Vibration Input Identification using Dynamic Strain Measurement

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ABSTRACT

Transfer Path Analysis (TPA) has been conducted in order to improve the noise and vibration quality of mechanical structures. However, the force identification in the TPA is still challenging problem and its accuracy has to be improved. The Matrix Inversion Method and the Apparent-Mass Matrix Method are approaches for force identification. The Matrix Inversion Method estimates the excitation force by the product of an inverse matrix of accelerance and a vector of actual operational acceleration. It is known that the Matrix Inversion Method is very sensitive to measurement noise especially at the resonance. Apparent-Mass Matrix Method has been recently proposed, which provides more accurate results than the Matrix Inversion Method. However, these methods are still insufficient in accuracy, and improvement of identification accuracy is required. This paper proposes a new force identification method by using strain measurement. The method estimates the force using the Strain Frequency Response Function (SFRF) instead of acceleration or Apparent-Mass Matrix. It is shown that SFRFs are more strongly affected by higher-order modes than accelerance FRF. Therefore, we considered that more accurate force identification can be obtained by using SFRF than the conventional method.

Keywords:Transfer Path Analysis, Force Identification, Matrix Inversion Method, Apparent Mass Method, Strain,

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1. INTRODUCTION

In recent years, the techniques for reducing the noise and vibration have become important for improvement of the product's performance. In order to find effective countermeasures, the contributions of the identified sources are evaluated. Especially, the transfer path analysis (TPA) has been established for vehicle noise and vibration problems. The TPA is a technique for calculating contributions to the evaluation response. In the TPA analysis, the target system is separated into active system and passive system. The active system generates an interface load, and the passive system reacts to the interface load. And each contribution is calculated as a product of excitation force spectra and frequency response functions. Hence, the accuracy of the input force identification influences TPA precision.

The excitation force needs to be measured by a load cell, but in many cases it is not possible to place a load cell in a proper manner due to space limitation or difficulty of keeping boundary condition. Therefore, It is necessary to conduct input identification using indirect measurements such as a matrix inversion method to obtain input. The matrix inversion method estimates the excitation force by the product of an inverse matrix of accelerance and a vector of actual operational acceleration. However, measurement errors in the accelerance matrix and acceleration vector are propagated to the identified force spectra through the inversion procedure[1]. In previous studies, an apparent-mass matrix method has been proposed[2]. In the apparent-mass matrix method an apparent mass matrix corresponding to an inverse matrix of the accelerance is directly estimated. Hence, inversion of the matrix is not necessary in estimation process. Consequently, the apparent-mass matrix method provides more accurate results than the matrix inversion method. Although the result improved, improvement of identification accuracy is still required.

This paper investigates a new force identification method by using strain measurement. The method estimates the excitation force using strain frequency response functions instead of the accelerance matrix or the apparent mass matrix. It is understood that the ill-condition of the frequency response

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function matrix deteriorates the accuracy of the input identification. It is expected that if at a target frequency the frequency response function receives an influence of high order modes, ill-condition of the frequency response function matrix is alleviated. In previous studies, it is suggested that strain frequency response functions are more strongly affected by higher-order modes than the accelerance[3,4]. However, a clear reason for it has not been proven in this study.

In this paper, numerical simulations are conducted to compare the results by the proposed method with the conventional method.

2. Theory

2.1 Matrix Inversion Method

In the matrix inversion method, frequency response functions *H*, response during the operating condition *y* and excitation force spectra *f* are related by the following expression.

$$\begin{Bmatrix} y_1(\omega) \\ \vdots \\ y_n(\omega) \end{Bmatrix} = \begin{bmatrix} H_{1,1}(\omega) & \cdots & H_{1,m}(\omega) \\ \vdots & \ddots & \vdots \\ H_{n,1}(\omega) & \cdots & H_{n,m}(\omega) \end{bmatrix} \begin{Bmatrix} f_1(\omega) \\ \vdots \\ f_m(\omega) \end{Bmatrix} \tag{1}$$

Where $f_j(\omega)$ is the fourier spectrum of the excitation force at point *j*, $y_i(\omega)$ is the fourier spectrum of the response at point *i*, $H_{i,j}(\omega)$ is the frequency response functions with input point *j* and response point *i*. Rewriting Eq.(1) gives

$$\mathbf{y}(\omega) = \mathbf{H}(\omega)\mathbf{f}(\omega) \tag{2}$$

If a number of excitation points *m* and a number of response points *n* are the same (*m* = *n*), force spectra are identified by premultiplying the inverse of the frequency response function matrix **H** to the acceleration vector as follows.

$$\mathbf{f}(\omega) = \mathbf{H}^{-1}(\omega)\mathbf{y}(\omega) \tag{3}$$

On the other hand, in order to improve identification accuracy of force spectra, it is common that the number of response points takes more than the number of excitation points(m < n). In this case, the excitation force is identified by the least squares method. And this condition is desirable to increase the identification accuracy. Then the excitation force is identified as follows.

$$\mathbf{f}(\omega) = \mathbf{H}^{+}(\omega)\mathbf{y}(\omega) \tag{4}$$

where $\mathbf{H}^{+}(\omega)$ is pseudo-inverse matrix $\left(\mathbf{H}^{+}(\omega) = \left(\mathbf{H}^{H}(\omega)\mathbf{H}(\omega)\right)^{-1}\mathbf{H}^{H}(\omega)\right)$.The superscript ^{H} indicates the conjugate transpose. The excitation force is estimated by using the Eq.(3) or Eq.(4)

2.2 Apparent-Mass Matrix Method

In the apparent-mass matrix method, the excitation force is identified as follows

$$\mathbf{f}(\omega) = \mathbf{G}(\omega)\mathbf{y}(\omega) \tag{5}$$

where $\mathbf{G}(\omega)$ is the apparent-mass matrix. The apparent-mass matrix estimation is the same as H2 estimation of FRF when the number of responses are equal to the number of input forces[5]. The identification accuracy is improved by avoiding calculation of the inverse matrix. And the apparent-mass matrix method estimates the input force so that the errors of the input is minimized. Therefore, the apparent-mass matrix method is more advantageous than the matrix inversion method to input identification.

2.3 Frequency Response Functions of the Beam

In this paper, the numerical simulation is conducted to compare the results by the proposed method with the conventional method. The target structure is a beam supported at both ends as shown in Fig.1. When considering the bending vibration of the both-ends supported beam, the accelerance (FRF) and strain frequency response functions (SFRF) are expressed as follows.

$$H(\omega) = -\omega^2 \sum_{r=1}^{n} \frac{W_r(x)W_r(\bar{x})}{\left(\omega_r^{\,2} - \omega^2 + 2\zeta\omega_r\omega j\right)\rho Al} \tag{6}$$

$$W_r(x) = \sin\left(\frac{r\pi x}{l}\right) \tag{7}$$

$$H_\varepsilon(\omega) = -z\sum_{r=1}^{n}\left(\frac{r\pi}{l}\right)^2 \frac{W_r^\varepsilon(x)W_r(\bar{x})}{(\omega_r^2 - \omega^2 + 2\zeta\omega_r\omega j)\rho Al} \tag{8}$$

$$W_r^\varepsilon(x) = -\sin\left(\frac{r\pi x}{l}\right) \tag{9}$$

Eq.(6) is the FRF, and the Eq.(8) is the SFRF, where W_r and W_r^ε are a displacement mode function, and a strain mode function, respectively. The strain mode function is obtained by differentiating the displacement mode function twice with respect to *x*. *r*, *z* and *n* are a mode order, a distance from the neutral axis to the surface in the bending of the beam, and number of adopted modes, respectively.

A numerical simulation is conducted to compare FRF with SFRF. The responses are calculated by changing the number of adopted modes *n*(*n*=5,10,50). The frequency range is from 0[Hz] to 800 [Hz].The bode diagrams of the FRF and SFRF (Driving Point Responses) are shown in Figs.2 and 3, respectively. The diagrams of Cross Responses are also shown in Figs.4 and 5.

These results indicate that frequency response functions are affected by higher-order modes in the case of driving point FRF. Especially, the SFRF are more strongly affected by higher-order modes than the FRF.

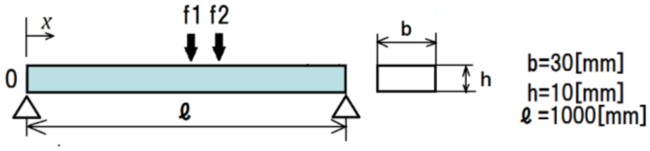


Fig. 1 – Both ends Supported Beam

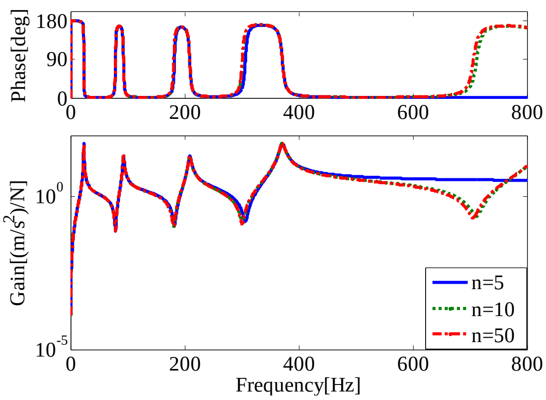


Fig.2– FRF (Driving Point Resp.)

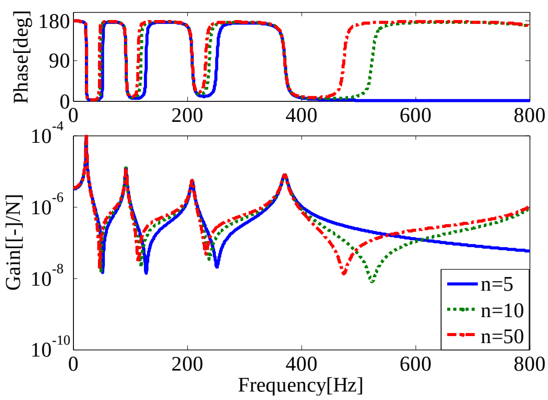


Fig.3 – SFRF (Driving Point Resp.)

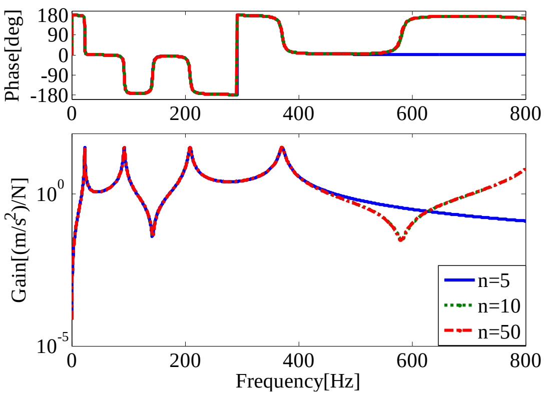


Fig.4 – FRF (Cross Resp.)

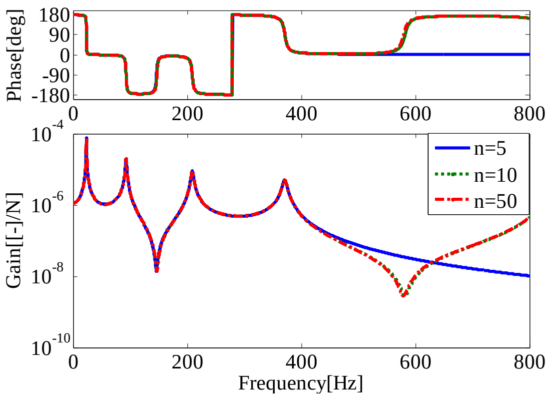


Fig.5 –SFRF(Cross Resp.)

3. Numerical Simulation

The numerical simulation is conducted to compare the input identification results by the proposed method with the conventional method.

3.1 Procedure of Numerical Simulation

The numerical simulation was performed as the following procedure.

(a)Prepare the excitation force, which is a random signal that follows the normal distribution with mean 0 and standard deviation 1.

(b)Obtain the response data (acceleration or strain) by multiplying frequency response functions (FRF or SFRF) to the excitation force generated in (a)

(c)Put errors to the excitation force and response data(acceleration or strain)obtained in (a) and (b),and estimate frequency response functions (FRF or SFRF) by regarding them as measured data. The error data is prepared as follows:

$$\mathbf{y}_{error,i}(\omega)=\mathbf{E}_i(\omega)\times\sqrt{\frac{a_{error}\times\left(\sum_k^N|y_i^{(k)}(\omega)|^2\right)/N}{\left(\sum_k^N|e_i^{(k)}(\omega)|^2\right)/N}} \tag{10}$$

where $\mathbf{y}_i(\omega)$ $\left(\mathbf{y_i}(\omega)=\lfloor y_i^{(1)}(\omega)\cdots y_i^{(N)}(\omega)\rfloor\right)$is a true response vector; $y_i^{(k)}(\omega)$is a response obtained in the k-th measurement at the point i; $\mathbf{E}_i(\omega)$ $\left(\mathbf{E}_i(\omega)=\lfloor e_i^{(1)}(\omega)\cdots e_i^{(N)}(\omega)\rfloor\right)$ is an error vector; $e_i^{(k)}(\omega)$ $\left(e_i^{(k)}(\omega)=e_{re,i}{}^{(k)}(\omega)+j\cdot e_{im,i}{}^{(k)}(\omega)\right)$is a random complex number with mean 0 and standard deviation 1 in the real and imaginary parts, respectively; a_{error} is an error adjustment factor; and N is an average number for input identification.

(d) Calculate the operational data without noise: the excitation force and response data (acceleration or strain). Put the error to the response data (acceleration or strain).

(e) Identify the excitation force using frequency response functions (FRF or SFRF) obtained in (c) and the response data (acceleration or strain) in (d).

(f) Evaluate the input identification accuracy. The frequency averaged error is an error index value by averaging the identification error at all frequencies. The frequency averaged error is calculated as follows.

$$\varepsilon_{ave.}(\omega)=\frac{1}{M}\sum_{u=1}^{M}\left(\frac{\sum_{k=1}^{N}\left|\hat{f}^{(k)}(\omega)-f^{(k)}(\omega)\right|^2}{\sum_{k=1}^{N}\left|f^{(k)}(\omega)\right|^2}\right) \tag{11}$$

where $\hat{f}^{(k)}(\omega)$ is the fourier spectrum of the excitation force that have been identified in the k-th frequency; $f^{(k)}(\omega)$ is the fourier spectrum of the true value of the excitation force; and M is a number of frequency lines.

3.2 Conditions of Numerical Simulation

Location of the response points and excitation points are summarized in Table 1. For Case A, the response data includes the driving point responses, whereas for Case B, they are not included. Other conditions are as follows: the frequency range is from 0[Hz] to 800 [Hz]; a frequency resolution is 0.25 [Hz]; all modal damping ratios are equal to 1%; and the number of adopted modes is 50. Two uncorrelated forces are applied to the system simultaneously. Frequency response functions are estimated with 50 averages. The input identification is iterated 50 times. The error adjustment factor is 0.1.

Table1 Response and Excitation Conditions

Case	Response Point[m]				Excitation Point[m]	
					f1	f2
A	Acceleration				0.5	0.6
	0.2	0.5	0.6	0.8		
	Strain					
	0.2	0.5	0.6	0.8		
B	Acceleration				0.5	0.6
	0.2	0.4	0.7	0.9		
	Strain					
	0.2	0.4	0.7	0.9		

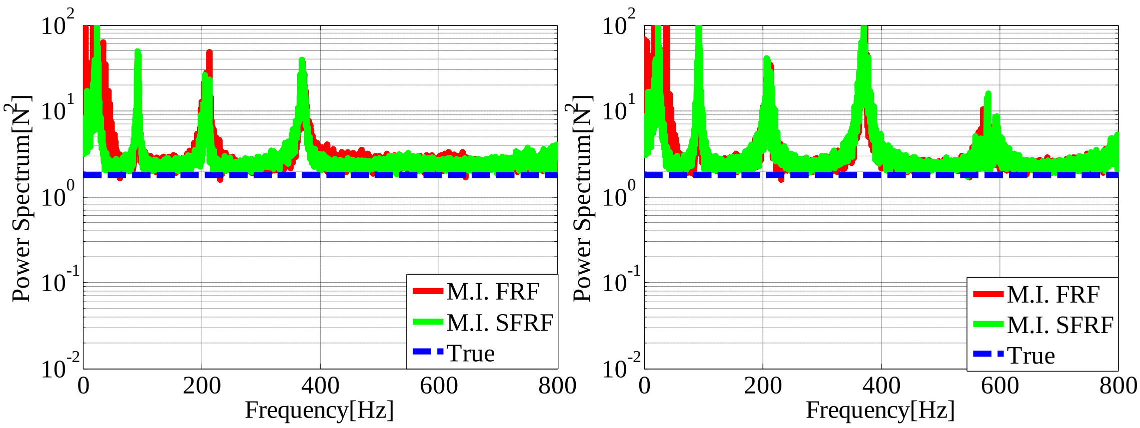
3.3 Results

The identified and true force power spectra by matrix inversion method (M.I.) are shown in Fig.6.

Those by the apparent-mass matrix method (A.M.) are shown in Fig.7. The frequency averaged error is shown in Figs.8 and 9.

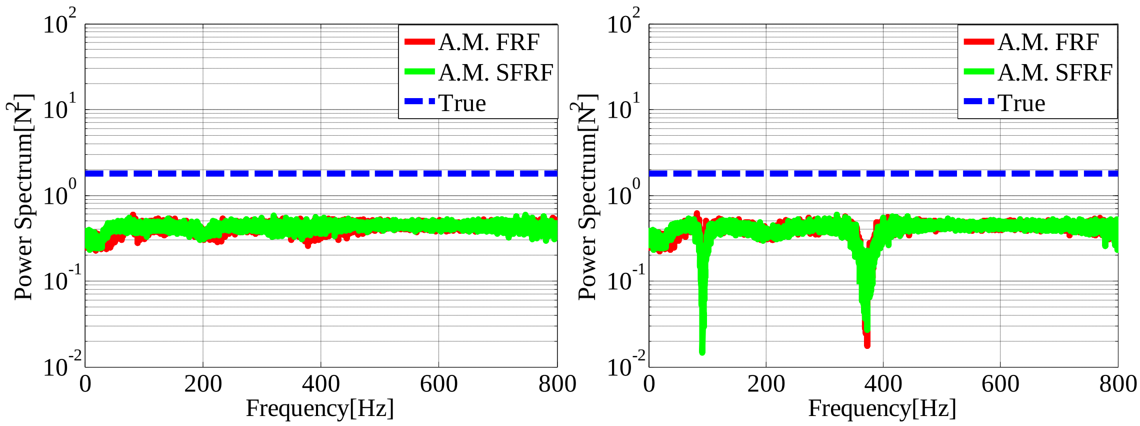
For M.I. case, Case A gives better identification results rather than Case B; and results of SFRF gives better results in Case A but vice versa in Case B.

For A.M. Case A gives better results than Case B although the difference is small; and SFRF gives better results in Case A but not valid in Case B. Furthermore, the apparent-mass matrix method (A.M.) gives much better results than the matrix inversion method (M.I.).



(a) Case A (b) Case B

Fig.6– Force Identification(M.I.)



(a) Case A (b) Case B

Fig.7– Force Identification(A.M.)

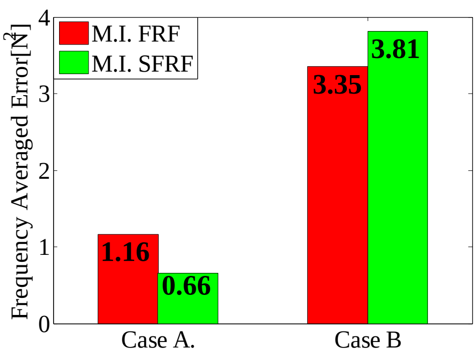


Fig.8 –Frequency Averaged Error(M.I.)

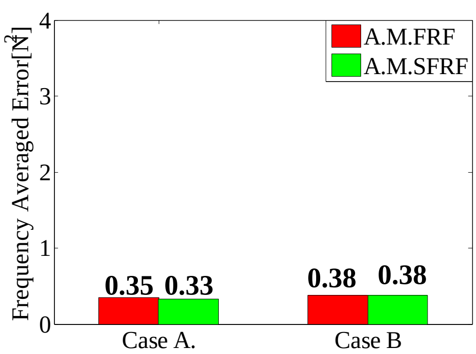


Fig.9 –Frequency Averaged Error(A.M.)

4. Discussion

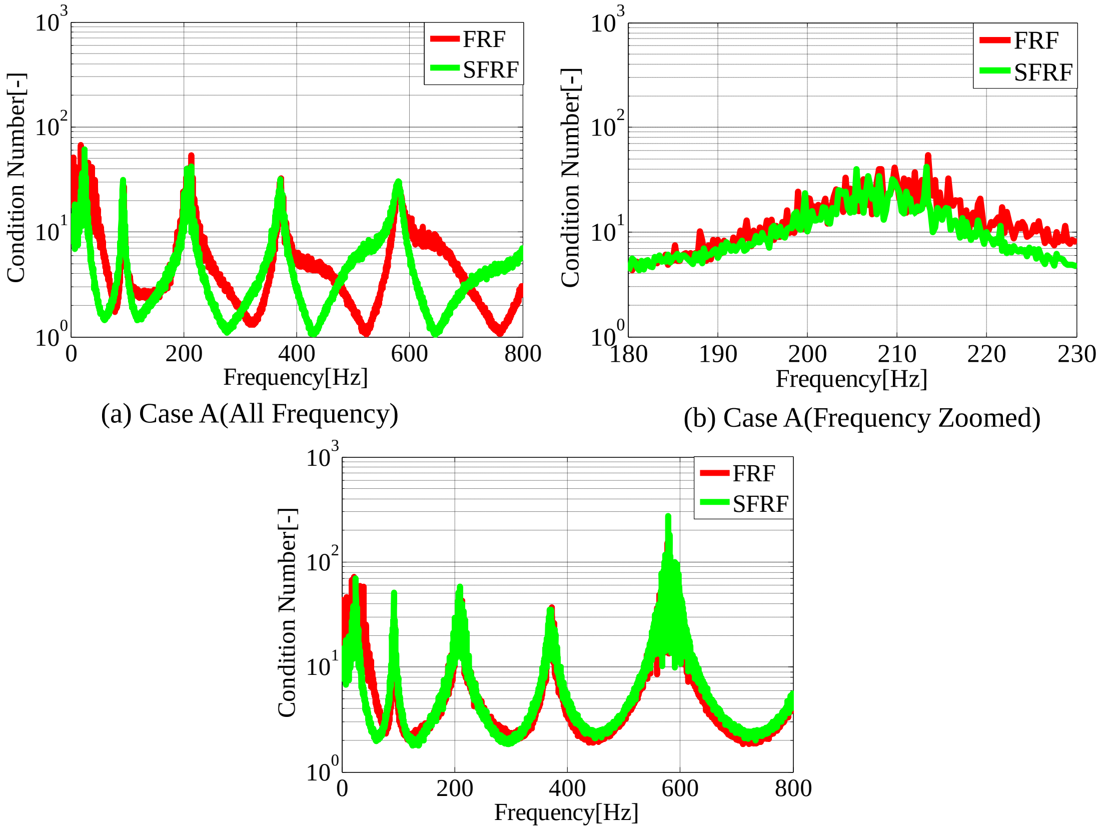
4.1 Condition Number

We compare the condition numbers of the SFRF and FRF matrices. The condition number is the factor that evaluates the error propagation in the excitation force that have been identified. The condition number $\kappa(H)$ is calculated as follows.

$$\kappa(H) = \frac{\mu_{max}}{\mu_{min}} \tag{12}$$

Where μ_{max} is a maximum singular value of the frequency response function matrix, μ_{min} is a minimum singular value of the frequency response function matrix. The condition number $\kappa(H)$ of the FRF and SFRF are shown in Fig.10.

Fig.10 indicates that the condition number value of the SFRF smaller than the condition number value of the FRF in the case of Case A. Therefore, the input identification using the SFRF is superior to identification using the FRF in Case A.



(a) Case A(All Frequency) (b) Case A(Frequency Zoomed)

(c) Case B

Fig.10–Condition Number

4.2 Hybrid Input Identification

This paper investigates a new force identification method by using both strain measurement and acceleration measurement. In this paper, This new force identification method is called ‘hybrid input identification’. In the hybrid input identification, strain is measured near the excitation point, and accelerations are measured at other points. It is abbreviated as Hmix in this paper.

The numerical simulation is conducted in the same manner as in Chapter 3. Location of the response points and excitation points are summarized in Table 2. In Case A-1,the excitation force is identified using the FRF or the SFRF, in Case A-2,the excitation force is identified by the Hmix.

The identified and measured force power spectra using matrix inversion method (M.I.) are shown in Fig.11. The identified and measured force power spectra using apparent-mass matrix method (A.M.) are shown in Fig.12. The frequency averaged error is shown in Fig.13.

These results indicate that the input identification using Hmix has a higher identification accuracy than the conventional method (FRF). In addition, the input identification accuracy using Hmix is almost the same as the input identification accuracy using SFRF. This is an effective method from a practical point of view.

Table2 Response and Excitation Conditions

Case	Response Point[m]				Excitation Point[m]	
					f1	f2
A-1	Acceleration				0.5	0.6
	0.2	0.5	0.6	0.8		
	Strain					
	0.2	0.5	0.6	0.8		
A-2	Acceleration		Strain		0.5	0.6
	0.2	0.8	0.5	0.6		

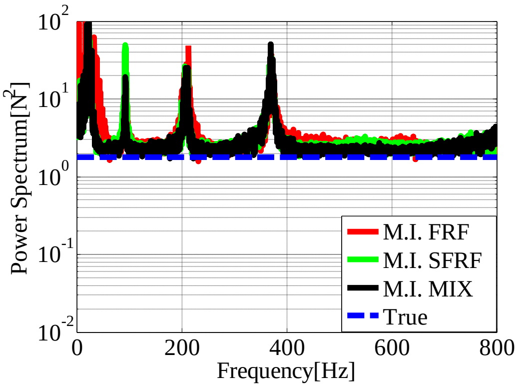


Fig.11– Force Identification(M.I.)

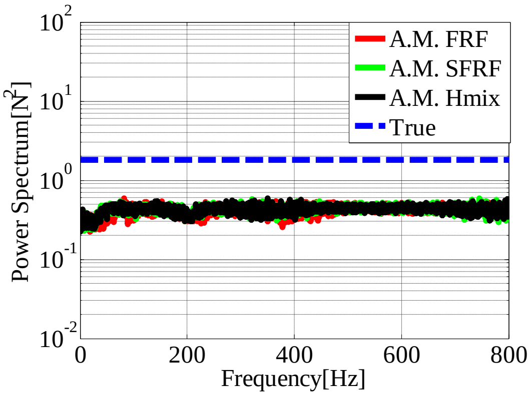


Fig.12– Force Identification(A.M.)

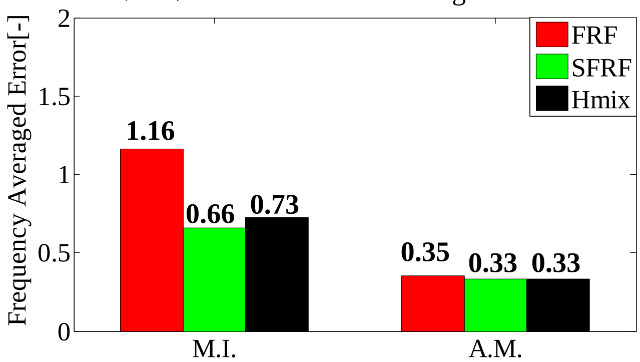


Fig.13–Frequency Averaged Error

5. CONCLUSIONS

- For driving point FRF, frequency response functions are affected by higher-order modes. Especially, the strain response (SFRF) is more affected by higher-order modes than acceleration response (FRF).
- The input identification accuracy is improved by adopting the driving point response. Especially, the input identification using strain response (SFRF) gives more accurate results than the acceleration response (FRF).
- This paper investigated a hybrid input identification method: both acceleration and strain are used for input indentification; strain are measured for driving point response, and accelerations are measured for other responses. This method has a higher identification accuracy than the conventional method.
- The identification accuracy of the hybrid input identification method is almost the same as the input identification accuracy using strain only response (SFRF).This is an effective method from a practical point of view.
- The input identification accuracy of the apparent-mass matrix method (A.M.)is much higher than that of the matrix inversion method(M.I.).

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