Estimation of pressure fluctuations in a turbulent boundary layer based on vibro-elastic models

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ABSTRACT

A simple, but scientifically rigorous, framework is proposed for estimation of effectiveness of elastic materials for the reduction of pressure fluctuations. The framework employs the intrinsic correspondence between a transformation of vorticity perturbations into acoustic waves at the elastic boundary and the conversion of transverse (shear) waves to radiated longitudinal (pressure) waves. The case of a significantly subsonic wall flow (such as a turbulent boundary layer) is considered and the relative effect of elastic boundaries on the turbulent pressure fluctuations is estimated. The framework does not allow calculation of the absolute radiation levels, but is reasonably straightforward to implement in comparison to full-scale CFD and aeroelasticity models, and is not computationally intensive. The proposed approach enables rapid estimation of numerous 'what-if' scenarios that become important for design of new vibro-absorbing materials and for prototyping studies.

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I-INCE Classification of Subjects Number: 21.5, 21.6, 23.6, 35.2

1. INTRODUCTION

The various aspects of flow noise have been conventionally investigated in the context of noise reduction from anthropogenic sources (aircrafts, naval ships, industrial equipment, etc. [1–3]) and for understanding the mechanisms of acoustic noise in global geophysical systems (ocean [4], atmosphere, [5] geosphere [6]). An interesting application of flow noise has been recently described in Ref. [7] where a prototype system was proposed for detecting emerging tornadoes based on their flow noise signature.

Starting with the seminal work of Lighthill in the 1950s (the so-called Lighthill analogy) the development of consistent models of flow noise has been the focus of both theoretical and experimental studies. There is a vast amount of literature devoted to this subject ([1–3] and references therein).

Although the main models for flow noise are nowadays discussed in textbooks it is still a challenging task and an area of active scientific research and engineering effort. The difficulty of this problem rests on the complex underlying phenomenology of flow noise and sophisticated experimental facilities for the related experimental studies. The analytical and/or numerical framework for flow noise estimation involves advanced models of a turbulent flow (CFD) coupled with the equations of acoustic wave generation and propagation in the presence of the flow (hydroacoustics and aeroacoustics). The phenomenology becomes significantly more complicated if there is a need to account for an elastic response of the boundaries coupled with the dynamics of turbulent flow (aeroelasticity [1]). These problems are intractable analytically and usually require intensive computer simulations. There are many software tools available that capture this phenomenology with different levels of realism. Unfortunately, the application of these tools very often requires an expert-level knowledge of CFD and advanced computing facilities in order to produce even the very basic 'what-if' estimates.

It is therefore difficult (and even impossible) to deploy advanced CFD models in the context of rapid prototyping, such as in the early stages of the design process. This motivated development of a simplified (but still rigorous) framework which is easier to implement (in comparison to the full-scale CFD models) while still capturing the complex phenomenology of the underlying process and producing quantitative results of acceptable accuracy.

The development and evaluation of such an approach is the main motivation of the present study. More

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specifically, an analytical framework is proposed that enables a consistent estimation of the effect of the elastic properties of an underlying surface on the turbulent boundary layer noise and associated vibrations. The analysis is restricted to low Mach numbers, i.e.,

$$M = U/c \ll 1, \tag{1}$$

where c is the speed of sound and U is the velocity of unperturbed flow (far from the the underlying surface). The methodology is used to provide relative evaluation between different surface materials and cannot be used for absolute estimates of flow noise.

2. THEORETICAL FRAMEWORK

The aim of the proposed framework is to find effective parameters of an elastic medium that correspond to a turbulent flow in a slightly compressible fluid medium. Conventional methods of the elastic wave transformation are then used to study the process of flow noise generation near the elastic boundary by considering the transformation of vorticity perturbations (shear waves) into sound waves (longitudinal waves) at the boundary.

It is well known that the velocity field of an arbitrary motion of any slightly compressible medium can be represented as a sum of two components, $\mathbf{v} = \mathbf{v}_{\parallel} + \mathbf{v}_{\perp}$, where $\mathbf{v}_{\parallel}$ is a potential component with $\mathbf{v}_{\parallel} = \nabla\varphi$ (φ is a scalar potential), and $\mathbf{v}_{\perp}$ is a rotational component with $\mathbf{v}_{\perp} = \nabla \times \mathscr{A}$ ($\mathscr{A}$ is a vector potential). For the case of elastic isotropic materials $\mathbf{v}_{\parallel}$ and $\mathbf{v}_{\perp}$ (and potentials φ and $\mathscr{A}$) satisfy the standard wave equations for the longitudinal and transverse waves

$$\frac{1}{c_l^2}\frac{\partial^2}{\partial t^2}\varphi + \nabla^2\varphi = 0, \tag{2}$$

$$\frac{1}{c_s^2}\frac{\partial^2}{\partial t^2}\mathscr{A} + \nabla^2\mathscr{A} = 0, \tag{3}$$

where

$$c_l = \sqrt{\lambda + 2\mu/\rho} \tag{4}$$

and

$$c_s = \sqrt{\mu/\rho} \tag{5}$$

are the speeds of longitudinal and transverse waves, respectively. The attenuation of these waves can be taken into account by assuming complex elastic moduli λ and μ. For a fluid with no mean flow these equations can still be applied but $c_l \gg c_s$ ($\lambda \gg \mu$) and the attenuation of transverse waves is high.

The next step is to deduce the effective moduli that correspond to the fluid with turbulent flow. According to seminal results of Chu and Kovasznay [8] for mode decomposition of compressible flows, and well known connections between elasticity and viscous flow ([9], page 337), in a slightly compressible fluid the dynamics of vorticity perturbations and sound waves can be decoupled. For vorticity perturbations

$$\frac{\partial}{\partial t}\Omega + (\mathbf{v}_{\perp}\nabla)\Omega = \nu\nabla^2\Omega, \tag{6}$$

while for sound waves the wave equation (2) applies. In the above equation $\Omega = \nabla \times \mathbf{v}_{\perp} = \nabla^2\mathscr{A}$ is the vorticity field and ν is the kinematic viscosity of the fluid. For a flow with constant velocity $\mathbf{U}$, the linear approximation leads to $(\mathbf{v}_{\perp}\nabla)\Omega \approx (\mathbf{U}\nabla)\Omega$.

Assuming the vortex perturbations move only in one direction, determined by $U = |\mathbf{U}|$ (which corresponds to boundary layer flow), and substituting a harmonic component of vorticity $\Omega \propto \exp[i(\mathbf{k_s} \cdot \mathbf{x} - \omega t)]$ into Eq. (6), leads to a dispersion relation for the vorticity perturbations. Considering an effective transverse velocity $c_s = \omega/k_s$, where $k_s = |\mathbf{k_s}|$, the matching of the dispersion relations leads to the following conjecture [10]

$$c_s = U - i\nu k_s, \tag{7}$$

where

$$c_s = \sqrt{(\mu_r + i\mu_i)/\rho}. \tag{8}$$

Matching of the real and imaginary parts of these equations leads to the effective shear modulus for the fluid in the presence of the flow. When $c_s \ll c_l$ and $U \ll c$ (slightly compressible limit), the components are $\mu_r \approx \rho U^2$ and $\mu_i \approx 2\rho\omega\nu$, providing $\nu k_s \ll U$ and $\nu\omega \ll 1$. The latter conditions are true for water at sonic frequencies, for example, but these modulus components need not be approximated if the conditions are not satisfied. Note that when $U = 0$ the effective shear modulus of the fluid is purely imaginary, as commonly used for reflection and transmission calculations using static fluid layers [11].

For low Mach number flow, the effective longitudinal wave speed of the fluid in the presence of the flow is simply

$$c_l \approx c, \tag{9}$$

with

$$k_l = \omega / c_l. \tag{10}$$

It is known that direct noise radiation from the scalar or vector potential components of the velocity field due to flow is generally weak, being quadrupole in nature [1–3]. The presence of a surface may enhance the radiation resulting in dipole sources created by unsteady forces on the flow boundary. In the simplest case of an absolutely rigid surface and ideal fluid the enhancement (due to 'scattering') can be understood in terms of reflected images, but for the viscous fluid and elastic surfaces the reflection requires some modification that accounts for the vorticity–sound transformation at the boundary, discussed above.

The 'correspondence' conditions given by Eqs. (7) and (9) will be employed in the modelling framework to estimate the relative effect of a surface on the noise generated by a turbulent boundary layer. It should be stressed that the conditions cannot be used to estimate the total amount of noise. The conversion of transverse (shear, or rotational) wave components into longitudinal (acoustic) wave components at the flow boundary will be considered. The same process could be used to estimate the reflection of incident longitudinal components at the boundary, but it is known [1] that this contribution is small because vorticity (rotational) components are dominant and the longitudinal wavenumber components within the flow itself largely mismatch propagating wavenumbers.

Transformation of elastic waves in layered structures has been well studied [11–14]. The coefficients of reflection, transmission and absorption are derived by requiring continuity of pressure, stress and displacement across the interfaces between the layers. The processes are modelled numerically by implementing known theory for plane-wave reflection [11–13]. Results for arbitrary waves could be modelled as linear combinations of plane waves but the important conclusions do not require this to be done. The boundary layer turbulence is modelled as an ensemble of transverse waves of arbitrary frequency distribution and propagation direction, with vector potential of one wave component

$$\mathscr{A} = \mathscr{A}_0 \exp(-i\omega t + i\xi x - i\eta_s z), \tag{11}$$

incident on the boundary adjacent to the flow from the halfspace $z > 0$, and a reflected longitudinal component with scalar potential

$$\varphi = \varphi_0 \exp(-i\omega t + i\xi x + i\eta_l z). \tag{12}$$

The direction z is normal to the layers and into the fluid, and x is parallel to the layers. The wavenumber components ξ, η_s and η_l are related by

$$\eta_s^2 = k_s^2 - \xi^2, \tag{13}$$

$$\eta_l^2 = k_l^2 - \xi^2, \tag{14}$$

with equivalent expressions for the lower layers. The incidence angle θ is related to the horizontal wavenumber component through

$$\xi = k_s \sin(\theta). \tag{15}$$

The transformation coefficients can be estimated as angle averaged values assuming uniform angle distribution.

As an initial simple example, the transformation to acoustic waves is considered for a single transverse wave 'harmonic' of frequency ω of Eq. (11). Danilov and Mironov [15] addressed the exact issue being considered here, but for a simple interface between two media. Assume the ratio of the scalar potential of the reflected longitudinal wave to the magnitude of the vector potential of the incident transverse wave is V. Then V represents the coefficient of conversion of transverse waves into longitudinal waves at a plane surface. For a given input medium, the reflected energy is proportional to $|V|^2$ (where $|\ |$ denotes taking the amplitude of a complex quantity). For a simple interface between a fluid and a fluid-like medium (such as a rubber) Ref. [15] gives the approximation

$$\frac{V}{V_*} = \frac{1 - \rho^{(1)}/\rho - 2(\eta_l^{(1)}/k_s)(-1 + \sqrt{\rho\mu^{(1)}/\rho^{(1)}\mu})}{(\rho^{(1)}/\rho + \eta_l^{(1)}/\eta_l)(1 + \sqrt{\rho\mu/\rho^{(1)}\mu^{(1)}})} \tag{16}$$

where $\rho^{(1)}$ and $\mu^{(1)}$ are the density and shear modulus in the reflecting fluid-like half-space, and ρ is the density of the fluid medium. Wave components η_l, $\eta_l^{(1)}$ and k_s are connected to the incidence angle, complex moduli and wave speeds through Eqs. (4), (5), (13), (14), and (15). V has been normalised here by V_*, which is the transformation coefficient at a rigid boundary (obtained by setting $\rho^{(1)} = \infty$ in the expression for V).

Eq. (16) provides insightful criteria for material selection for flow noise reduction that would be very difficult to deduce by other means (for experimental results see [16]). When $c_s^{(1)} \gg c_s$, as with a water-rubber boundary, Eq. (16) at angles close to normal incidence can be simplified to

$$\frac{V}{V_*} \simeq 1 - \frac{\rho^{(1)}}{\rho}, \tag{17}$$

implying that the intensity of turbulent boundary layer noise can be significantly decreased provided the material underlying the turbulent boundary layer has fluid-like properties (such as with rubber) and its density is close to the density of the fluid. This is strikingly different from an intuitive assumption of impedance match, $\rho^{(1)}c_s \simeq \rho c$. The equation is, however, only valid for an infinite half-space boundary. More complex multi-layer-material calculations, which avoid this assumption, can be made using the theory of Levesque and Piche [11]. Some numerical results are shown in the next section.

3. NUMERICAL RESULTS

In this section the formalism proposed above is applied to estimate the effect on turbulent boundary noise of application of elastic materials at the flow boundary. The modelling scenario corresponds to the flow noise generated by a turbulent boundary layer over a surface moving at 3 m/s relative to the fluid.

The conversion coefficient V of transverse waves into longitudinal (pressure) waves, equivalent to Eq. (16), has been calculated numerically using well-known matrix formalism [11] and compared with the approximation of [15], Eq. (16), in Fig. 1. The physical parameters used for the calculation are shown in Table 1. The interface approximates water over a rubber half-space, in order to satisfy the assumptions of [15], and good agreement is obtained. The red curve is unaltered if power reflection coefficients are used instead of simple pressure ratios (which do not account for the fluid velocity). The vertical green line is the critical angle, defined by $\sin\theta_c \simeq c_s/c_l \ll 1$. All transverse wave components of fluid turbulence which reflect longitudinal pressure waves into the incident fluid half-space have incidence angles less than the critical angle, which is 0.12 degrees in this example. Stated another way, the incident angle range 0–0.12° produces reflected pressure components in the range 0–90°. Values of V/V_* in the figures greater than the critical angle are not relevant for scattering into the incident half-space. The rubber density here is 1100 kg/m^3, and the reflection is reduced to less than -17 dB relative to a rigid interface, in rough agreement with the simplified form of Eq. (17), which gives -20 dB. The calculation is at 10 kHz but there is only a slight dependence on frequency because the viscosity term which contributes to the imaginary part of the modulus for water, from Eqs. (7) and (8), is frequency dependent.

Table 1 – Material Properties for Fig. 1

Material	Parameter	Value	Units
Water	ρ	1000	kg/m^3
	c_l	1500	m/s
	ν	1×10^{-6}	m^2/s
	U	3	m/s
Rubber	$\rho^{(1)}$	1100	kg/m^3
	$c_l^{(1)}$	1650	m/s
	$c_s^{(1)}$	165	m/s

Realistic surfaces adjacent to flow will have finite thickness. On purely physical grounds it would be expected that the performance indicated by Fig. 1 would fail below some frequency value. The consideration is restricted to three-layer elastic structures (water-rubber-steel). Figs. 2 and 3 show the reflection of longitudinal waves V due to transverse wave input from layers of material of finite thickness covering 40 mm of steel, relative to the reflection from the steel alone, V_{steel}, at 1 and 5 kHz. In all cases the steel is air-backed. The calculation again uses the theory of [11]. The blue curves are for an actual rubber material, with known frequency-dependent elastic moduli, and the red curves are for a nominal rubber material designed as a good

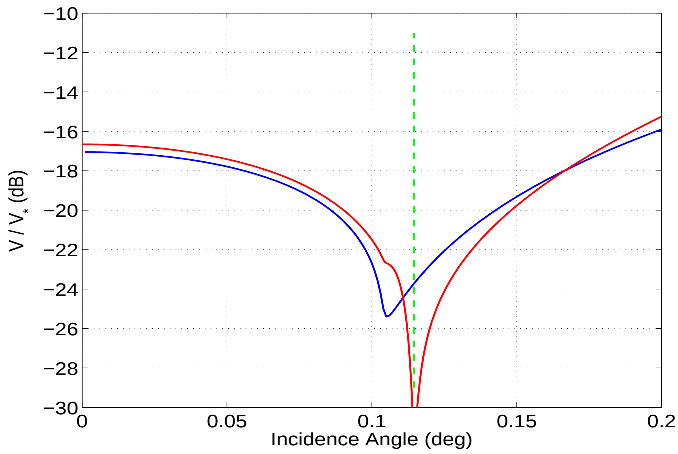

Figure 1 – Relative reflection efficiency at 10 kHz of a rubber half-space as a function of transverse wave incidence angle. *Red line,* Eq. (16); *blue line,* formalism of [11].

absorber of longitudinal pressure waves. The green line indicates the critical angle, as before. It can be seen that the transverse wave reflection is also reduced by about 10 dB at 5 kHz relative to a pure steel surface. At 1 kHz there is little improvement.

Note that, although not shown, the absolute value of V for conversion of transverse to longitudinal components is typically small at about -50 to -10 dB, being lower at lower frequencies and smaller incidence angles. At incidence angle zero there is no conversion of a plane transverse wave to a pressure wave, giving $V = 0$ ($-\infty$ dB).

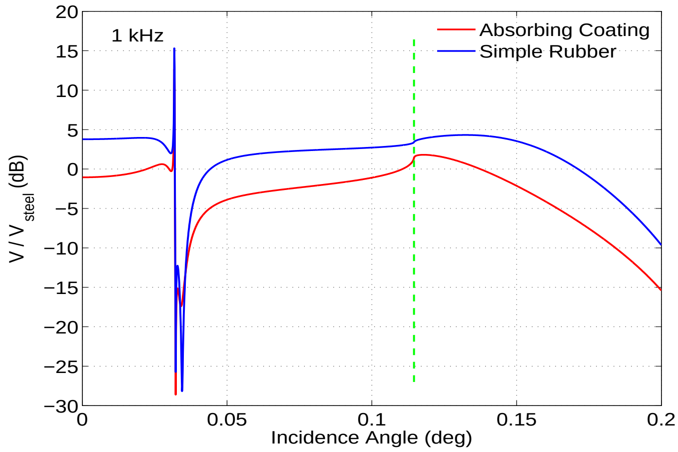

Figure 2 – Relative reflection efficiency at 1 kHz of two rubber coating layers as a function of transverse wave incidence angle.

The frequency dependence of the reflection efficiency is more clearly shown by solid-angle averaging over the incidence angle up to the critical angle, assuming a random ensemble of transverse-wave directions, as in Eq. (11). Fig. 4 shows the relative reflection efficiency of the coatings, as a function of source frequency, obtained by this averaging process. It can be seen that the reduction in reflection occurs only above 1 kHz, and below 1 kHz the effect of the rubber is to increase the reflection relative to bare steel.

Several points must be made about the assumptions needed to associate the changes in efficiency presented in this paper with actual changes in radiated noise from the surfaces due to flow. First, it is assumed that the effect of the surface itself has little effect on the flow sources that are the starting point for this analysis. For situations where there is strong fluid–structure coupling, such as singing, this would not be the case. Second, plane wave reflection coefficients have been used for flow sources that are clearly not planar. However, arbitrary sources can often be decomposed into linear combinations of plane waves so if a consistent trend in reduced reflection is observed this is not likely to be an issue. The actual reflection of a random selection of flow noise sources involves much phase cancellation and will not be V, but should still be proportional to V providing the flow noise sources are unaltered by the surface. Of more significance in this regard is the thickness of the reflecting surface layers relative to the flow noise source distributions. If the layers are thin

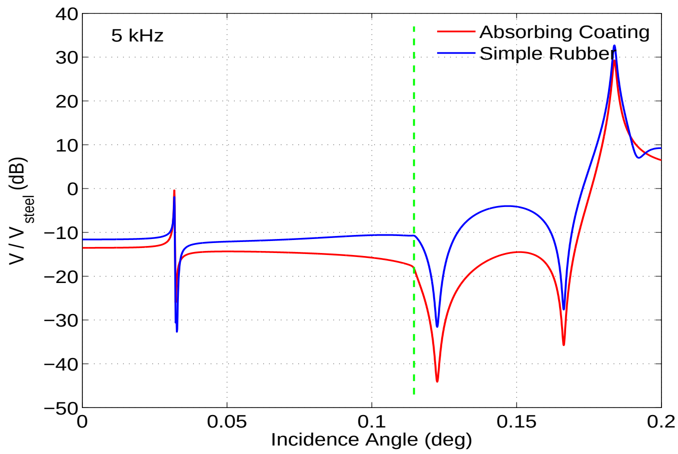

Figure 3 – Relative reflection efficiency at 5 kHz of two rubber coating layers as a function of transverse wave incidence angle.

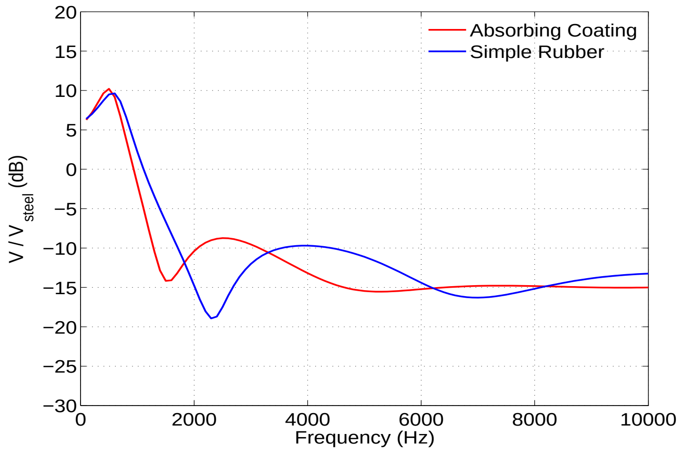

Figure 4 – Relative efficiency of two rubber coating layers as a function of flow noise frequency.

compared with the thickness of the flow noise region that contributes most to the reflection then the spatial phasing of the reflection laterally will be maintained and this argument still applies. If the noise source region is thin compared with the layers then the lateral phasing could be completely changed with a different set of layers, and it would not be generally true that the reduction is proportional to V.

4. CONCLUDING REMARKS

A theoretical framework is presented that allows an estimation of the effect of elastic properties of flow boundaries on the intensity of flow noise. While this framework cannot estimate the total amount of flow noise generated from a particular boundary surface, it quantitatively models the modification of the radiated acoustic pressure from turbulent wall flow (turbulent boundary layer). The proposed framework is based on a number of simplified assumptions such as significantly subsonic velocity of the flow and that flow noise sources are essentially unaffected by the different surface type.

The formalism described here enables consistent design and evaluation of various types of elastic materials for the reduction of flow noise without needing to undertake laborious CFD and aeroelasticity simulations.

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