



## Vibration of a curved subsea pipeline due to internal slug flow

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### ABSTRACT

Subsea oil and gas pipelines undergo vibration due to “slug” flow within the internal fluid contents of the pipeline. This slug flow is generated by the differences in density of the internal fluid. It acts as a traversing force along the length of the pipeline and causes structural vibration of unsupported pipeline spans. The resulting vibration of the pipeline may cause high cycle fatigue due to these fluctuating forces. Previous modelling of a moving slug within pipelines has been undertaken for straight pipe span sections. As unsupported pipeline spans are often curved, understanding the impact this curvature has on the traversing fluid load is important. This paper presents a Finite Element model to investigate the effect pipeline curvature has on the slug flow induced forces, resulting in vibration and hence possible fatigue damage to the pipeline structure. Furthermore, the paper presents a technique for using commercial finite element packages for analysing the dynamic response of curved beams to time variant moving loads.

**Keywords:** Dynamic Modelling, Slug Flow Induced Fatigue, Moving Force over a Curved Beam, High Cycle Fatigue, Slugging at Sleepers/support.

### 1. INTRODUCTION

As the oil and gas industry moves towards deep-water development, subsea pipelines are being increasingly required to operate at higher levels of pressure and temperature and with processing facilities further away from the well head. This has led to two design challenges,

1. high pressure and temperature require large axial pipeline expansion and can result in uncontrolled lateral buckling if not properly mitigated
2. larger distances between the well-head and processing plant can cause multiphase internal ‘slug’ flow resulting in dynamic forces as the oil and gas passes through the pipeline

In terms of the first design challenge, a subsea pipeline laid on a seabed tends to axially expand and contract under the repeated operating cycle of start-ups and shut-downs. The axial expansion is the result of the internal operating pressure as well as the raised wall temperature in relation to the seabed ambient temperature. As the axial expansion is restrained by the frictional restraint of the seabed, an effective axial force can develop in the pipeline. If the effective axial force exceeds the buckle initiation force, the pipeline will undergo Euler buckling to relieve the resulting high axial forces in the pipe wall.

Uncontrolled buckling can have serious implications on the integrity of the pipeline. Buckle mitigation strategies can therefore be achieved by providing controlled lateral pipeline movements where it is demonstrated that the response of the in service buckled pipeline exceeds any ultimate and fatigue limit states.

Buckle mitigation measures are intended to induce lateral deformations at designated locations; therefore expansion is shared between the adjacent buckle locations. A number of buckle mitigation measures have been employed, or are currently proposed to initiate buckling at the defined locations, these include:

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- Local vertical out-of straightness (i.e. Sleeper and Zero Radius Bend)
- Imposed local curvatures (Snake Lay)
- Local buoyancy

This paper will focus on the buckle mitigation measures of local vertical out-of straightness using a sleeper underneath the laid pipeline. The obstacle placed under the pipeline is usually called a sleeper and is sometimes made from a section of a pipe, the diameter of which could be in the range of 0.5m to 1.2 m; the higher the sleeper the greater the probability of a lateral buckle being formed, but also the greater the length of unsupported span along the pipeline on either side of the sleeper. A pipeline laid over a sleeper acting as a lateral buckling initiator can be seen in Figure 1.

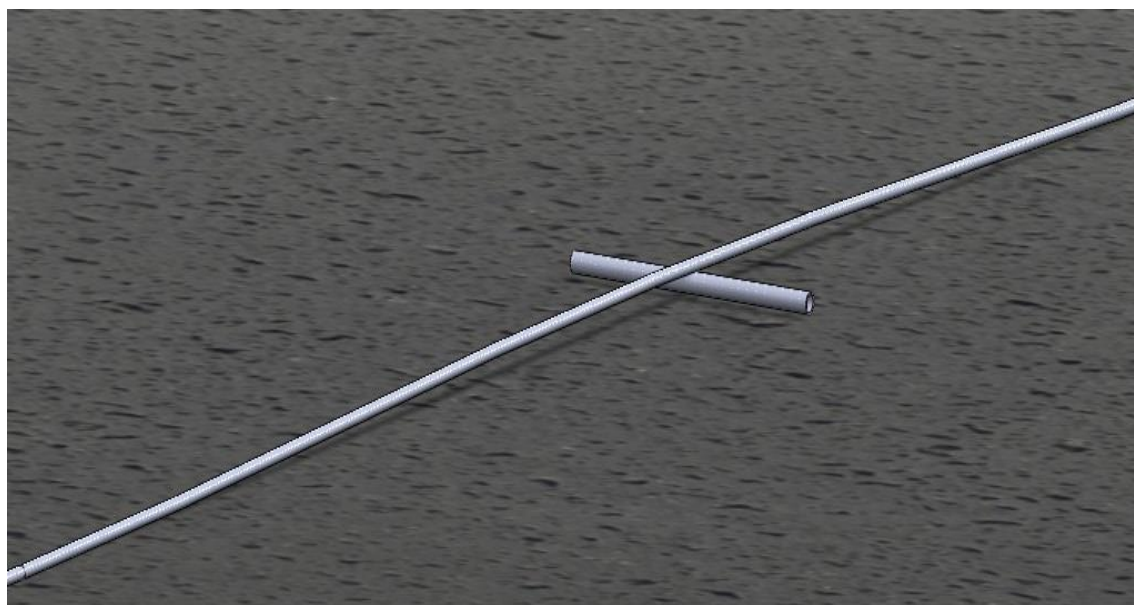


Figure 1 Schematic diagram of a pipeline over a sleeper type buckle initiator

Laying the pipeline over a vertical buckle initiator may generate significant unsupported span lengths as shown in Figure 1, depending on the height of the vertical initiator. Multiphase internal flow in the pipeline can cause ‘slugs’ of differing density fluid to pass through these unsupported span lengths causing dynamic motion, cyclic stress and fatigue events in the pipeline. It is the combination of both of these pipeline design challenges which are investigated within this paper, with particular attention given to the curved path the internal flow must pass as it traverses the unsupported span over a sleeper type buckling mitigation design.

This paper will present the modelling of an unsupported pipeline span length under slug flow conditions which can lead to cyclic fatigue. Previous modelling of a moving slug flow in a pipeline has generally been presented for straight (non-curved) spans, References (1-2). The work included in this paper will highlight the effect of the span vertical out-of-straightness, due to the sleeper, as well as show when this vertical out-of-straightness has a significant influence. The inclusion of the pipeline curvature introduces centrifugal forces and produces additional vertical and axial forces in the pipeline that vary when the slug traverses the unsupported pipe span. This variation in the force, produced by the moving slug across a curved span, changes the dynamic motion and thus the stress within the pipeline during the slug motion.

## 2. SLUG FLOW

Slug flow is an unstable flow phenomenon that occurs in multiphase pipelines under certain conditions and can pose considerable challenges in terms of pipeline design and operability. Near horizontal pipelines, at low to moderate gas and liquid flow rates, may operate in one or more of the slugging flow regimes which are characterized by alternate periods of high liquid production rates

(slugs) followed by high gas production rates (gas bubbles) as shown in Figure 2. The likelihood of slug flow is both a function of the incoming fluid as well as the pipeline layout. Processing of these slugs on topside separator facilities can be extremely difficult if the slugs become excessively long.

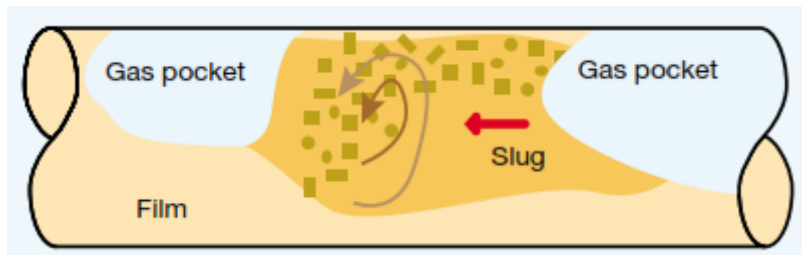


Figure 2 Internal slug flow schematic (Reference (7))

### 3. MOVING POINT FORCE

In order to investigate the vibration and dynamic motion of a pipe span due to internal slug flow, the moving slug will be modelled as a moving point force across a beam span. This modelling procedure has been previously shown to be an adequate simplification of the real distributed moving mass of internal pipe slug flow, References (4-5).

#### 3.1 Moving point force across a straight beam

For a moving point force across a single span straight beam, shown schematically in Figure 3, the general form of the finite element equations of motion are given by,

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F(t)\} \tag{1}$$

The force and moment vectors  $\{F(t)\}$  has been previously derived by Reference (3), where the force and moment vectors due to a moving point force, can be written as:

$$\{F(t)\} = \{000\dots f_1^{(s)}(t) f_2^{(s)}(t) f_3^{(s)}(t) f_4^{(s)}(t) \dots 000\} \tag{2}$$

where  $f_i^{(s)}(t)$  ( $i=1-4$ ) represents the equivalent nodal forces and the equivalent nodal moments on element 's' which the force is traversing at that particular time and where  $\{N\}$  is the element shape function matrix such that, Reference (3):

$$\{f^{(s)}(t)\} = [f_1^{(s)}(t) f_2^{(s)}(t) f_3^{(s)}(t) f_4^{(s)}(t)]^T = P\{N\} \tag{3}$$

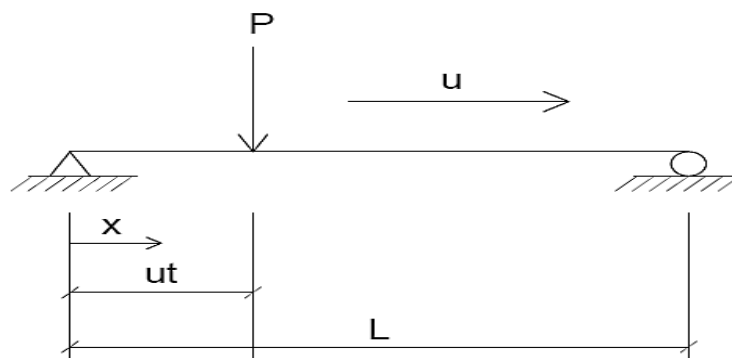


Figure 3 Schematic of a moving force 'P' moving across a simple straight beam span

### 3.2 Moving point load on a curved beam

With a moving force across a curved beam of an arbitrary shape, the distance in which the force between two sets of nodes on the finite element model needs to be calculated in a more sophisticated manner than that of a simple straight beam. Additionally, the nodal force needs to be rotated into the local coordinate system for the element of interest and the inclusion of an axial deformation term in the element formulation needs to be considered, as shown in Figure 4. The derivation of the force vector due to a curved beam with a moving vertical force will be derived below.

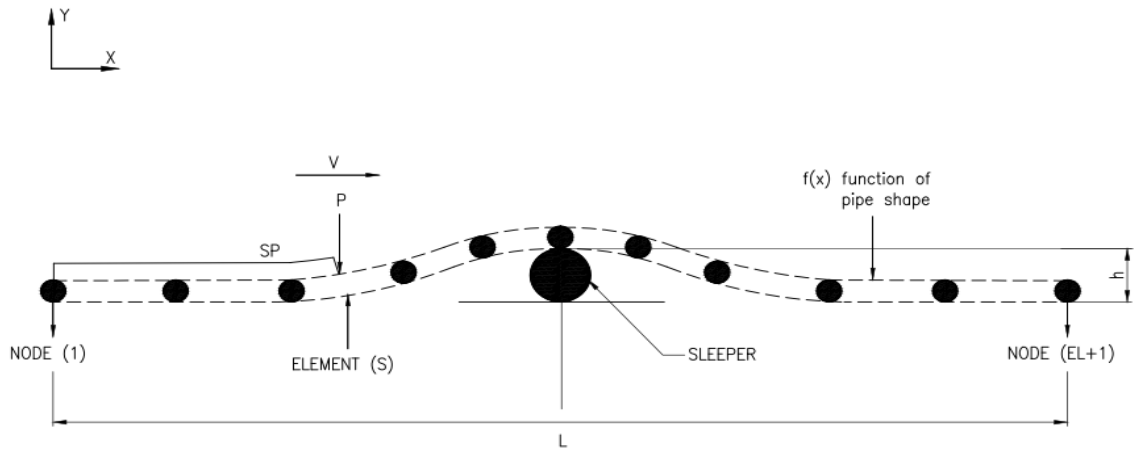


Figure 4 Curved beam subjected to a slug load, P, moving with velocity, V.

For a curved beam, as shown in Figure 4, under the influence of a Force, P, traveling at a constant speed (relative to the pipe) the distance travelled by the force at time step ‘m’ can be shown to be:

$$S_p(t) = mV\Delta t \tag{4}$$

where:

V = Slug Load Speed

$\Delta t$  = Time Step Size

In order to find the distance that the force is across element ‘s’ directly below the force at any given time, the length of the curve needs to be calculated. The length of each element ‘j’ can be shown to be (See Figure 5 for definition of arc length as a function of node x and y coordinate:

$$arc_{length}(s) = \sqrt{\Delta x(k)^2 + \Delta y(k)^2} \tag{5}$$

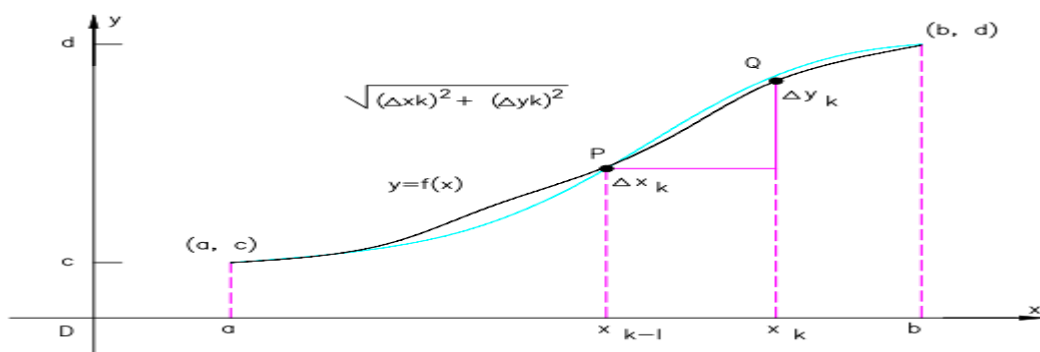


Figure 5 Schematic of arc length of a curve

The non-dimensional distance that force F has traveled across element 's' can now be given as:

$$\xi = \frac{S_p(t) - \sum_{j=1}^s \text{arc}_{length}(j)}{\text{arc}_{length}(s)} \quad (6)$$

Now that the location of the force and moment at any given time have been derived, the force and moment vectors for all nodes can be calculated as the vertical force traverses the curved span as shown in Figure 6:

$$\{F(t)\} = \{0 \dots F(t)_1^Q \ F(t)_2^Q \ F(t)_3^Q \ F(t)_4^Q \ F(t)_5^Q \ F(t)_6^Q \dots 0\} \quad (7)$$

Noting that  $F(t)_1^Q$ ,  $F(t)_2^Q$ ,  $F(t)_4^Q$  and  $F(t)_5^Q$  represent the force vectors whereas  $F(t)_3^Q$  and  $F(t)_6^Q$  represent the moment vectors.

where:

$$F(t)_1^Q = (P \sin(\phi)). N_1 \quad (8)$$

$$F(t)_2^Q = (F_C + P \cos(\phi)). N_2 \quad (9)$$

$$F(t)_3^Q = (F_C + P \cos(\phi)). N_3 \quad (10)$$

$$F(t)_4^Q = (P \sin(\phi)). N_4 \quad (11)$$

$$F(t)_5^Q = (F_C + P \cos(\phi)). N_5 \quad (12)$$

$$F(t)_6^Q = (F_C + P \cos(\phi)). N_6 \quad (13)$$

Here we have introduced two coordinates systems: a local one (X,Y) directed along the length of the element and a global one ( $\bar{X}, \bar{Y}$ ). The global coordinate is selected to be best suited with respect to the whole sleeper model. The forces shown above were derived in the local coordinates, where P &  $F_C$  are the magnitude of the slug force and the centrifugal force respectively. Letting the mass of the moving slug be  $m_{slug}$  and the tangential speed V, with the radius of curvature of the span  $\rho$ , then the force terms P &  $F_C$  can be determined by:

$$P = m_{slug} \cdot g \quad (14)$$

$$F_C = \frac{m_{slug} \cdot V^2}{\rho} \quad (15)$$

$N_i$  ( $i = 1 - 6$ ) represents the shape function for a straight beam element.

Shape functions for a cubic beam element can be defined as, Reference (6):

$$N_1 = 1 - \xi \quad (16)$$

$$N_2 = 1 - 3\xi^2 + 2\xi^3 \quad (17)$$

$$N_3 = (\xi - 2\xi^2 + \xi^3).L \quad (18)$$

$$N_4 = \xi \quad (19)$$

$$N_5 = 3\xi^2 - 2\xi^3 \quad (20)$$

$$N_6 = (-\xi^2 + \xi^3).L \quad (21)$$

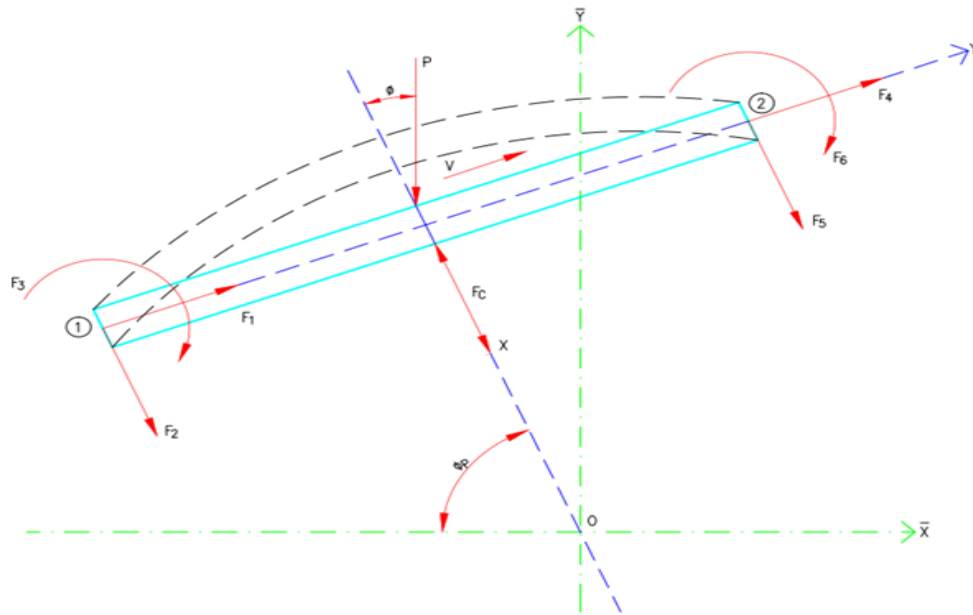


Figure 6 External and nodal forces on element 's'

The local forces will be transformed to the global coordinates. The global forces are dependent on the location of the slug. Assuming that the slug load is located between node 1 and node 2 shown in Figure 6, the global forces can be determined by:

$$F_{\bar{x}} = F(t)_1^Q \cdot \cos(\varnothing) - F(t)_2^Q \cdot \sin(\varnothing) + F(t)_4^Q \cdot \cos(\varnothing) - F(t)_5^Q \cdot \sin(\varnothing) \quad (22)$$

$$F_{\bar{y}} = F(t)_1^Q \cdot \sin(\varnothing) + F(t)_2^Q \cdot \cos(\varnothing) + F(t)_4^Q \cdot \sin(\varnothing) + F(t)_5^Q \cdot \cos(\varnothing) \quad (23)$$

The moments are equal to zero when the slug load is located between node 1 and node 2. The moment is only considered on the slug load is located at either the node (1) or node (EL+1) illustrated in Figure 6.

### 3.3 Non-dimensional centrifugal force parameter

The difference in nodal forces between a straight beam and a curved beam come from both the curvature of the beam changing the direction of the applied vertical load, as well as the centrifugal force due to the fluid flow now following a curved path. This suggests that the effect of curvature on the nodal forces will be a function of both the rate of curvature and the velocity of the traversing force.

In order to provide a means of assessing the effect of curvature on the applied nodal forces, and considering Figure 3, a simple half sine wave shape will now be assumed for the pipe span to provide a non-dimensional indication of the span curvature/velocity effects.

The centrifugal force is given as:

$$F_c = \frac{mv^2}{\rho} \quad (24)$$

For a simple half sine wave, the shape of the pipeline lying over the sleeper is defined as:

$$f(x) = \frac{h}{2} \left[ 1 - \cos\left(\frac{2\pi x}{L}\right) \right] \quad (25)$$

The radius of curvature is simply the inverse of the second derivation of the pipeline shape, and can be shown to be given as:

$$\rho = \frac{2}{h} \left(\frac{L}{2\pi}\right)^2 \sec\left(\frac{2\pi x}{L}\right) \quad (26)$$

Therefore, the maximum centrifugal force exerted by the slug as it passes through the curved pipeline will occur at the center of the span and have the value of:

$$F_{c\max} = \frac{2mV^2h\pi^2}{L^2} \quad (27)$$

A new non-dimensional parameter,  $\gamma$ , is introduced to describe the percentage contribution of the maximum centrifugal force compared to the original vertical force of the slug, being,

$$\gamma = \frac{F_c}{mg} = \frac{2V^2h\pi^2}{gL^2} * 100 \% \quad (28)$$

It can be seen that it is a function of the span curvature ( $h/L^2$ ) and the force traversing speed ( $V^2$ ). For small  $\gamma$  values, the effect of curvature will not have any significant changes on the applied nodal forces as compared to a straight beam. This will be shown in the results in section 5 below.

#### 4. FINITE ELEMENT MODEL

Notwithstanding the sophistication of the commercial finite element packages such as ABAQUS and ANSYS, a large amount of effort is still required to model a moving force within these commercial software packages. The results shown in this paper utilizes the commercial finite element package ABAQUS to model the pipeline and run the FE analysis, but the input force was applied using an in-house subroutine using the above derivation of the nodal forces.

Firstly, the general-purpose FE implicit solver ABAQUS is used to lay the pipeline over the sleeper and on the seabed. This is in order to obtain the correct as-laid shape for the pipeline over a span (note: that the previous derivation for the centrifugal force was for a sinusoidal type lay shape). Once the static analysis is completed, the pipeline coordinates are exported to a subroutine, developed in-house, to generate a force input file.

Once, the static analysis is completed, a dynamic analysis is restarted to investigate the dynamic behavior of an unbuckled pipeline over the buckle initiator sleeper. The dynamic analysis is undertaken using the implicit Hiller Hughes Taylor operator for integration of the equation of motion.

The pipe was modeled using PIPE31H, a 2-noded hybrid formulation pipe element. The reason for selecting this pipe element is that it is well suited to modeling long slender pipelines with better convergence behavior than standard pipe elements. An element length of 0.5m is used.

The seabed and sleeper are modeled using an analytical rigid cylindrical surface. Contact between the pipe and the seabed is modeled as soft contact. Friction, between the pipeline and the seabed, is assigned to the seabed in the axial and lateral directions. The standard ABAQUS friction model is employed in the analyses. The friction between the pipe and the sleeper is modeled using a simple

Coulomb friction model. The contact between the pipeline and sleeper is modeled using contact elements. Figure 7 shows a representation of the resulting finite element model.

The following loading sequence is employed in the finite element model to investigate the dynamic response of pipeline span to time variant moving loads:

1. Apply a small lateral out of straightness (OOS) to the straight pipe

The value considered in the analysis is the maximum allowed OOS in a single pipe joint as defined below. This is consistent with DNV-OS-F101.

$$OOS \leq 0.15 \% \text{ Pipe Joint}$$

2. Apply external pressure.
3. Apply gravity to settle the pipe on the seabed.
4. Reset the boundary conditions.
5. Apply the internal operating pressure (Internal pressure assumed to be equal to the external pressure).
6. Restart dynamic analysis to determine the stress ranges and span displacements.

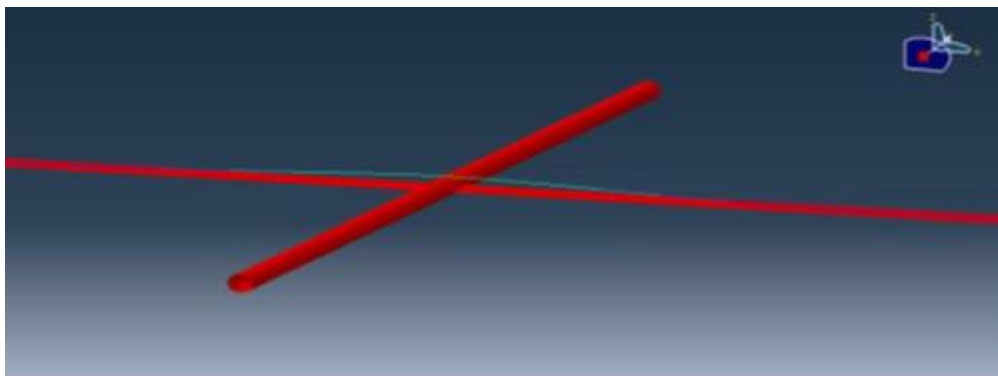


Figure 7 ABAQUS finite element model showing the sleeper and ground contact surface in red. The pipeline is shown in blue

## 5. INPUT DATA AND ASSUMPTIONS

The input data used in this paper is shown in Table 1. The following assumptions have been included:

- The pipe is of a constant cross section and constant unit mass per length.
- The mass of the moving load is smaller than that of the mass of the beam.
- The velocity of the slug is constant along the span.
- Slug is modeled as a point load.
- Vibration due to slugs passing through the pipeline span will only incur in the vertical pipe plane.



Table 1 - Input Data

Parameter	Unit	Value
Pipe Outside Diameter	mm	219.1
Wall Thickness	mm	15.9
Steel Density	kg/m <sup>3</sup>	7850
Pipe Unit Submerged Weight	N/m	456.6
Slug Load	kg	100
Modulus of Elasticity	GPa	207
Sleeper Height	m	1.2
Slug Speed	m/s	8 , 15 & 30

## 6. RESULTS

A modal analysis to determine the mode shapes and the natural frequency of the span was first conducted. The first 3 natural frequencies are listed in Table 2, with the mode direction being either transverse to the span plane (In-line) or the mode shape in-plane with the span (Cross-flow). The mode shapes generally came in In-line/Cross-flow pairs with only a slight difference in frequency as would be expected for an axi-symmetric structure with only a small amount of out of straightness due to the pipeline bend over the sleeper. The mode shapes for the corresponding In-line and Cross-flow modes are almost identical, as can be seen with mode 1 and mode 2 laying directly on top of each other in Figure 8.

Table 2 - Mode shape natural frequencies

Mode #	frequency (Hz)	Mode direction
1	0.85175	Inline
2	0.88617	Cross-flow
3	1.1634	Inline

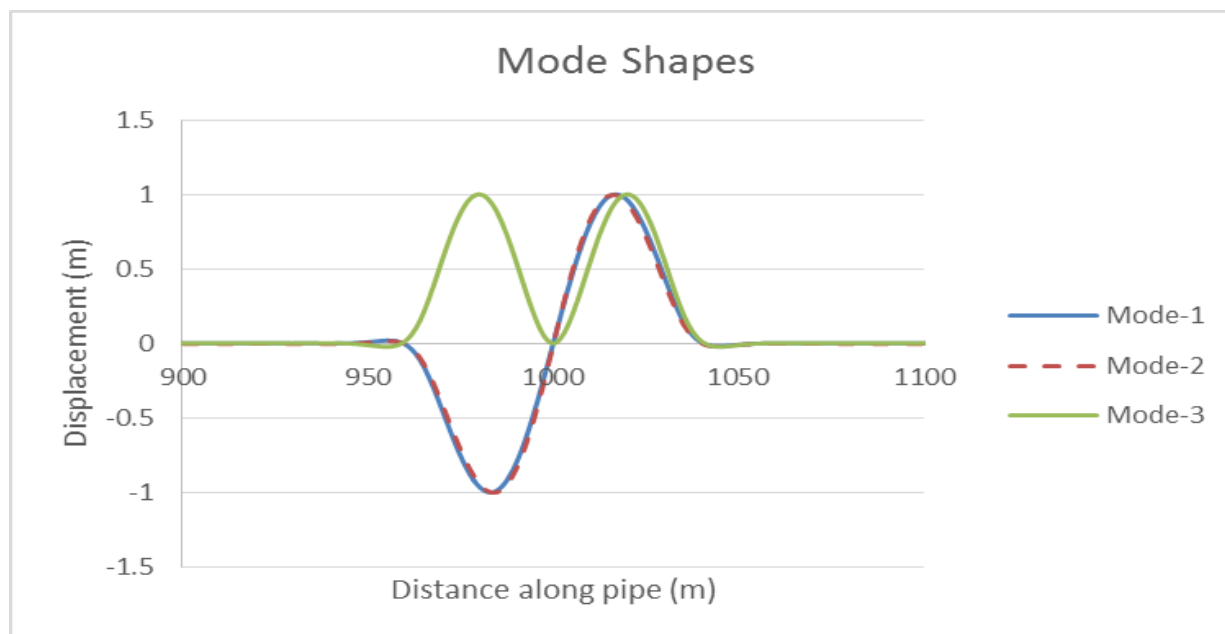


Figure 8 First 3 mode shapes of pipeline

The developed finite element model was run for three individual pipe span and velocity cases to determine the effect of the pipeline out of straightness on the dynamic response of the pipeline, with parameters as shown in Table 3. For each velocity case the centrifugal force parameter was removed so that the effect of span out of straightness on this parameter could be assessed. The vertical and axial displacement and axial stress in the pipeline with and without the centrifugal force included are given from Figure 9 to Figure 14. It can be seen that the inclusion of the centrifugal force does not have any significant impact on either the maximum stress or deflection at any location along the length of the span for the 8m/s and 15m/s speeds. This result is expected due to the fact that the non-dimensional centrifugal force parameter for both of these cases was approximately less than 10%. Or in other words, the effect of the combination of the out of straightness and the slug velocity only changes the applied force approximately 10% as compared to a simple straight span.

A larger effect from the slug speed and span out of straightness can be seen for the results when the slug speed is at 30m/s. For the stress at the center span location, the stress is reduced due to the increasing force slug speed, due to the centrifugal force vector being in the positive vertical direction and thus offsetting some of the weight force of the slug. In particular for the axial stress seen in Figure 13 and Figure 14, the axial stress is only changed a negligible amount for results within and without the centrifugal force being included for 8m/s and 15m/s slug speeds. However the stress is reduced by approximately 40% for 30m/s slug speed. This is in reasonable correlation with the non-dimensional force parameter of  $\gamma = 34\%$ .

Finally the stress pattern of the pipeline span is reasonably complex for all slug speeds and is not easily described by the simplified non-dimensional force parameter and suggests the need for finite element modeling of the pipeline span and force interaction if detailed stress values are required.

Table 3 - Results case parameters

	<b>v (m/s)</b>	<b>L (m)</b>	<b>h (m)</b>	<b>f(x)</b>	<b><math>\gamma</math></b>	<b>Fc included</b>
Case 1	8	80	1.2	natural lay	2.4%	Yes
Case 2	8	80	1.2	natural lay	2.4%	No
Case 3	15	80	1.2	natural lay	8.5%	Yes
Case 4	15	80	1.2	natural lay	8.5%	No
Case 5	30	80	1.2	natural lay	34%	Yes
Case 6	30	80	1.2	natural lay	34%	No

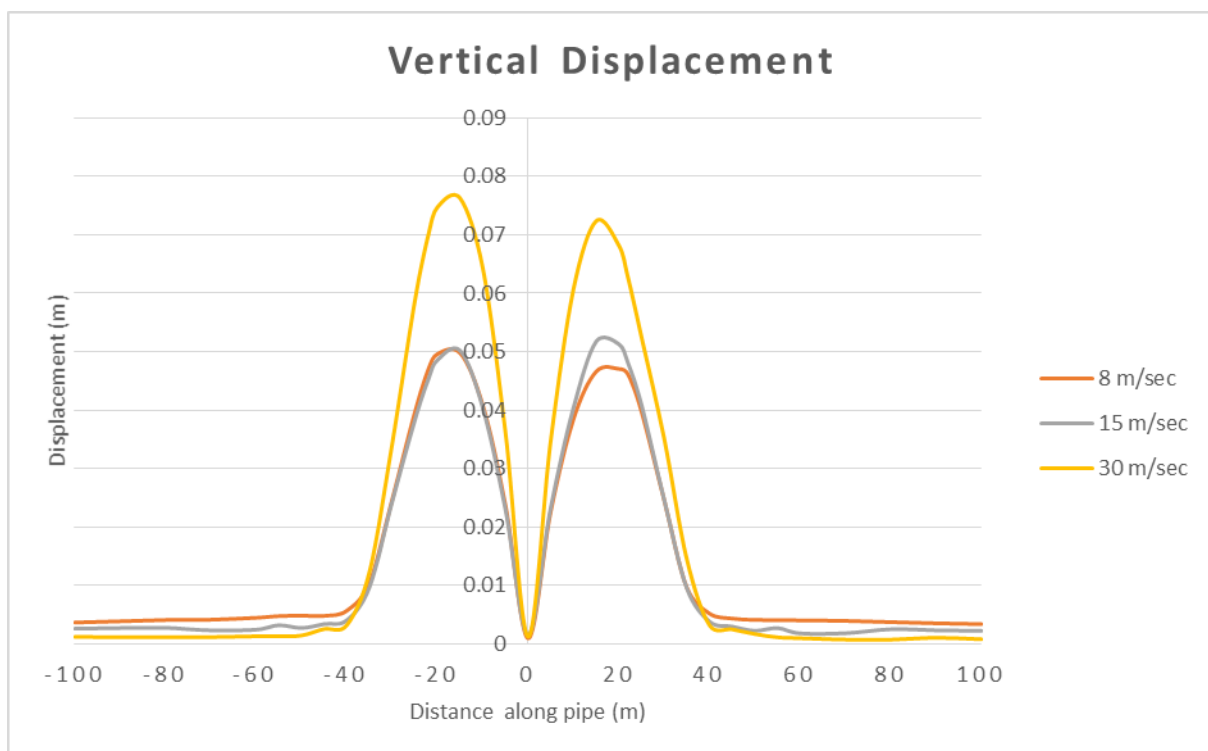


Figure 9 Maximum vertical span deflection across the span length as the force traverses the span.

Included centrifugal force

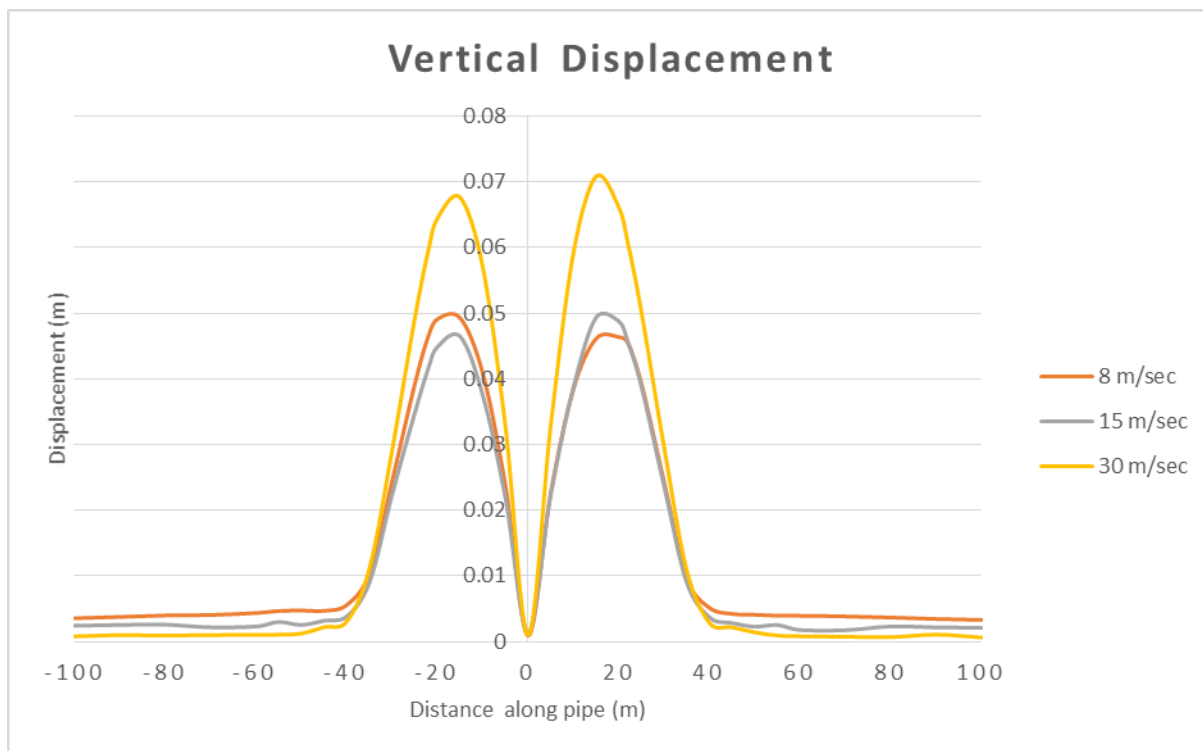


Figure 10 Maximum vertical span deflection across the span length as the force traverses the span.  
Centrifugal force not included

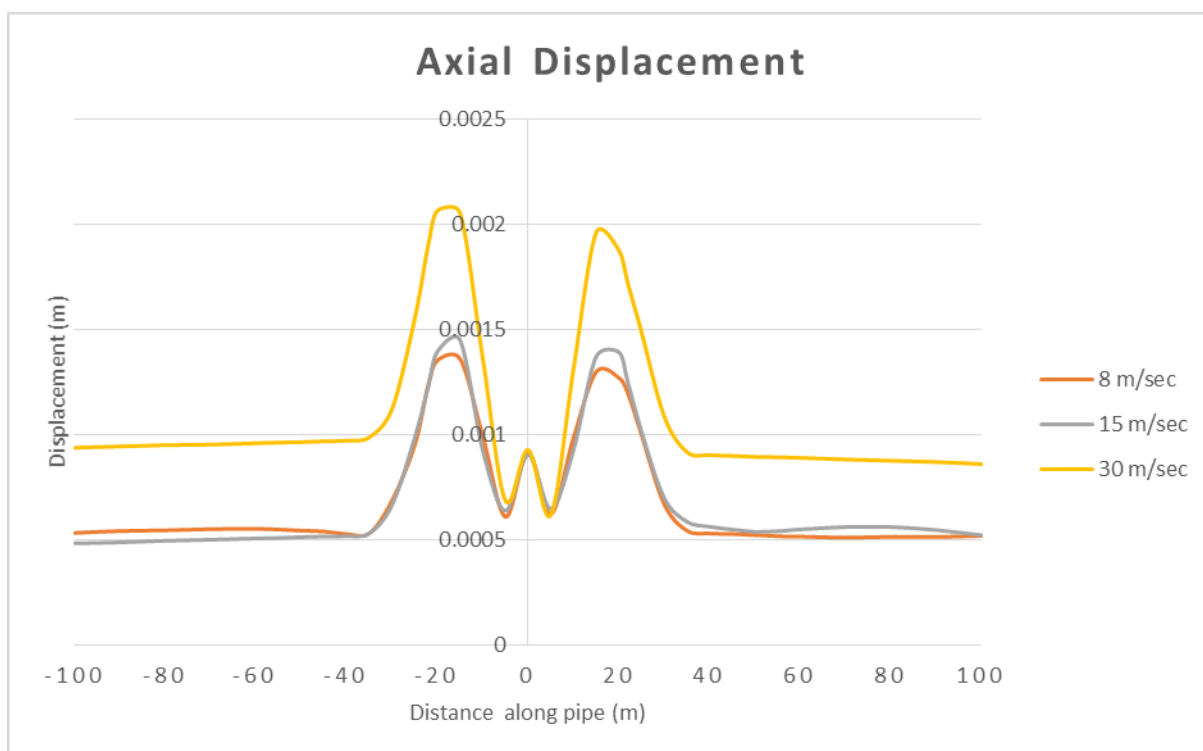


Figure 11 Maximum horizontal span deflection across the span length as the force traverses the span.  
Included centrifugal force

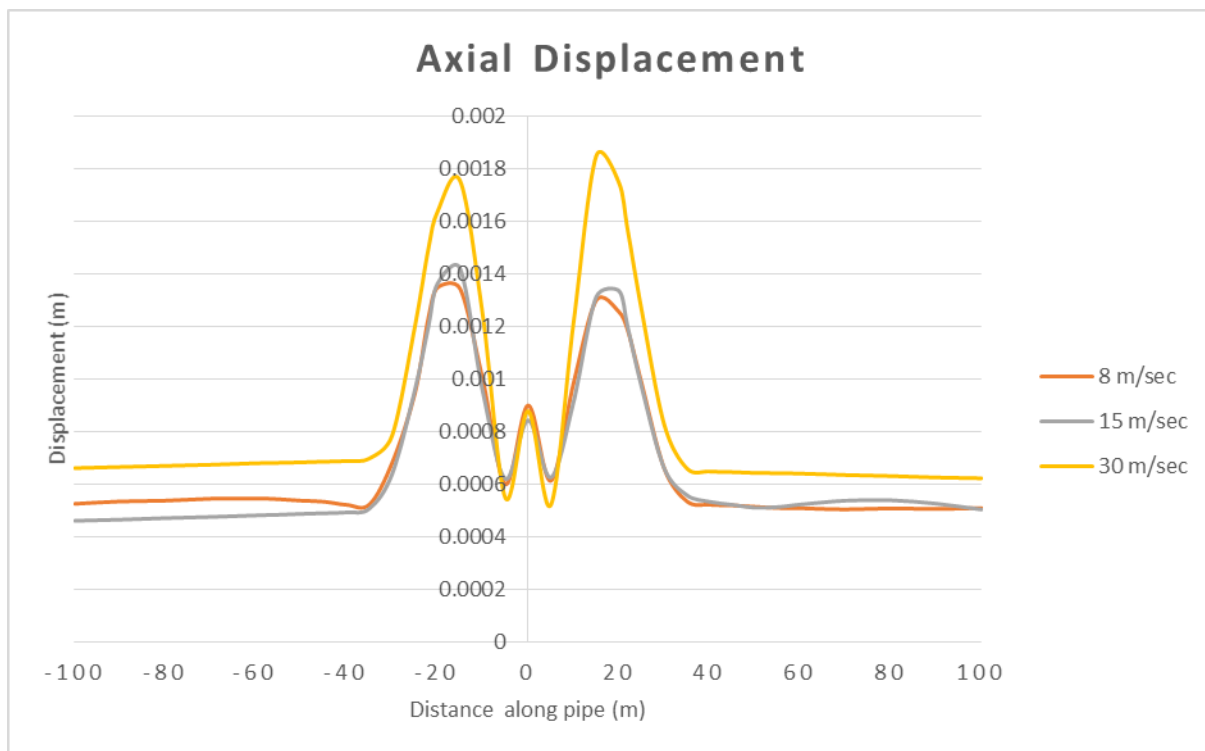


Figure 12 Maximum horizontal span deflection across the span length as the force traverses the span.  
Centrifugal force not included

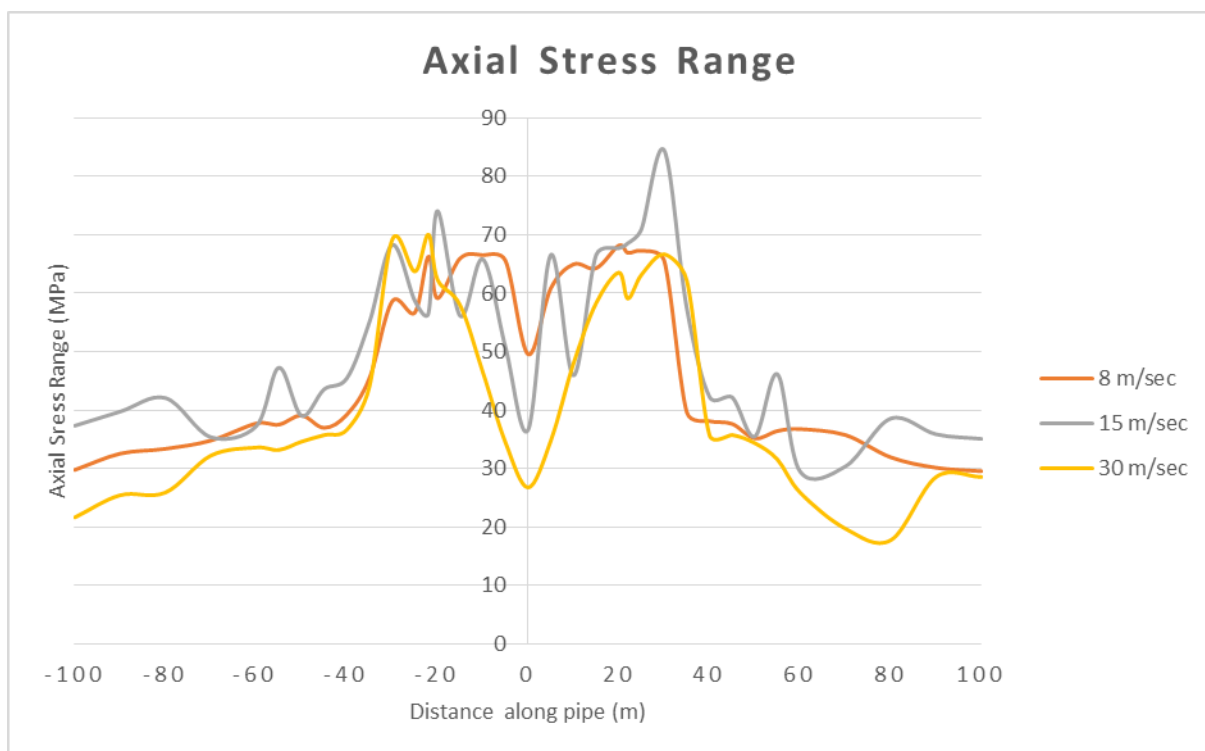


Figure 13 Maximum axial stress range across the span length as the force traverses the span. Included centrifugal force

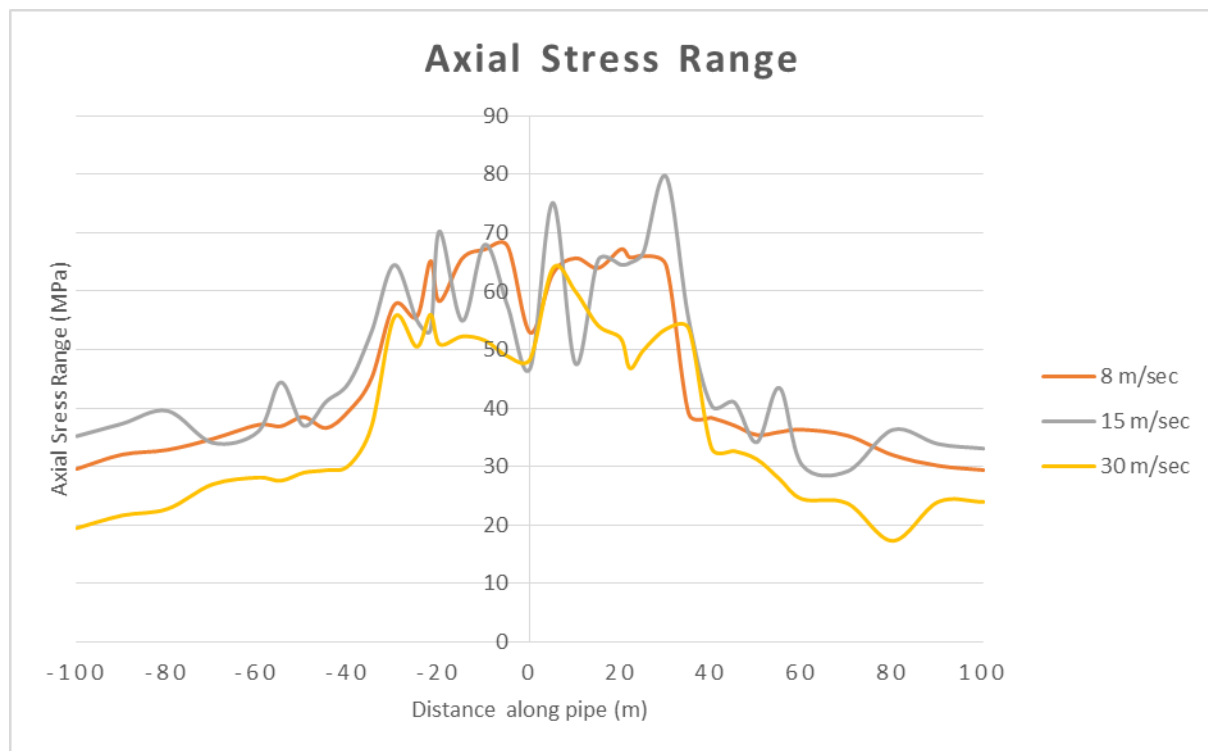


Figure 14 Maximum axial stress across the span length as the force traverses the span. Centrifugal force not included

## 7. CONCLUSIONS

Multiphase slug flow inside oil and gas pipelines cause fluctuating forces on the pipeline. When an un-supported span of pipeline develops these fluctuating forces, the resulting pipeline vibration and cyclic stress can cause fatigue. Within this paper the effects of internal pipeline slug flow were modelled as a moving force across a pipeline span, with the effect of the span out of straightness specifically investigated. The effect of span out of straightness causes centrifugal forces is to be included due to the moving slug following a curved path. The severity of the centrifugal force impact was shown to be a function of both the slug velocity and the span out of straightness. A non-dimensional centrifugal force parameter has been developed which can be used to assess whether the combination of out of straightness and slug velocity will have any appreciable influence on the pipeline vibration over and above that of a straight pipe span. Results showed that for a non-dimensional centrifugal force parameter,  $\gamma < 10\%$ , the out of straightness has little effect. Additionally the stress pattern over the pipeline span is relatively complex even if  $\gamma < 10\%$ , this indicates that if detailed stress values are required across the pipeline, a finite element analysis with a transient force applied due to the slug motion should be conducted.

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