High-resolution nearfield acoustic holography based on iterative weighted equivalent source method

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ABSTRACT

Because of the ill-posed nature of equivalent source method (ESM)-based nearfield acoustic holography (NAH), regularization methods based on the minimum L2-norm criterion such as the Tikhonov regularization are usually employed to stabilize its reconstruction procedure. However, the minimum L2-norm constraint sometimes makes the reconstruction result too smooth and thus degrades the spatial resolution of NAH, because this kind of constraint tends to disperse the acoustic field energy on the hologram surface to all the equivalent sources. To deal with this problem, an iterative weighted ESM (IWESM) is proposed, in which an iterative weighted regularization procedure is developed to solve the strengths of equivalent sources. The IWESM can reduce the dispersion of acoustic field energy on the hologram surface to those equivalent sources that contribute little to the acoustic field, and get more reasonable solution of source strengths. The validity of the IWESM is proven by a numerical simulation, and the result shows that comparing with ESM-based NAH, the proposed method can significantly improve the spatial resolution of reconstruction results.

Keywords: High-resolution, Nearfield acoustic holography, Weighted norm, Equivalent source method
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1. INTRODUCTION

The equivalent source method (ESM) (also known as the superposition method) (1-3) is an alternative approach to realize the spatial transformation of acoustic field in nearfield acoustic holography (NAH). The basic idea of the ESM-based NAH is to approximate the actual acoustic field by superposing the field produced by a series of equivalent sources. The strengths of equivalent sources are determined by matching the pressure measured on the hologram surface, then the acoustic quantities in the acoustic field can be obtained by the source strengths and the transfer matrices constructed by the equivalent sources. ESM-based NAH is easy to implement and can be used to deal with NAH problem of arbitrarily shaped source (1-2). Furthermore, ESM-based NAH avoid the windowing effect which relates to the use of the DFT and usually can get a more accuracy result (4). However, because of the ill-posed nature of NAH problem, regularization methods based on the minimum L2-norm criterion such as the Tikhonov regularization (5) are needed to stabilize the strengths solving procedure in ESM-based NAH (1-3). But the minimum L2-norm constraint tends to disperse the signal energy on the hologram surface to all the equivalent sources, the energy dispersion sometimes leads an over-smoothed reconstruction result and thus degrades the accuracy and the spatial resolution of NAH, especially when the true distribution of signal energy is highly concentrated.

To deal with this problem, an iterative weighted ESM (IWESM) is proposed in this paper, in which an iterative weighted regularization procedure is developed to reduce the dispersion of acoustic field energy on the hologram surface to those equivalent sources that contribute little to the acoustic field and get more reasonable solution of source strengths. In Section2, the ESM-based NAH is briefly introduced. In Section 3, the IWESM-based NAH is described in details. In Section 4, a numerical simulation for sound pressure reconstruction was presented to demonstrate that the IWESM-based NAH can obtain a higher spatial resolution reconstructed result than the ESM-based NAH.

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2. ESM based-NAH

Assume that there are M points pressure measured on the hologram surface and the equivalent sources number is N. According to the principle of ESM, the points measured pressure vector $\boldsymbol{P}_h$ can be represented in matrix form as

$$\boldsymbol{P}_h = i\rho_0 ck\boldsymbol{G}_{hp}\boldsymbol{Q} \tag{1}$$

where $i=\sqrt{-1}$, ρ_0 is the density of the medium, c is the speed of sound, $k=\omega/c$ is the wave number, ω is the angular frequency, $\boldsymbol{Q}=[q(\boldsymbol{r}_{o1}), q(\boldsymbol{r}_{o2}),\cdots, \ q(\boldsymbol{r}_{oN})]^{\mathrm{T}}$ is the column vector with the strengths of the equivalent sources $q(\boldsymbol{r}_{on})$, $\boldsymbol{r}_{on}$ is the location vector of the nth equivalent source, and $\boldsymbol{G}_{hp}$ is the complex transfer matrix formed by Green's function,

$$\boldsymbol{G}_{hp|m,n} = g(\boldsymbol{r}_{hm},\boldsymbol{r}_{on}) = \frac{\mathrm{e}^{\mathrm{i}k|\boldsymbol{r}_{hm}-\boldsymbol{r}_{on}|}}{4\pi\,|\,\boldsymbol{r}_{hm}-\boldsymbol{r}_{on}\,|} \tag{2}$$

in which $\boldsymbol{r}_{hm}$ is the location vectors of the mth measurement point and g is the free space Green's function. The unknown source strength vector $\boldsymbol{Q}$ can be obtained from the expression

$$\boldsymbol{Q} = \boldsymbol{G}_{hp}^{+}\boldsymbol{P}\big/ i\rho_0 ck \tag{3}$$

where $\boldsymbol{G}_{hp}^{+}$ is the pseudo-inverse matrix of $\boldsymbol{G}_{hp}$. Once the equivalent source strength vector has been determined, the pressure reconstruction in ESM-based NAH can be realized by the equation (4).

$$\boldsymbol{P}_s = i\rho_0 ck\boldsymbol{G}_{sp}\boldsymbol{Q} \tag{4}$$

where $\boldsymbol{P}_s$ are the reconstructed pressure vectors on the reconstruction surface, and G_{sp} is the complex transfer matrices

$$\boldsymbol{G}_{sp|mn} = g(\boldsymbol{r}_{sm},\boldsymbol{r}_{on}) \tag{5}$$

Because of the ill-posed nature of NAH problem, the inverting procedure for calculating equivalent source strength vector in equation (3) is not stable. Thus, the accuracy of $\boldsymbol{Q}$ is seriously affected by measurement noise which is ineluctable in practice. Regularization methods based on the minimum L2-norm criterion such as the Tikhonov regularization are usually employed to overcome the ill-posedness and to stabilize the reconstruction procedure. Instead of directly solving equation (3), $\boldsymbol{Q}$ is sought by minimizing formulation (6) in Tikhonov regularization.

$$\mathbf{min}\{\left\| i\rho_0 ck\boldsymbol{G}_{hp}\boldsymbol{Q} - \boldsymbol{p}_h \right\|^2 + \lambda^2 \left\|\boldsymbol{Q}\right\|^2\} \tag{6}$$

where $\left\|\bullet\right\|$ is the L-2 norm, $\left\| i\rho_0 ck\boldsymbol{G}_{hp}\boldsymbol{Q} - \boldsymbol{p}_h \right\|^2$ is the residual terms which grantees the solution of equation (6) is a good approximant of equation (3), $\left\|\boldsymbol{Q}\right\|^2$ is the penalty term which makes sure the solution has the minim energy, the regularization factor λ provides flexibility to trade off among the two terms.

The Tikhonov regularization method based on minimum L2-norm criterion is popular in dealing with ill-posedness in NAH problem because its computation is straightforward and it requires no prior information about equivalent source strengths. However, since the minimum L2-norm constraint gives all equivalent sources equal likelihood to contribute $\left\|\boldsymbol{Q}\right\|^2$, it tends to disperse the acoustical field energy on the hologram surface to all the equivalent sources. Actually in many case the energy distribution between the equivalent sources of the true acoustical field is not so smooth, and the minimum L2-norm constraint leads to an accuracy loss and thus degrades the spatial resolution of NAH.

3. IWESM-based NAH

To deal with this problem, considering the minimum weighted norm criterion (MWNC) (6,7) which has an expression as

$$\min\{\|\boldsymbol{Q}\|_w^2\} = \min\{\sum_{n=1}^{N}\frac{|q_{on}|^2}{w_n{}^2}\} \tag{7}$$

where $\|\bullet\|_w$ denotes the weighted norm, $|q_{on}|$ is the amplitude of the nth equivalent source, w_n $n \in 1,2,\cdots N$ is the weight given for the nth equivalent source . The matrix form of weighted norm can be expressed as

$$\|\boldsymbol{Q}\|_w^2 = \boldsymbol{Q}^H\boldsymbol{W}^{-2}\boldsymbol{Q} = \|\boldsymbol{W}^{-1}\boldsymbol{Q}\|^2 \tag{8}$$

where $\boldsymbol{W}$ is a diagonal weight matrix with w_n on its main diagonal.

If we replace the minimum L-2 norm constraint by MWNC, the equation (6) can be rewritten as

$$\min\{\|\boldsymbol{G}_{hp}\boldsymbol{Q}-\boldsymbol{p}_h\|^2 + \lambda^2\|\boldsymbol{Q}\|_w^2\} = \min\{\|\boldsymbol{G}_{hp}\boldsymbol{Q}-\boldsymbol{p}_h\|^2 + \lambda^2\|\boldsymbol{W}^{-1}\boldsymbol{Q}\|^2\} \tag{9}$$

According to the definition of weighted norm in equation (7), it can be found that the equivalent sources of small weight have main contribution to weighted norm and that of large weight contribute little to weighted norm. Therefore, minimization of formulation (9) leads to the result that the energy (or strength) of equivalent source of larger weights would be enhanced, and that of smaller weights would be suppressed. Thus the regularized solution obtained by formulation (9) would have a similar energy distribution of w_n. So w_n can be used as a tool to control energy distribution of equivalent sources. If any information about equivalent source energy distribution is known priorly, it can be introduce into regularization to help us to achieve a more suitable solution by reasonable selecting of w_n.

Formulation (9) is a Tikhonov regularization problem of nonstandard form. If we define $\boldsymbol{W}^{-1}\boldsymbol{Q} \equiv \boldsymbol{X}$, the formulation (10) can be recast as

$$\min\{\|\boldsymbol{G}_{hp}\boldsymbol{W}\boldsymbol{X}-\boldsymbol{p}_h\|^2 + \lambda^2\|\boldsymbol{X}\|^2\} \tag{10}$$

Formulation (10) is the Tikhonov regularization problem of standard form. Singular value decomposition (SVD) method can be used to obtain a regularized solution of $\boldsymbol{X}$, in conjunction with generalized cross-validation (GCV). Then, the estimation of $\boldsymbol{Q}$ can be obtained by equation (11)

$$\boldsymbol{Q} = \boldsymbol{W}\boldsymbol{X} \tag{11}$$

Ideally, the best choice of w_n is let w_n equal to the true energy distribution of equivalent source, i.e. w_n should be equal to $\boldsymbol{Q}$. However, $\boldsymbol{Q}$ is what we seek for and not available priorly. An iterative procedure is built to deal with this problem. The weight w_n^j, at an iterative step j, is constructed by the solution $\boldsymbol{Q}^{j-1}$ obtained in the previous step.

The IWESM based NAH can be realized iteratively by following steps:

Step1: the ESM is used to estimate the initial solution $\boldsymbol{Q}^0$ and set j=1.

Step2: Constructing the weight matrix

$$W^j = diag(w_n^j) = diag(\boldsymbol{Q}^{j-1}) \tag{12}$$

Step3: solving the formulation (10) for $\boldsymbol{X}^j$.

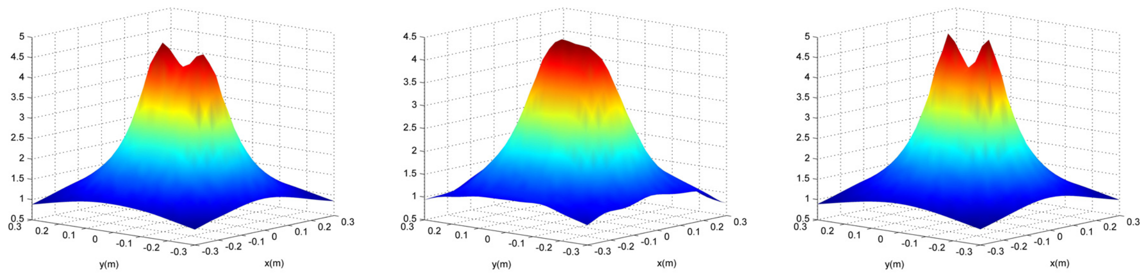
Step4: Computing $\boldsymbol{Q}^j$ by equation (11).

Step5: Setting iterative number $j = j+1$. The iteration will be stopped if $j = J$ (J is the total iterative number, usually J is less than 10 and in this paper J=3), and using $\boldsymbol{Q}^j$ to compute reconstructed pressure $\boldsymbol{P}_s$ by equation (4). Otherwise go to step 2.

4. SIMULATION

In order to demonstrate the advantage of high resolution of IWESM-based NAH, numerical simulation of sound pressure reconstruction was performed. The acoustic field was generated by two pulsating spheres with a radius of 0.05m and a velocity of 0.1m/s placed at the points of (0, 0, 0.1m) and (0, 0, 0.1m), respectively. The hologram plane was located at z=0.09m and distributed with a grid 25×25 points with lattice spacing in the x- and y-direction of 0.025 m. The reconstruction plane was located at z=0.05 m and distributed with a same grid as the hologram plane.

The hologram pressure was computed by utilizing the pressure radiation formula of pulsating sphere. To make the simulation more realistic, Gauss random noise was added to the hologram pressure data and SNR was 20dB.Then the hologram pressure was used to calculate the pressure in the reconstruction plane by the IWESM-based NAH and the ESM-based NAH, respectively. The reconstructed pressure at 100Hz obtained by the IWESM-based NAH is shown in Figure 1 (a) and that obtained by ESM-based NAH is shown in Figure 1 (b). The true pressure at 100Hz is shown in Figure 1 (c).

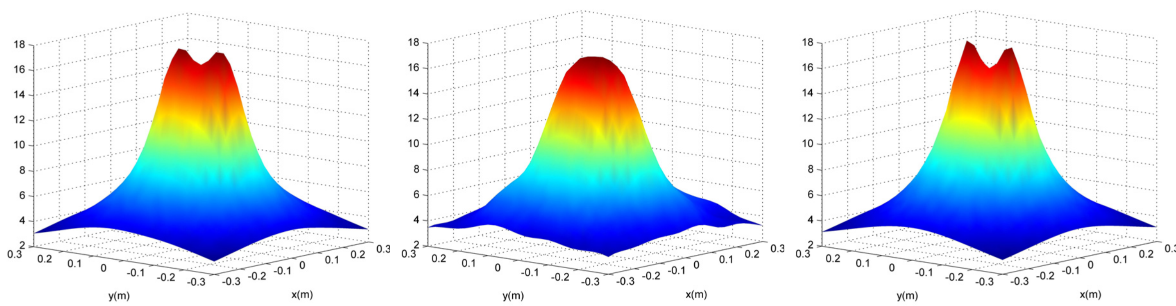


(a) (b) (c)

Figure 1– Comparison of reconstructed pressure obtained by IWESM-based NAH and ESM-based NAH with the true pressure at frequency of 100Hz: (a) the reconstructed pressure obtained by the proposed method; (b) the reconstructed pressure obtained by the one-step patch NAH; (c) the true pressure on reconstruction surface.

Two peaks of reconstructed pressure value are clearly shown in Figure 2 (a) and we can easily identify that there are two sound sources in the field. But in Figure 2 (b), only a single and smooth pink is found. By comparing Figure 2 (c) with Figure 2 (a) and (b), it can be found that the reconstructed pressure obtained by IWESM-based NAH is more similar to the true pressure, which demonstrated that IWESM-based NAH has a higher spatial resolution than ESM-based NAH.

A similar case was done at 400Hz.



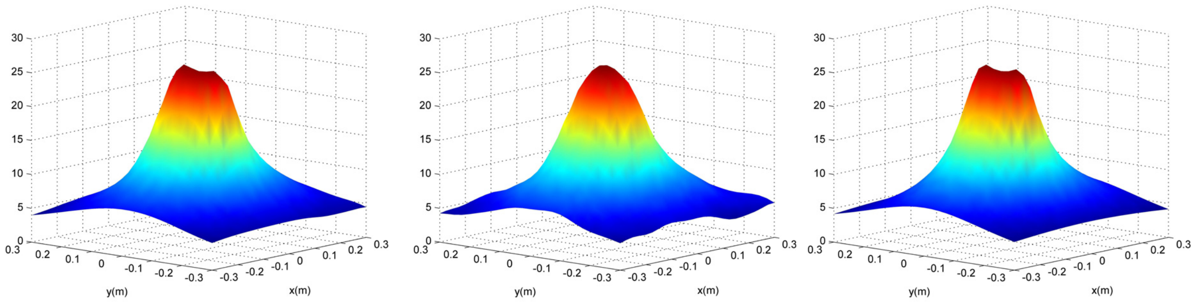
(a) (b) (c)

Figure 2 – Comparison of reconstructed pressure obtained by IWESM-based NAH and ESM-based NAH

with the true pressure at frequency of 400Hz: (a) the reconstructed pressure obtained by the proposed method; (b) the reconstructed pressure obtained by the one-step patch NAH; (c) the true pressure on reconstruction surface.

Figure 2 shows the reconstructed pressure at 400Hz obtained by the IWESM-based NAH. Figure 2 (b) shows the reconstructed pressure at 400Hz obtained by the ESM-based NAH. Figure 2 (c) shows the true pressure at 400Hz. It can be seen that two peaks which depict the positions of pulsating spheres are clearly shown in Figure 2 (a) and (b), but two pulsating spheres can not be distinguished in Figure 2 (c). The comparison also shows that IWESM-based NAH has a higher spatial resolution than ESM-based NAH at frequency of 400Hz.

Figure 3 shows a same comparison at 700Hz.



(a) (b) (c)

Figure 3 – Comparison of reconstructed pressure obtained by IWESM-based NAH and ESM-based NAH with the true pressure at frequency of 700Hz: (a) shows the reconstructed pressure obtained by the IWESM-based NAH; (b) shows the reconstructed pressure obtained by the ESM-based NAH; (c) shows the true pressure at 700Hz.

Comparing Figure 3 (c) with Figure 3 (a) and (b), it is found that the reconstructed pressure obtained by IWESM-based NAH is more agrees well with the true pressure and details of pressure field is still kept during NAH computation. But the reconstructed pressure obtained by ESM-based NAH is too smooth and the details of pressure field are lost.

In order to demonstrate the accuracy of IWESM-based NAH, the reconstructed error was calculated by equation (13) in the frequency range from 100Hz to 1600Hz with 100Hz interval. To show the advantage of IWESM-based NAH, a comparison of reconstructed error between IWESM-based NAH and ESM-based NAH is shown in Figure 4. It is clear that IWESM-based NAH gets better results than the ESM-based NAH, and the reconstructed errors of IWESM-based NAH almost keep in a half level of ESM-based NAH. It can be concluded that IWESM-based NAH has higher accuracy than ESM-based NAH, and more suitable for application.

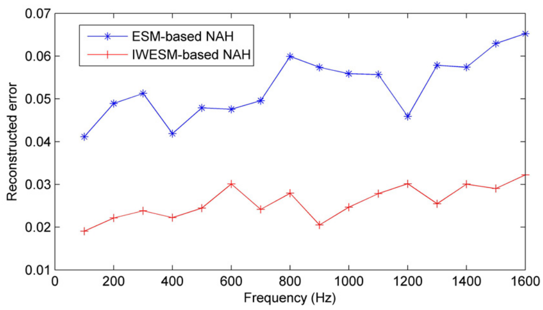


Figure 4 – The comparison of reconstructed error between IWESM-based NAH and ESM-based NAH.

$$E_p = \left\| P_{True} - P_{reconstructed} \right\| / \left\| P_{True} \right\| \tag{13}$$

where P_{True} and $P_{reconstructed}$ are the true pressure and the reconstructed pressure, respectively.

5. CONCLUSIONS

To improve the spatial resolution and accuracy of ESM-based NAH, a high resolution NAH method named IWESM-based NAH is proposed, in which an iterative weighted regularization procedure is developed to reduce the dispersion of acoustic field energy on the hologram surface to those equivalent sources that contribute little to the acoustic field, and get more reasonable solution of source strengths. A simulation of sound pressure reconstruction was performed to prove the validity of the proposed method and the result shows that proposed method has a higher spatial resolution and accuracy than ESM-based NAH.

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