Response occur t=0 because of Initial condition and force start to excite the system at t=0.1s so it moves again from that time.

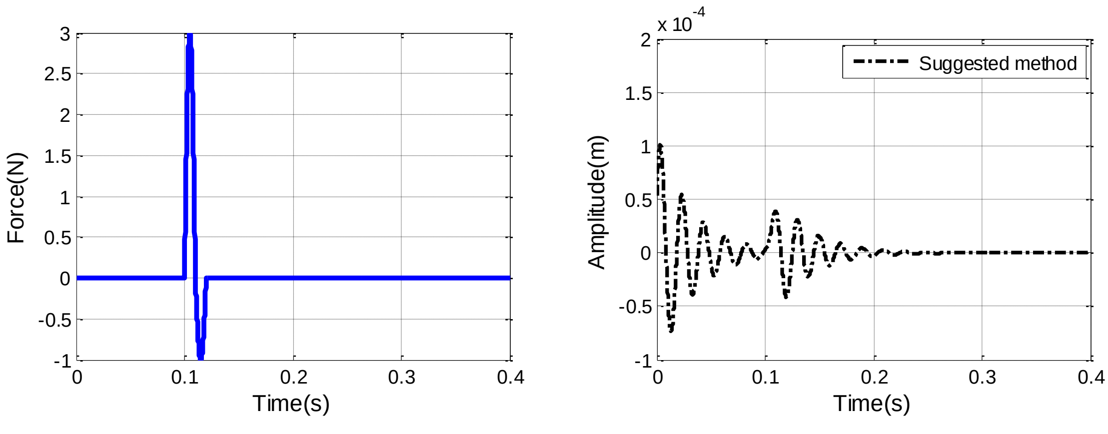

Figure 7 – Transient response with initial condition

4.5 Transient response of triangular force

To check the accuracy of response, let us compare the impulse response of FRF. Discrete Fourier transform of impulse response and transfer function have almost same magnitude but it has phase difference at low frequency. In Fig. 8 two line's gap is a phase difference ϕ. These phase differences make a response to distort and delay thus phase differences function help to modify it. A phase difference is derived by subtracting the phase tangent of each transfer function. Phase difference of complex stiffness system and suggest Eq. (46) are given by

$$\tan(\phi) = \frac{2a(\omega_n^2 - \omega^2)\omega + {\omega_n}^2(-s + \omega^2)\eta}{(\omega_n^2 - \omega^2)(s - \omega^2) + 2a{\omega_n}^2\omega\eta} \quad (46)$$

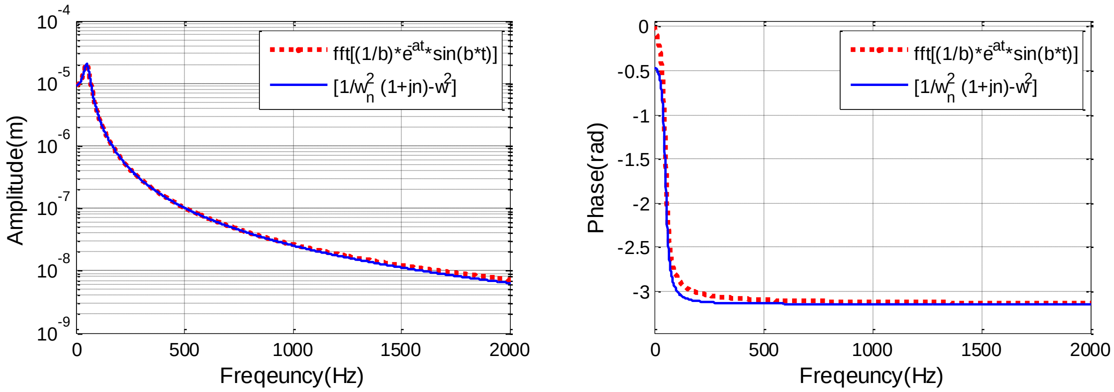

Figure 8 – Transient response with initial condition

Fig. 9 show the effeteness of phase difference function, transient response is modified small amount. Because both phase functions have almost same phase except for low frequency. And even though it has the highest phase difference in low frequency, normal excitation which consist of width frequency range periodic force effect was not affected heavily. But in case of only low frequency force, transient response should be modified by phase differences function.

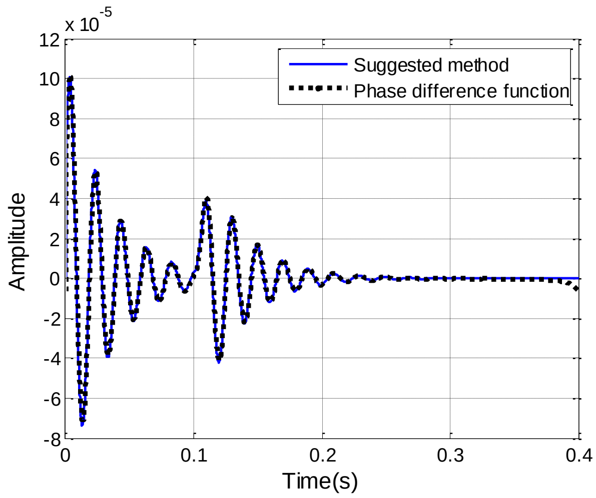

Figure 9 – The effect of phase difference function

5. CONCLUSIONS

The method to get transient response of complex stiffness system which contain initial condition problem was suggested in this paper, the Hilbert Transform and the Steady Space Technic are used to get free vibration response of this system and derived equation give us green function of convolution integral. As a way of numerical integration, convolution integral is used for small discrete time step integration to predict next time step value and superposition method and zero and first order approximation method are introduced and compared. The results of comparison show that first order approximation is the most accurate in those methods.

Checking the mag and phase graph, Suggest method has a phase difference but it is affect little in case of normal excitation or can be modify by phase difference function.

ACKNOWLEDGEMENTS

This work was supported by the Human Resources Development of the Korea Institute of Energy Technology Evaluation and Planning (KETEP) grant funded by the Korea Ministry of Knowledge Economy (No. 20113020020010). The financial support for this work through World Class 300 from Ministry of Knowledge Economy of Korea is acknowledged.

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