inter.noise 2014
MELBOURNE AUSTRALIA
16-19 NOVEMBER

Analysis of frequency-domain active noise control algorithm with parallel structure

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ABSTRACT

Active noise control is a noise reduction technique introducing a canceling anti-noise wave. Recently, it is widely used in industrial operations, manufacturing, and consumer products. In many cases, periodic noise occurs because most applications include engines, compressors, motors, fans, and propellers which have reciprocating motion. The noise usually contains tones at the fundamental frequency and at several higher harmonic frequencies in practice. For this type of noise, we developed a frequency-domain active noise control algorithm and determined that it's effective before. In practice, however, some errors can occur and deteriorate the performance of the algorithm. In this paper, we consider that practical condition and have an analytical approach to the property of frequency-domain active noise control algorithm. We achieved the dynamics of the algorithm and predict the performance in practice.

Keywords: Active noise control, Error analysis I-INCE Classification of Subjects Number(s): 38.2

1. INTRODUCTION

Active noise control is achieved by introducing a canceling anti-noise wave through an appropriate array of secondary sources and an effective to attenuate noise that is difficult problem to control using passive means (1). It's widely used for reducing some acoustic in many industrial operations, manufacturing, and consumer products. In many cases, periodic noise occurs because most applications include engines, compressors, motors, fans, and propellers which have reciprocating motion. Periodic noise usually contains tones at the fundamental frequency and at several harmonic frequencies in practice. This type of noise can be attenuated by multiple-frequency active noise control methods (2, 3, and 4). In general, realization of multiple notches requires higher-order filters. Instead, it can be realized by parallel or cascade connection of multiple second-order sections. We developed another multiple-frequency active noise control algorithm (5). The algorithm structure is parallel, and each part is updated independently in the frequency domain. We applied it for some periodic noises, and the result showed it is effective for periodic noise. However, some errors can occur and deteriorate the performance in practice. In this paper, we assume that the noise is not exactly periodic, but quasi-periodic which means Fourier coefficients of the noise signal are locally constant, but slowly changed. And we also assume that there is background noise and some frequency error between noise and anti-noise. In this case, we have an analytical approach to the property of frequency-domain active noise control algorithm and predict the performance in practice.

2. ANALYSIS CONDITION

2.1 Assumptions

We have an analytical approach on following assumptions.

First, the system is linear. Second, the noise is quasi-periodic which means Fourier coefficients are linearly changing by little. Third, there is some frequency error between noise and anti-noise. Fourth, the background noise is zero-mean white Gaussian noise.

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2.2 Noise signal

We consider that noise $d(t)$ contains tones at the fundamental frequency ω_1, several harmonic frequencies and background noise w.

$$d(t) = \sum_{l=1}^{\infty}\{\alpha_l(t)\cos(\omega_l t) + \beta_l(t)\sin(\omega_l t)\} + w(t) \tag{1}$$

α_l and β_l are Fourier coefficients of each frequency component of noise signal. l is the index of each frequency component. And each frequency is multiple of ω_1 as $\omega_l = l \cdot \omega_1$. This noise is measured at error microphone and filtered by anti-aliasing filter.

$$\breve{d}(t) = \sum_{l=1}^{L}\{\alpha_l(t)\cos(\omega_l t) + \beta_l(t)\sin(\omega_l t)\} + \breve{w}(t) \tag{2}$$

L is the number of frequency components. And then, the filtered signal is converted to digital signal.

$$\breve{d}(\mathrm{nT_s}) = \sum_{l=1}^{L}\{\alpha_l(nT_s)\cos(\omega_l nT_s) + \beta_l(nT_s)\sin(\omega_l nT_s)\} + \breve{w}(nT_s) \tag{3}$$

T_s is sampling period. Finally, digital signal $d(n)$ is achieved as follows.

$$d[\mathrm{n}] = \sum_{l=1}^{L}\{\alpha_l[n]x_{l,c}[n] + \beta_l[n]x_{l,s}[n]\} + w[n] \tag{4}$$

$x_{l,c}$ is $\cos[\omega_l nT_s]$ and $x_{l,s}$ is $\sin[\omega_l nT_s]$. As we mentioned, α_l and β_l are linearly changing so that we consider as follows.

$$\alpha_l[n] = \alpha_{l,1} nT_s + \alpha_{l,0}\,,\ \beta_l[n] = \beta_{l,1} nT_s + \beta_{l,0} \tag{5}$$

As the first assumption, $\alpha_{l,1}$ and $\beta_{l,1}$ have very small value.

2.3 Control signal

Control signal $y(n)$ is the output of the algorithm and input to the secondary source generating anti-noise. It also contains tones at the fundamental frequency ω_1' and several harmonic frequencies.

$$y[\mathrm{n}] = \sum_{m=1}^{M}\{Y_{m,c}[n]x'_{m,c}[n] + Y_{m,s}[n]x'_{m,s}[n]\} \tag{6}$$

M is the number of frequency components. $x'_{\mathrm{m,c}}$ is $\cos[\omega'_m nT_s]$ and $x'_{\mathrm{m,s}}$ is $\sin[\omega'_m nT_s]$. And each frequency is multiple of ω_1 as $\omega_m = m \cdot \omega_m$. $Y_{m,c}$ and $Y_{m,s}$ are Fourier coefficients of each frequency component of control signal. They have constant values in each update period. As the third assumption, there is some error between ω_m and ω'_m as $\Delta_m \equiv \omega_m - \omega'_m$, $\Delta_m \ll 1$.

The control signal wave through secondary path and converted to $\hat{y}(n)$. It is a digital signal of anti-noise which is included in error signal measured at error microphone.

$$\hat{y}[\mathrm{n}] = \sum_{m=1}^{M}\{Y_{m,c}[n]\hat{x}'_{m,c}[n] + Y_{m,s}[n]\hat{x}'_{m,s}[n]\} \tag{7}$$

$\hat{x}'_{m,c}$ and $\hat{x}'_{m,\mathrm{s}}$ are filtered-x as $\hat{x}'_{m,c}[n] = s * x'_{m,c}$ and $\hat{x}'_{m,\mathrm{s}}[n] = s * x'_{m,\mathrm{s}}$.

2.4 Error signal

The signal measured at error microphone called error signal. It is the sum of noise and anti-noise as follows.

$$e[n] = d[n] - \hat{y}[n] \tag{8}$$

As we mentioned, we update this algorithm in the frequency domain. So, the error signal is transformed to frequency-domain values $E_{m,c}$ and $E_{m,s}$. Meanwhile, the frequency-domain values of noise are $D_{m,c}$ and $D_{m,s}$. For anti-noise, they are $Y_{m,c}$ and $Y_{m,s}$, and we called them adaptive filters.

2.5 Update algorithm

In this algorithm, the Fourier coefficients of control signal are updated as follows.

$$\mathbf{Y}[K+1] = \mathbf{Y}[K] + \mathbf{E}[K] \tag{9}$$

$\mathbf{Y}$ and $\mathbf{E}$ are defined as $\mathbf{Y} \equiv \left[Y_{1,c}\ \ Y_{1,s}\ \ \cdots\ \ Y_{M,c}\ \ Y_{M,s}\right]^T$ and $\mathbf{E} \equiv \left[E_{1,c}\ \ E_{1,s}\ \ \cdots\ \ E_{M,c}\ \ E_{M,s}\right]^T$. K is the index of update period. For each frequency component, the update algorithm is expressed as follows.

$$\begin{aligned} Y_{m,c}[K+1] &= Y_{m,c}[K] + E_{m,c}[K] \\ &= Y_{m,c}[K] + (D_{m,c}[K] - Y_{m,c}[K]) \\ &= \bar{D}_{m,c,CT}[K] + \tilde{D}_{m,c,CT}[K] + D_{m,c,NT}[K] + W_{m,c}[K] \end{aligned} \tag{10}$$

$D_{m,c}$ is divided into four parts. $\bar{D}_{m,c,CT}$ is the mean of $D_{m,c}$. $\tilde{D}_{m,c,CT}$ occurs by quasi-periodicity and frequency error. $D_{m,c,NT}$ occurs by the other frequency components. $W_{m,c}$ occurs by background noise. It's the same equation for sine component.

3. IDEAL CASE

3.1 Adaptive filter dynamics

In ideal case, the noise signal is perfectly periodic and there is no frequency error. So, the update algorithm can be expressed as follows.

$$\begin{aligned} Y_{m,c}[K+1] &= \bar{D}_{m,c,CT}[K] + \tilde{D}_{m,c,\mathrm{CT}}[K] + D_{m,c,NT}[K] + W_{m,c}[K] \\ &= \hat{\alpha}_{m0} + W_{m,c}[K] \end{aligned} \tag{11}$$

$\hat{\alpha}_{m0}$ is a constant value which differs from α_{m0}. It's because of secondary path effect. Likewise, $Y_{m,s}[K+1] = \hat{\beta}_{m0} + W_{m,s}[K]$ is achieved. Because of that background noise parts, we have a stochastic analysis. That background noise parts group into $\mathbf{W}_m$ and the stochastic properties is achieved as follows (6).

$$\begin{aligned} &\mathbf{W}_m \triangleq W_{m,c} + jW_{m,s} \\ &\text{mean: } \overline{\mathbf{W}_m} = 0, \text{ variance: } \overline{\left|\mathbf{W}_m\right|^2} - \left|\overline{\mathbf{W}_m}\right|^2 = \sigma_w^2 / P_b \end{aligned} \tag{12}$$

P_b is the number of samples for transforming error signal to frequency-domain values. As the stochastic properties of background noise parts, those of adaptive filter is achieved as follows.

$$\begin{aligned} &\mathbf{Y}_m[K+1] \triangleq Y_{m,c}[K+1] + jY_{m,s}[K+1] \\ &\text{mean: } \overline{\mathbf{Y}_m[K+1]} = \overline{Y_{m,c}[K+1] + jY_{m,s}[K+1]} = \hat{\alpha}_{m0} + j\hat{\beta}_{m0} \\ &\text{variance: } \overline{\left|\mathbf{Y}_m[K+1] - \overline{\mathbf{Y}_m[K+1]}\right|^2} = \sigma_w^2 / P_b \end{aligned} \tag{13}$$

The mean of adaptive filter for each frequency components is fixed. And the bigger the variance of background noise and the smaller P_b is, the bigger the variance of $\mathbf{Y}_m$ gets.

3.2 Error dynamics

Using the update algorithm, we can get the error dynamics as follows.

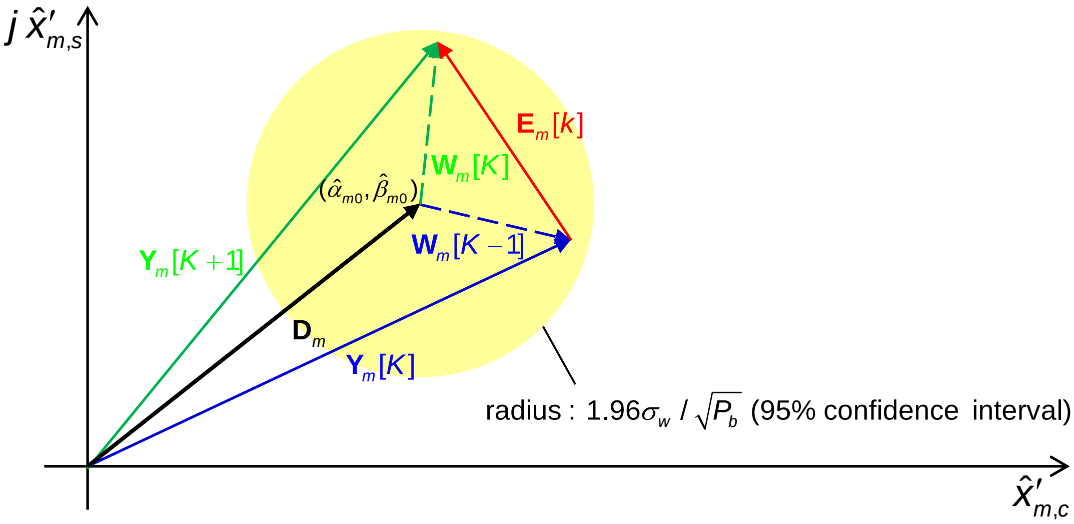

Figure 1 – The conceptual diagram of total dynamics in ideal case

$$\begin{aligned} E_{m,c}[K+1] &= \hat{\alpha}_{m0} + W_{m,c}[K+1] - Y_{m,c}[K+1] \\ &= \hat{\alpha}_{m0} + W_{m,c}[K+1] - (\hat{\alpha}_{m0} + W_{m,c}[K]) \\ &= W_{m,c}[K+1] - W_{m,c}[K] \end{aligned} \tag{14}$$

Likewise, we can get the sine component as follows.

$$E_{m,s}[K+1] = W_{m,s}[K+1] - W_{m,s}[K] \tag{15}$$

As previous analysis, we have a stochastic analysis as follows.

$$\begin{aligned} &\mathbf{E}_m[K+1] \triangleq E_{m,c}[K+1] + jE_{m,s}[K+1] = \mathbf{W}_m[K+1] - \mathbf{W}_m[K] \\ &\text{mean : } \overline{\mathbf{E}_m[K+1]} = \overline{\mathbf{W}_m[K+1]} - \overline{\mathbf{W}_m[K]} = 0 \\ &\text{variance : } \overline{\left|\mathbf{E}_m[K+1]\right|^2} - \left|\overline{\mathbf{E}_m[K+1]}\right|^2 = 2\sigma_w^2 / P_b \end{aligned} \tag{16}$$

The mean of error is zero, and the variance twice as those of adaptive filter.

3.3 Total dynamics

For each frequency component, we achieve total dynamics in Figure 1. The optimal vector $\mathbf{D}_m$, which is defined as $\mathbf{D}_m \triangleq D_{m,c} + jD_{m,s}$, is fixed at point $(\hat{\alpha}_{m0}, \hat{\beta}_{m0})$. Surrounding it in a circle with a radius of $1.96\sigma_w / \sqrt{P_b}$, the adaptive filter $\mathbf{Y}_m$ varies. And the differential shows the error dynamics.

4. QUASI-PERIODIC CASE

4.1 Adaptive filter dynamics

In quasi-periodic case, the noise signal is quasi-periodic and there is no frequency error. So, the update algorithms can be expressed as follows.

$$Y_{m,c}[K+1] = \bar{D}_{m,c,CT}[K] + \tilde{D}_{m,c,CT}[K] + D_{m,c,NT}[K] + W_{m,c}[K] \tag{17}$$

Each part has a function of time, noise spectrum and how fast the coefficients varies. It means that the adaptive filter follows the Fourier coefficients of noise, but has some error.

4.2 Error dynamics

In the update algorithm, we can achieve error dynamics as follows.

(18)

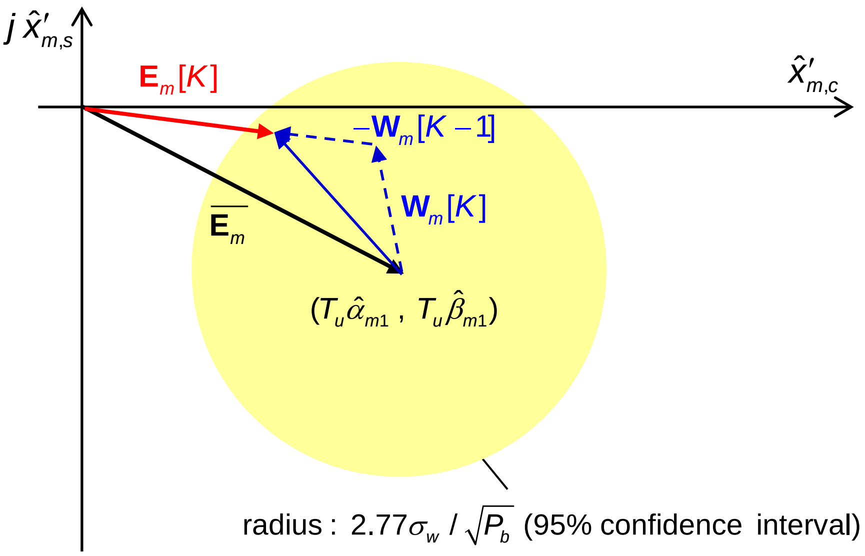

Figure 2 – The conceptual diagram of error dynamics in quasi-periodic case

$$\begin{aligned}
E_{m,c}[K] &= Y_{m,c}[K+1] - Y_{m,c}[K] \\
&= (\bar{D}_{m,c,CT}[K] + \tilde{D}_{m,c,CT}[K] + D_{m,c,NT}[K] + W_{m,c}[K]) \\
&\qquad - (\bar{D}_{m,c,CT}[K-1] + \tilde{D}_{m,c,CT}[K-1] + D_{m,c,NT}[K-1] + W_{m,c}[K-1]) \\
&= (\bar{D}_{m,c,CT}[K] - \bar{D}_{m,c,CT}[K-1]) \\
&\quad + (\tilde{D}_{m,c,CT}[K] - \tilde{D}_{m,c,CT}[K-1]) \\
&\quad + (D_{m,c,NT}[K] - D_{m,c,NT}[K]) \\
&\quad + (W_{m,c}[K] - W_{m,c}[K-1])
\end{aligned} \tag{19}$$

The error has four parts. The first part is differential of $\bar{D}_{m,c,CT}$. The mean of $D_{m,c}$ linearly varies, so this part is simplified as $T_u\hat{\alpha}_{m1}$. T_u is update period, and $\hat{\alpha}_{m1}$ is constant value which differs from α_{m1} due to secondary path effect. The second and third part are differential of $\tilde{D}_{m,c,CT}[K]$ and $D_{m,c,NT}[K]$. This parts of adaptive filter has some value, but those of error can be zero if the update period is a multiple of base period $T_b = 2\pi / \omega_1$. Then, the equation is simplified as follows.

$$E_{m,c}[K] = T_u\hat{\alpha}_{m1} + W_{m,c}[K] - W_{m,c}[K-1] \tag{20}$$

Likewise, the sine component is achieved as follows.

$$E_{m,s}[K] = T_u\hat{\beta}_{m1} + W_{m,s}[K] - W_{m,s}[K-1] \tag{21}$$

With these, we have a stochastic analysis as follows.

$$\begin{aligned}
&\text{mean : } \overline{\mathbf{E}_m[K]} = (T_u\hat{\alpha}_{m1} + jT_u\hat{\beta}_{m1}) + \overline{\mathbf{W}_m[K]} - \overline{\mathbf{W}_m[K-1]} = T_u(\hat{\alpha}_{m1} + j\hat{\beta}_{m1}) \\
&\text{variance : } \overline{\left|\mathbf{E}_m[K] - \overline{\mathbf{E}_m[K]}\right|^2} = \overline{\left|\mathbf{W}_m[K] - \mathbf{W}_m[K-1]\right|^2} = 2\sigma_w^2 / P_b
\end{aligned} \tag{22}$$

The mean is not zero but constant, and the variance is the same with ideal case. For each frequency component, we have error dynamics in Figure 2.

4.3 Total dynamics

For each frequency component, we can achieve total dynamics in Figure 3. The optimal vector $\mathbf{D}_m$ is linearly varying and the trace is parallel to the mean of $\mathbf{E}_m$. The adaptive filter chases the trace of $\mathbf{D}_m$ with some error which includes fixed error by $\tilde{D}_{m,c,CT}$, $D_{m,c,NT}$ and the differential of $\bar{D}_{m,c,CT}$ and random error by $\mathbf{W}_m$.

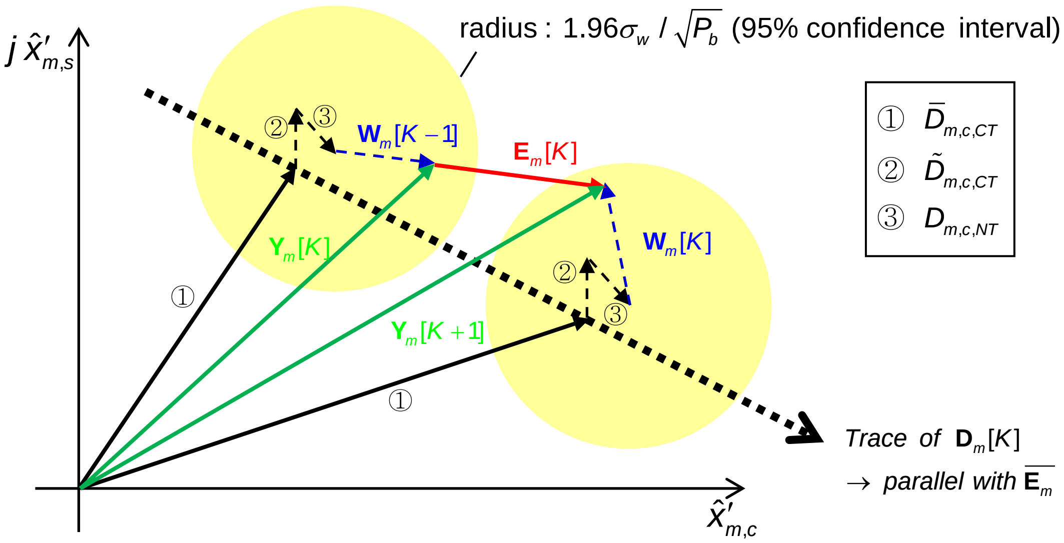

Figure 3 - The conceptual diagram of total dynamics in quasi-periodic case

5. CONCLUSIONS

We analyzed the property of frequency-domain active noise control algorithm in ideal and quasi-periodic case. In ideal case, the optimal vector is fixed and the estimation depends on only background noise. In quasi-periodic case, the optimal vector linearly varies, and the estimation chases it with fixed error and random error. The faster the coefficient of noise signal varies, the more error the estimation gets. But, we already analyze the dynamics and can estimate the fixed error using error signal. For this aspect, further research presently carried out with a view to compensate the error. Also, the error analysis for other cases will be carried out it.

ACKNOWLEDGEMENTS

This work was supported by the National Research Foundation of Korea(NRF) grant funded by the Korea government(MSIP) (No. 2010-0028680), the Ministry of Knowledge Economy grant funded by the Korea government (No. 10037244) and BK21 Plus project.

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