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Research on eigenfrequency shifts due to cracks in cylindrical structures and the application in non-destructive testing

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ABSTRACT

This paper presents numerical and experimental studies on modal behavior of cylindrical, beam-like lightly damped structures having circumferential cracks with variable position, width and depth. In the numerical investigation utilizing Finite-Elements a discretization strategy is worked out that enables a three-dimensional crack representation. The experimental setup uses a Laser Scanning Vibrometer in the frequency range between 0 – 40 kHz. The major challenges to achieve adequate precision of the experimental data exist in the realization of a broadband impulse excitation and support conditions for repeatable measurement conditions. Numerically as experimentally the first 15 mode shape pairs with the corresponding eigenfrequencies are identified. The model updating is performed for the elastic parameters and appropriate boundary conditions to minimize the deviation between measured and calculated eigenfrequencies. The acquired data serves for the implementation of a damage identification procedure in which the geometrical crack properties are identified. The deviation between real and determined crack positions is lower than 0.3 % for crack cross-section ratios less than 7 %.

Keywords: Modal analysis, Finite-Element-Method, Eigenfrequency, Non-destructive testing, Damage Identification I-INCE Classification of Subjects Number(s): 04.3

1. INTRODUCTION

At first we investigate finite element models of a cylindrical structure having circumferential cracks of different position, depth and width. In a second step test objects with analogous geometry to the numeric models are analyzed by the use of a Scanning Vibrometer. Thirdly the gained data serves for the implementation of a damage identification procedure in which crack position and severity are determined.

Vibration based damage identification has diverse applications in Structural Health Monitoring (SHM), Condition Monitoring (CM) and Non-Destructive-Testing (NDT). As major advantage of testing methods based on the alteration of eigenfrequencies appears the global character and therefore the faster data extraction in comparison to classical methods like ultra sonic. Defects change the physical properties of structures and cause a measurable modification of their dynamic behavior in relation to an undamaged system. Disadvantages are often mentioned in terms of non-uniqueness [1, 2]. It is commonly known that especially in rotationally symmetric parts the results of the damage position is ideally ambiguous [3, 4]. To overcome this drawback it is suggested to connect global methods with local methods such as in multi-criterion optimization. Herein eigenfrequency-based data is combined with high resolution local information e. g. eigenvectors.

The problem of damage identification based on eigenfrequencies is a well studied subject with a large amount of literature [5, 6, 7, 8]. By contrast concrete information regarding the specification of precision or detecting small damage scenarios is very sparse. Investigations fail for the following reasons:

- eigenfrequencies have high dependency on geometric tolerances. It is difficult to differentiate between damage-induced and tolerance-induced alterations,
- less effect of especially small damage levels on the eigenfrequencies of lower rank,
- the lack of high frequency measurement data because of big part dimensions or limitations of excitation and measurement equipment,

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- non reproducible support conditions and
- highly damped structures.

In this paper we present a damage identification scheme using numerical and experimental eigenfrequencies in a range up to 40 kHz. The principle of damage identification is briefly described as follows: the minimization of an eigenfrequency-based objective function, in which numerically generated modal responses of a parametric model are compared with experimentally generated modal data of a certain test object, serves for the computation of geometric crack parameters. It has to be emphasized that only modal features e. g. eigenfrequencies which are sensitive to the existing damage pattern should be involved in the objective function. The crack is assumed to be open, therefore bilinear effects due to closing crack sides are neglected. Only bending mode shapes are considered. Main focus was on the development of fine meshed numeric models in combination with highly accurate experimental data which both influence the predictive accuracy of the implemented damage identification procedure directly. This provides the ability to identify crack cross section ratios less than 7 %, which is not presented in the available literature.

2. Investigation of finite element models

The geometric dimensions of the cylindrical structure are l = 300 mm and d = 6 mm. Tab. 1 contains the material specifications. Fig. 1 presents the crack dimensions where x represents the position, b the width and t the depth.

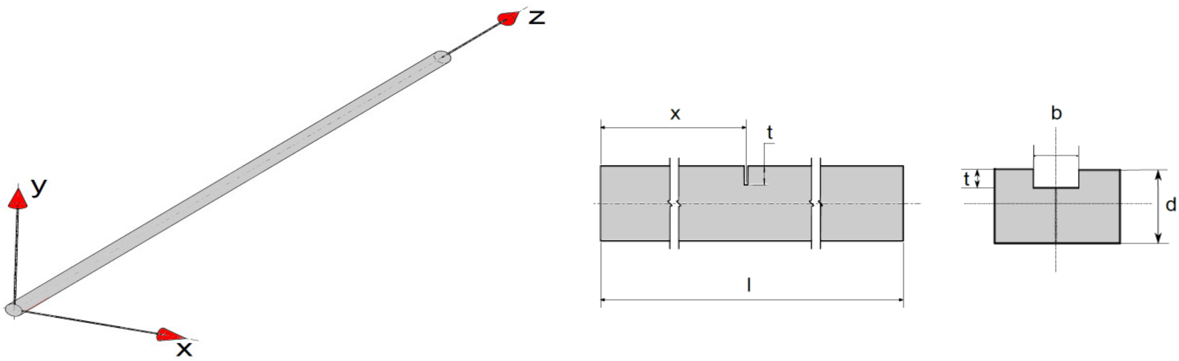

Figure 1 - Structure model

Table 1 - Material parameters

Density [kg/m^3]	Young´s modulus [GPa]	Poission´s ratio
14450	566.5	0.19

For the numeric analyses the commercial software *ABAQUS* is used. The presented discretization strategy considers on the one hand that large stress gradients due to the crack geometry have to be dissolved and on the other hand the ability to parameterize the structural model, which will be discussed later.

As presented in Fig. 2 the crack free partitions are discretized with second order 20-node brick elements with reduced integration. To avoid mesh distortion the crack partition is modeled with second order 15-node tetrahedrical elements. To eliminate the stress gradients due to the crack geometry the crack partition is meshed with factor 2 more elements, than the uncracked partitions. Tab. 2 presents the mesh specifications of the structure models including type of element, number of elements and nodes, degrees of freedom and maximal element side length. The connection between the partitions of different element density and type is realized with tie-conditions. It exist the necessity to transfer the displacement of adjacent nodes, which do not coincide for the reason of not-conform mesh structure. Tie Constraints generate new nodes on the adjoining sides and tie these together [9].

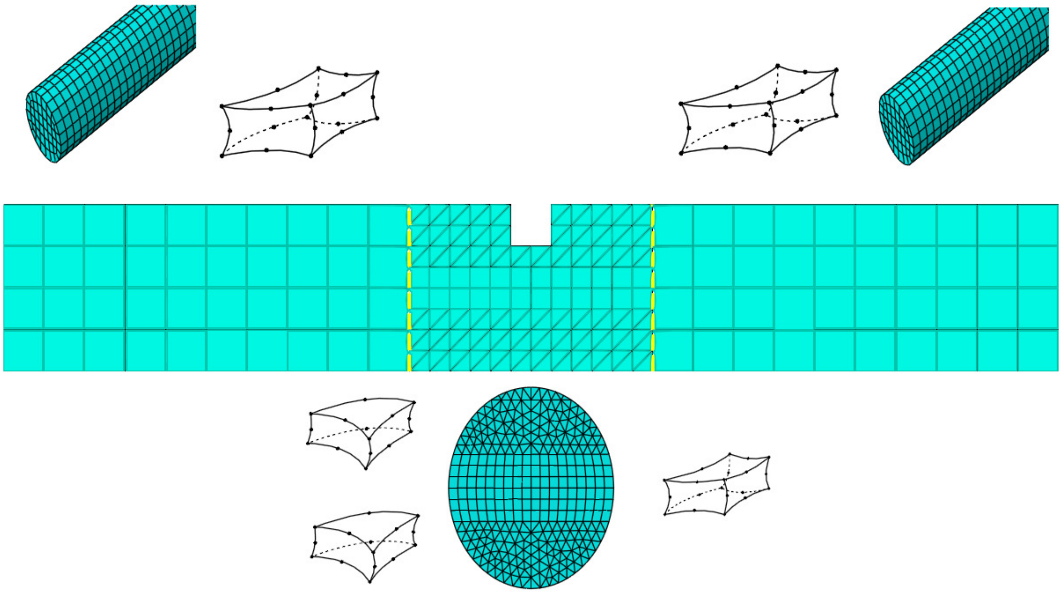

Figure 2 - Mesh description

A crack causes a local reduction of the moment of inertia. This reduction is equivalent to the decrease of local bending stiffness and results in a lowering of eigenfrequencies. The reduction increases if damage is positioned in cross section of higher bending moments or damage size rises. For this reason a large crack near vibration nodes can influence the eigenfrequency of a observe mode equally like a small crack positioned in antinodes. To overcome this fact it is suggested to observe at the minimum two consecutive eigenfrequencies in the intended identification procedure.

One cross section of the two mass centroid axes being perpendicular to each other is less affected by the introduced damage scenario. As a result one eigenfrequency of the mode shape pair decreases less and they spilt up. Fig. 3 present the absolute difference of eigenfrequencies in Hz over the 15 identified mode shape pairs in the frequency range up to 40 kHz of the 9 considered models (cf. Tab.3).

Table 2 - Mesh parameters

Type of element	C3D15/C3D20R
Number of elements	60000
hex	35000
wed	25000
Number of nodes	1075000
hex	700000
wed	375000
Degrees of freedom	3225000
hex	2100000
wed	1125000
Maximal element side length	$8.62 \cdot 10^{-4}$ m

Table 3 – Geometrical parameters of the crack models

Model	Width w [mm]	Position x [mm]	Depth d [mm]
1	0.50	150.0	0.375
2	0.50	150.0	0.750
3	0.50	150.0	1.500
4	0.50	150.0	0.375
5	0.50	150.0	0.750
6	0.50	150.0	1.500
7	0.50	150.0	0.375
8	0.50	150.0	0.750
9	0.50	150.0	1.500

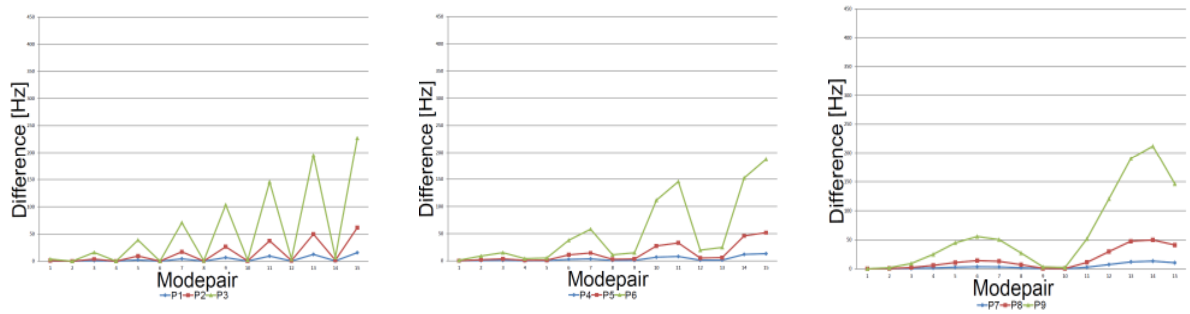

Figure 3 - Difference of numerical eigenfrequencies for the first 15 bending mode shapes (left: models 1 – 3, center: models 4 – 6, right: models 7 – 9)

3. Experimental Investigation

This section contains the selected measurement methodology for the system responses of the vibrating cylindrical test objects. A Scanning Vibrometer and a stationary excitation source are employed to measure surface velocities at 39 points of the two mass centroid axes. A post-processing software is used to extract the eigenvectors and eigenfrequencies. The first 15 bending mode shape pairs are identified in a frequency range up to 40 kHz (Fig. 4).

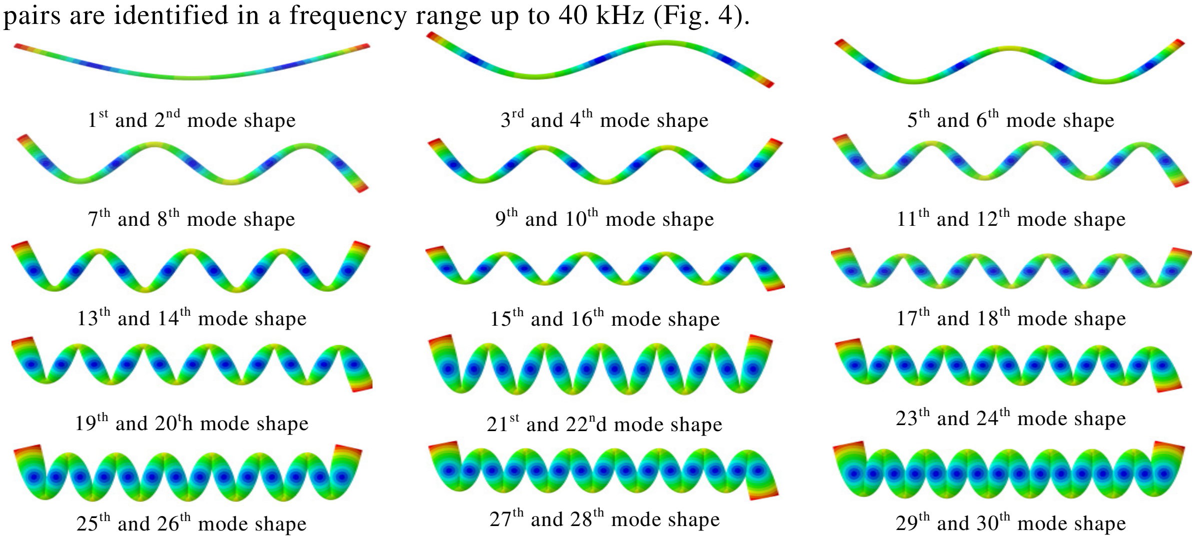

Figure 4 - First 30 bending mode shapes of an undamaged test object

Main focus was on a fast as well as sufficient intense excitation to stimulate all mode shapes in the intended frequency range and the generation of support conditions for repeatable measurements.

Fig. 5 presents the utilized measurement setup with the three main components:

- the excitation unit that contains a electro dynamical shaker. After a transient signal the test objects decay freely,
- a signal transfer system that captures excitation force and duration. On output side a Scanning Vibrometer measures surface velocities on the test objects and
- an analyzer unit that digitizes, processes and visualizes the captured signals.

The ends of the test objects lay on foam to approximate free-free boundary conditions.

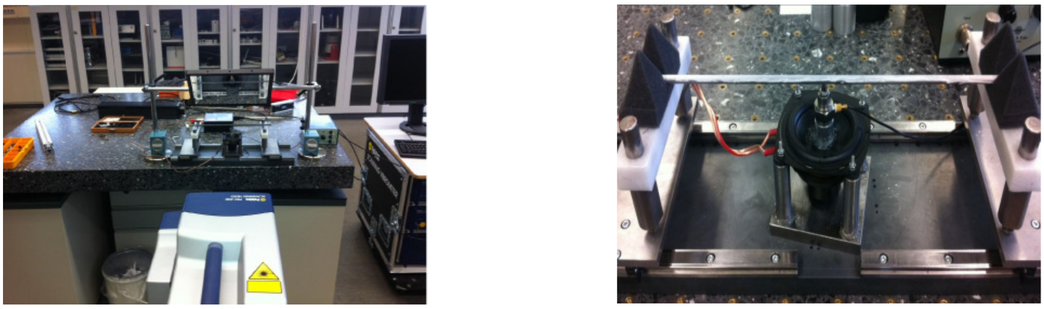

Figure 5 – Measurement setup

4. Validation of the structure model

Real structures possess an infinite number of degrees of freedom (Dof), whose reaction to an excitation is measurable only at finite points. The number of measurement points is normally much smaller than the number of numerically modeled Dof´s ($m \ll n$). The progression in computational power allows for the generation of structure models with many of million Dof´s. However, engineers are faced with the choice of adjusting the numeric model to the experiment (modal reduction) or vice versa (modal expansion) [10]. In this study we use a modal reduction strategy in which the numeric model is adopted to the experiment (39 measurement points). The calculated displacement is transferred to master-nodes with the same position measured in the experiment. Furthermore a mode-tracking procedure [11] is implemented to consider possible mode shifts due to the varying crack parameters.

After ensuring the availability of numeric and experimental eigenvectors of coincident position, there is a need for modal validation. Modal validation verifies the quality of estimated modal parameters. One possibility is to use the Modal Assurance Criterion (MAC) [12, 13] to check correlation between calculated and measured eigenvectors, that is

$$MAC = \frac{(\phi_m^T \phi_n)^2}{(\phi_m^T \phi_m)^2 (\phi_n^T \phi_n)^2}. \quad (1)$$

The MAC can attain values between 0 and 1. The higher the value between comparative eigenvectors, the more similar the mode shapes. In this study the MAC-values of the main diagonal are between 0.78 – 0.98. Side entries account for values between 0.001 – 0.075.

If the experimental data serves as reference, one strives for the validation of modal assumptions and boundary conditions. Possible deviations between numeric and experimental results can be explained due to uncertainties of elastic material parameters or elastic support. To reduce these deviations, firstly, the elastic support is modeled. Secondly, the Young´s modulus is updated manually. These steps consequently increase the predictive accuracy in the process of damage identification. Fig. 6 presents the elastic foundation [14] and the experimental support conditions.

Table 4 contains the percental deviations of the first 30 experimental and numerical eigenfrequencies for the nine test objects after the consideration of an elastic support and the update of Young´s modulus. The test objects exhibit a constant crack width of 0.5 mm and vary in crack position as well as depth. They serve as basis for the damage identification procedure.

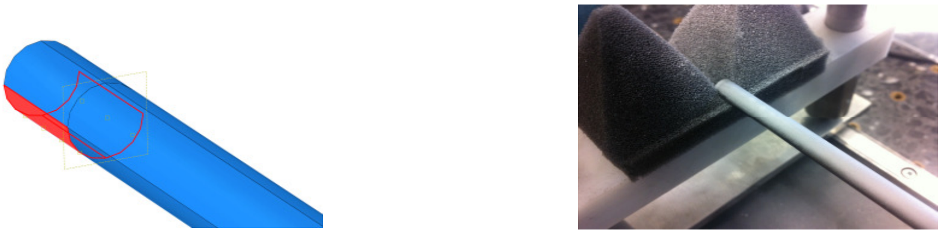

Figure 6 - Numerical and experimental support conditions

Table 4 - Percental deviations of the first 30 experimental and numerical eigenfrequencies for test objects 1 – 9

Mode	1	2	3	4	5	6	7	8	9
1	0.149	0.140	0.257	0.143	0.095	0.150	0.096	0.075	0.094
2	0.141	0.126	0.091	0.125	0.047	0.049	0.121	0.094	0.062
3	0.075	0.056	0.012	0.086	0.077	0.136	0.091	0.094	0.119
4	0.076	0.065	0.024	0.077	0.034	0.079	0.081	0.071	0.089
5	0.062	0.079	0.100	0.067	0.042	0.124	0.082	0.069	0.135
6	0.047	0.049	0.016	0.050	0.003	0.046	0.060	0.045	0.066
7	0.063	0.076	0.013	0.072	0.047	0.067	0.088	0.095	0.199
8	0.066	0.075	0.018	0.065	0.016	0.055	0.058	0.053	0.077
9	0.087	0.103	0.128	0.079	0.051	0.087	0.093	0.110	0.231
10	0.074	0.072	0.050	0.075	0.024	0.060	0.074	0.071	0.094
11	0.074	0.070	0.039	0.077	0.057	0.032	0.075	0.100	0.235
12	0.073	0.071	0.040	0.072	0.022	0.063	0.069	0.060	0.084
13	0.076	0.089	0.114	0.069	0.048	0.128	0.072	0.088	0.148
14	0.061	0.057	0.032	0.062	0.017	0.064	0.061	0.055	0.073
15	0.061	0.062	0.022	0.067	0.030	0.055	0.066	0.073	0.093
16	0.062	0.061	0.013	0.061	0.006	0.042	0.055	0.048	0.058
17	0.064	0.078	0.092	0.056	0.027	0.071	0.047	0.055	0.059
18	0.048	0.051	0.019	0.051	0.002	0.038	0.048	0.049	0.047
19	0.047	0.051	0.013	0.050	0.035	0.129	0.039	0.043	0.058
20	0.047	0.047	0.001	0.045	0.004	0.040	0.041	0.039	0.047
21	0.050	0.063	0.076	0.045	0.028	0.115	0.039	0.051	0.101
22	0.037	0.035	0.005	0.037	0.009	0.030	0.035	0.030	0.039
23	0.034	0.044	0.005	0.036	0.005	0.027	0.042	0.052	0.125
24	0.034	0.025	0.021	0.031	0.016	0.019	0.029	0.025	0.040
25	0.035	0.046	0.062	0.027	0.002	0.044	0.018	0.048	0.139
26	0.020	0.016	0.010	0.022	0.023	0.013	0.022	0.017	0.034
27	0.017	0.030	0.063	0.016	0.003	0.019	0.014	0.038	0.120
28	0.015	0.004	0.052	0.014	0.033	0.009	0.015	0.008	0.023
29	0.017	0.027	0.011	0.005	0.007	0.045	0.008	0.027	0.056
30	0.001	0.001	0.028	0.004	0.042	0.002	0.008	0.001	0.009

5. Damage Identification

With view to the damage identification procedure a parametric structure model has to be generated whose composition and meshing depend on the sought damage parameters. Fig. 7 shows the three dimensional model description. With the damage parameters crack depth t and position x the FE-preprocessor generates a variable structure model that is meshed automatically. Maintaining the physical consistency and avoiding mesh distortion, the variables should move in an adequate value range. Crack depth t fluctuates between 0.2 – 1.75 mm and crack position between 15 – 285 mm. Tab. 2 contains element specifications for the structure models used in the damage identification.

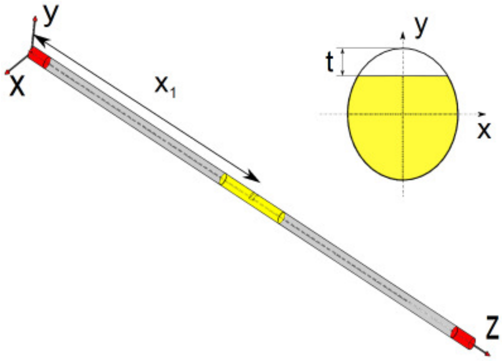

Figure 7 - Parametric model for damage identification

An optimization process is mathematically defined as the minimization of an objective function

$$\min\left\{f(\vec{\vartheta})\right\}; \vartheta \in \Re^n, \tag{1}$$

in which $\vec{\vartheta}$ describes the vector of the design/search parameters. $g_j(\vec{\vartheta})$ and $h_k(\vec{\vartheta})$ are the equality and inequality restrictions.

$$\begin{aligned} &g_j(\vec{\vartheta}) \leq 0; j = 1, m, \\ &h_k(\vec{\vartheta}) \leq 0; k = 1, q, \quad , \\ &\vartheta_i^L \leq \vartheta_i \leq \vartheta_i^U; i = 1, n. \end{aligned} \tag{1}$$

The restrictions on the design variables ϑ_i with the lower ϑ_i^L and upper ϑ_i^U limits are called explicit restrictions.

To reduce the number of time intense FE-analyses during the optimization a local approximation method is used to calculate new parameter vectors. The results of such local methods possess only validity in the proximity of the actual design point [15]. The objective function to be minimized consists of a quadratic error sum between the experimental and numerical eigenfrequencies

$$f(\vartheta) = \sum_{i=1}^{30} \frac{(f_{\exp_i} - f_{num_i})^2}{f_{\exp_i}^{\;2}}. \tag{1}$$

To avoid adverse value ranges, it is suggested to scale the objective function as well as the design parameters.

The optimization algorithm uses the Method of Moving Asymptotes (MMA) [16] that already proved applicability in other structural dynamic problems [17, 18]. During the optimization the MMA generates in each iteration step a strictly convex approximating subproblem that is controlled by the moving asymptotes. They serve for stabilization and an increasing velocity of convergence in the general optimization procedure. In each iteration, the current design point is given. Then an approximating explicit subproblem is generated. In this subproblem, the exact objective function is replaced by an approximating convex function. This approximation is based mainly on gradient information at the current iteration point and also implicitly on information from previous iteration points. The subproblem is solved and the unique optimal solution becomes the next iteration point. After that a new subproblem is generated.

Fig. 8 presents a flow chart of the developed damage identification procedure. *Python* is used as coding language to connect *ABAQUS* with the external *MATLAB*-based optimizer. Therein the first step consist in the definition of starting variables for crack position x and depth t as well as the step sizes for the differential quotients. Also the eigenfrequencies that are included in the objective function are selected. Secondly *ABAQUS* starts successively one objective function and two gradient analyses. The results – eigenvectors and eigenfrequencies – are saved to an output file. The third step contains the modal reduction with the objective of comparing experimental und numerical eigenfrequencies of same mode shapes. After that objective function, gradient as well as restriction function values are computed and committed to the external optimizer. Fourthly the optimization algorithm starts which results in a new vector of parameters that is utilized for the next iteration. A maximum number of iterations completed or the lower deviation of a predefined criterion of convergence stop the process.

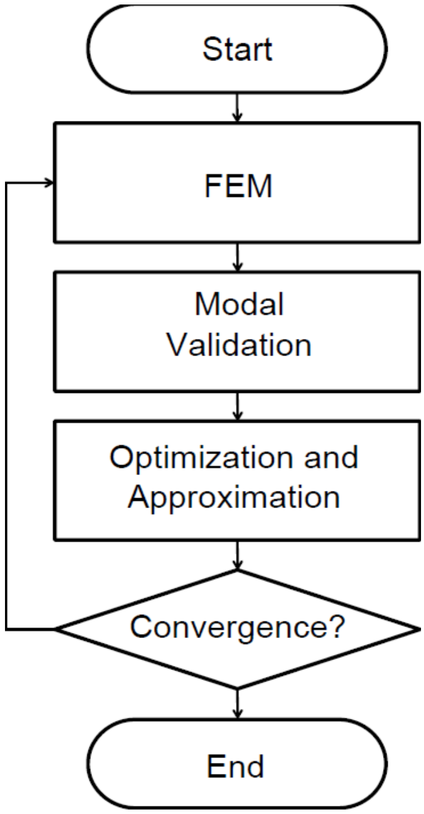

Figure 8 - Flow chart of optimization scheme

The results of damage identification could only be as precise as the underlying data. The optimizer searches for the minimum of the objective function. In lots of application in damage identification the presence of many local minima complicates the identification of the global minimum [19]. Furthermore, modal features have to be carefully selected. Only eigenfrequencies that are sensible the certain damage pattern yield to the determination of the sought damage parameters. Therefore, in this study the four highest eigenfrequencies are chosen in the objective function to detect even the smallest damage levels

$$f(\vartheta) = \sum_{i=27}^{30} \frac{(f_{\exp_i} - f_{num_i})^2}{f_{\exp_i}^{\;2}} . \qquad (1)$$

The success of the presented damage identification highly depends on the presence of appropriate start values for t and x. Trail and error often results in useless outcomes because of convergence against the bounds of search space. For the identification of proper start values a two step optimization strategy is worked out. At first objective function plots of the nine test objects are compiled (Fig. 9). Secondly the parameter combinations with the smallest deviation of objective function are utilized as starting values for the optimization. Tab. 5 contains the results of damage identification for the nine test objects 1 – 9 with the used eigenfrequencies 27 – 30. The percental deviations for crack depth t vary between 0.968 – 7.613 %. The results for the determination of crack position x fluctuate between 0.046 – 0.280 %. The test objects 14 and 15 show the most precise results in damage identification because of the smallest deviations of eigenfrequencies (cf. Tab. 4).

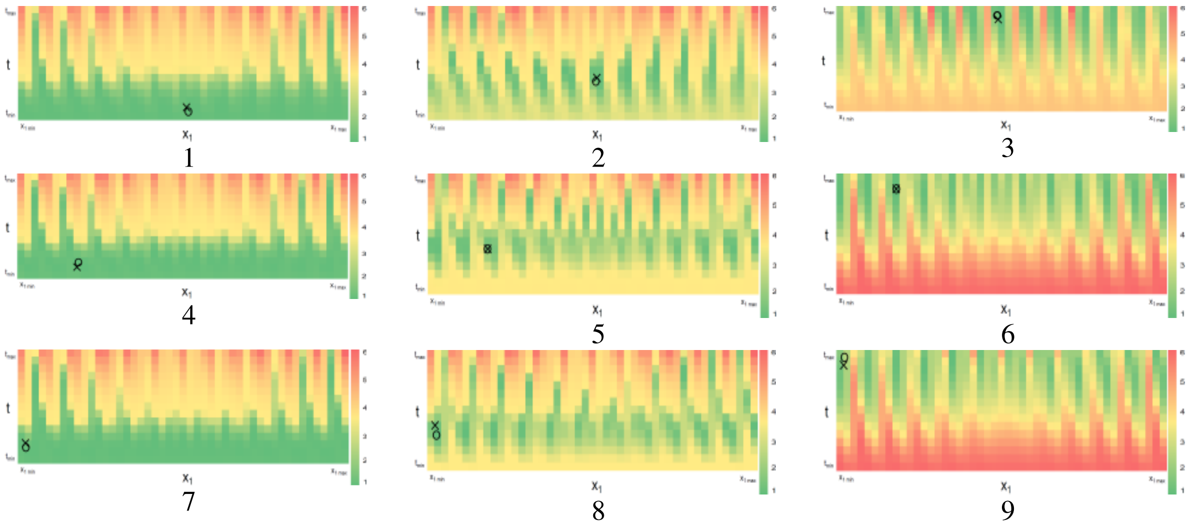

Figure 9 - Objective function plots for test objects 1 – 9 (cross denotes the real parameter vector; circle the computer parameter vector)

Table 5 – Results of damage identification

Test object	1	2	3	4	5	6	7	8	9
t_{real} [mm]	0.375	0.75	1.500	0.375	0.750	1.500	0.375	0.750	1.500
t_{opt} [mm]	0.309	0.699	1.551	0.429	0.731	1.485	0.316	.0632	1.610
%-dev.	4.258	3.290	3.290	3.484	1.226	0.968	3.806	7.613	7.097
x_{real} [mm]	150.000	150.000	150.000	75.000	75.000	75.000	37.500	37.500	37.500
x_{opt} [mm]	150.757	149.416	149.457	75.451	74.875	75.294	37.955	38.231	37.862
%-dev.	0.280	0.216	0.201	0.167	0.046	0.109	0.169	0.271	0.134

6. CONCLUSIONS

In the presented study a damage identification procedure has been worked out, that has a possible application in non destructive testing. By the use numerically and experimentally determined eigenvectors and eigenfrequencies, geometrical crack parameters are computed. It is demonstrated that for detecting small damage levels (crack cross-section ratios < 10 %) it is vital to utilize high frequency data with adequate precision. As might have been expected the smaller the deviations between computed and measured eigenfrequencies, the more accurate are the damage identification results. Critical considerations are required if it comes to generally valid start variables in the process of optimization. As there is no a-priori knowledge in real testing scenarios, further investigation has to be invested. In the light of increasing usage of composites in all technical sectors, engineers are faced with the challenge to apply frequency based damage identification methods on structures with high structural damping. This is accompanied by difficulties in the modal analysis of highly damped materials, those in turn prevent the identification of small damage levels.

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