On the effect of shear and bias flow on the performance of acoustic liners

L. M. B. C. CAMPOS[1]; C. LEGENDRE[2]; C. SAMBUC[3]

[1] LAETA, (Centro de Ciências e Tecnologias Aeronáuticas e Espaciais – CCTAE), Instituto Superior Técnico (IST), Universidade de Lisboa, Portugal

[2] Free Field Technologies S. A., Anic Park Louvain-la-Neuve, 9 rue Emile Franqui, Mont-Saint-Guibert, Belgium

[3] Free Field Technologies S. A., Anic Park Louvain-la-Neuve, 9 rue Emile Franqui, Mont-Saint-Guibert, Belgium

ABSTRACT

The acoustic-vortical wave equation is derived describing the propagation of sound in (i) a uni-directional shear flow with a linear velocity profile upon which is superimposed (ii) a uniform cross flow; together with an impedance wall boundary condition representing the effect of a locally reacting acoustic liner in the presence of bias and shear flow. This leads to a third-order differential equation in the presence of cross flow, and in its absence simplifies to the Pridmore-brown equation (second-order); also the singularity of the Pridmore-brown equation for zero Doppler-shifted frequency is removed by the cross flow. Because the third-order wave equation has no singularities (except at the sonic condition), its general solution is a linear combination of three linearly independent MacLaurin series in powers of the distance from the wall. The acoustic field in the boundary layers is matched through the pressure and horizontal and vertical velocity components to the acoustic field in a uniform free stream consisting of incident and reflected waves. The scattering coefficients are plotted for several values of the five parameters of the problem, namely the angle of incidence, free stream and cross-flow Mach numbers, specific wall impedance and Helmholtz number using the boundary layer thickness.

Keywords: acoustic liners, engine nozzles, shear and bias flow.

1. INTRODUCTION

The air inlets and exhaust nozzles of jet engines make extensive use of liners to absorb or attenuate sound. A locally reacting acoustic liner can be represented by an impedance wall condition. In the case of a duct with mean flow, the boundary layer near the wall leads to interaction between acoustic and vortical modes that also affects the pressure perturbation and energy [1]. In addition, a perforated plate can have a bias flow, thus superimposing a cross-flow to the shear flow in the boundary layer. This paper addresses the combination of the three effects, namely (i) a plane flow over a flat impedance wall (figure 1); (ii) a boundary layer with a unidirectional shear flow consisting of a linear velocity profile matched to a uniform stream; and (iii) in addition, a uniform cross-flow representing the bias flow out of the perforated liner. The pressure perturbation in the free stream consists of incident and reflected plane waves; it must be matched to the pressure field in the boundary layer (§2) in order to apply the impedance boundary condition at the wall (§3); the latter specifies the reflection coefficient, and thus the wave pressure perturbation in the whole flow, inside and outside the boundary layer.

[1] luis.campos@ist.utl.pt
[2] Cesar.Legendre@fft.be
[3] Cristophe.sambuc@fft.be

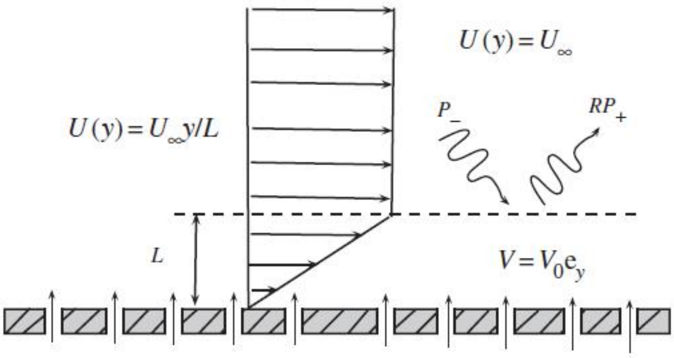

Figure 1. Sound propagation in a linear shear and cross layer profile.

2. LINEAR UNIDIRECTIONAL SHEAR WITH UNIFORM CROSS-FLOW

The fundamental equations of fluid mechanics are considered as (i) the continuity equation for the mass conservation:

$$\frac{D\tilde{\rho}}{Dt} + \tilde{\rho}\nabla \cdot \tilde{\mathbf{v}} = 0, \tag{2.1}$$

where ρ˜ is the mass density and v˜ is the velocity; (ii) the inviscid momentum equation:

$$\tilde{\rho}\frac{D\tilde{\mathbf{v}}}{Dt} + \nabla\tilde{p} = 0, \tag{2.2}$$

where ˜p is the pressure; and (iii) the adiabatic equation:

$$\frac{D\tilde{p}}{Dt} = \tilde{c}^2\frac{D\tilde{\rho}}{Dt}, \tag{2.3}$$

where ˜c is the adiabatic sound speed. In all three equations appears the material derivative:

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \tilde{\mathbf{v}} \cdot \nabla. \tag{2.4}$$

The tilde notation is used to denote the total flow variables, the subscript '0' the mean flow variables and neither is used for the perturbations. The mean flow is assumed to be plane and consists (figure 1) of: (i) a uniform bias or cross-flow orthogonal to a straightwall with velocity V0:

$$\mathbf{v}_0 = \mathbf{e}_y V_0 + \mathbf{e}_x U_0(y) \tag{2.5}$$

(ii) a unidirectional shear flow parallel to the wall with a linear velocity profile in a boundary layer of thickness L:

$$U_0(y) = \begin{cases} U_\infty \frac{y}{L}, & \text{if } 0 \le y \le L, \\ U_\infty, & \text{if } L \le y < +\infty, \end{cases} \tag{2.6a, b}$$

matched to a uniform stream with velocity U∞.

Denoting with a subscript '0' the mean flow quantities, the momentum equation (2.2) becomes

$$-\nabla p_0 = (\mathbf{v}_0 \cdot \nabla)\mathbf{v}_0 = \rho_0 \left[U_0(y)\frac{\partial}{\partial x} + V_0\frac{\partial}{\partial y}\right]\mathbf{v}_0. \tag{2.7}$$

Using equation (2.5), the y- and x-components of the momentum equation for the mean flow (2.2) are respectively

$$\frac{\partial p_0}{\partial y} = 0 \tag{2.8a}$$

and

$$-\frac{\partial p_0}{\partial x} = \rho_0 V_0 \frac{\mathrm{d}U_0}{\mathrm{d}y}. \tag{2.8b}$$

From (2.8a) follows that the mean flow pressure does not depend on the distance from the wall, but it does depend on the distance along the wall (2.8b) for a shear flow. For a shear flow with a linear velocity profile (2.9a) from (2.8b) follows (2.9b):

$$\frac{\mathrm{d}U_0}{\mathrm{d}y} = \text{const.}: \quad 0 = \frac{\mathrm{d}}{\mathrm{d}y}\left(V_0 \frac{\mathrm{d}U_0}{\mathrm{d}y}\right) = -\frac{\partial}{\partial y}\left(\frac{1}{\rho_0}\frac{\partial p_0}{\partial x}\right) = \frac{1}{\rho_0{}^2}\frac{\partial p_0}{\partial x}\frac{\partial \rho_0}{\partial y} \tag{2.9a,b}$$

showing that the mass density does not depend on the distance from the wall (2.10a); using (2.10b) in the continuity equation (2.1) for the mean flow (2.9a) shows that

$$\frac{\partial \rho_0}{\partial y} = 0; \tag{2.10a}$$

and

$$0 = \mathbf{v}_0 \cdot \nabla \rho_0 = U_0(y)\frac{\partial \rho_0}{\partial x} + V_0 \frac{\partial \rho_0}{\partial y} = U_0(y)\frac{\partial \rho_0}{\partial x}, \tag{2.10b}$$

where the mass density is constant (2.11a); because V0 and dU0/dy are constants (2.5), (2.9a) the two components (2.8a,b) of the momentum equation show that the mean flow pressure is a linear function of the distance along the wall (2.11b):

$$\rho_0 = \text{const.}; \tag{2.11a}$$

and

$$p_0(x) = p_0(0) - \rho_0 V_0 \frac{\mathrm{d}U_0}{\mathrm{d}y} x. \tag{2.11b}$$

Considering a limited length of the wall satisfying (2.12a), the mean flow pressure is approximately constant (2.12b):

$$x \ll \frac{p_0(0)}{(\rho_0 V_0 \,\mathrm{d}U_0/\mathrm{d}y)}: \quad p_0 = \text{const.} \tag{2.12a,b}$$

Under the same conditions, the mean flow is adiabatic (2.13a), and the sound speed is constant (2.13b):

$$\mathbf{v}_0 \cdot \nabla p_0 = c_0{}^2 \left(\mathbf{v}_0 \cdot \nabla \rho_0\right) = 0, \tag{2.13a}$$

and

$$c_0 = \text{const.} \tag{2.13b}$$

These conditions simplify the following (§2b) derivation of the acoustic-vortical wave equation in a linear unidirectional shear flow (2.6a,b) superimposed on a uniform cross-flow (2.5) for a limited wall distance (2.12a).

3. THIRD-ORDER ACOUSTIC-VORTICAL WAVE EQUATION

An unsteady, non-uniform perturbation of the mean flow (2.5) is considered

$$\tilde{u} = U_0(y) + u(x,y,t), \tag{3.1a}$$

$$\tilde{v} = V_0 + v(x,y,t), \tag{3.1b}$$

$$\tilde{p} = p_0 + p(x,y,t) \tag{3.1c}$$

and

$$\tilde{\rho} = \rho_0 + \rho(x,y,t), \tag{3.1d}$$

and the momentum equation (2.2) is linearized

$$\rho_0\left[\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}+(\mathbf{v}\cdot\nabla)\,\mathbf{v}_0\right]+\rho\,(\mathbf{v}_0\cdot\nabla)\,\mathbf{v}_0+\nabla p=0, \tag{3.2}$$

using the linearized material derivative:

$$\frac{\mathrm{d}}{\mathrm{d}t}\equiv\frac{\partial}{\partial t}+\mathbf{v}_0\cdot\nabla=\frac{\partial}{\partial t}+U_0(y)\frac{\partial}{\partial x}+V_0\frac{\partial}{\partial y}. \tag{3.3}$$

The adiabatic (2.3) and continuity (2.1) equations are combined

$$\frac{D\tilde{p}}{Dt}+\tilde{\rho}\tilde{c}^2\nabla\cdot\tilde{\mathbf{v}}=0, \tag{3.4}$$

and are linearized

$$\frac{\mathrm{d}p}{\mathrm{d}t}+\rho_0{c_0}^2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0. \tag{3.5}$$

The linearized material derivative (3.3) commutes (3.6a) with the x-component gradient, but not with the y-component gradient (3.6b):

$$\frac{\partial}{\partial x}\frac{\mathrm{d}}{\mathrm{d}t}=\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial}{\partial x} \tag{3.6a}$$

and

$$\frac{\partial}{\partial y}\frac{\mathrm{d}}{\mathrm{d}t}=\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial}{\partial y}+\frac{\mathrm{d}U_0}{\mathrm{d}y}\frac{\partial}{\partial x}. \tag{3.6b}$$

For example, applying d/dt to equation (3.5) gives

$$\frac{1}{{c_0}^2}\frac{\mathrm{d}^2p}{\mathrm{d}t^2}+\rho_0\frac{\partial}{\partial x}\left(\frac{\mathrm{d}u}{\mathrm{d}t}\right)+\rho_0\frac{\partial}{\partial y}\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)=\rho_0\frac{\mathrm{d}U_0}{\mathrm{d}y}\frac{\partial v}{\partial x}, \tag{3.7}$$

where were used the commutation relations (3.6a, b).

The x- and y-components of the momentum equation (3.2) are

$$\rho_0\frac{\mathrm{d}u}{\mathrm{d}t}+\rho_0 v\frac{\mathrm{d}U_0}{\mathrm{d}y}+\rho V_0\frac{\mathrm{d}U_0}{\mathrm{d}y}+\frac{\partial p}{\partial x}=0, \tag{3.3a}$$

and

$$\rho_0\frac{\mathrm{d}v}{\mathrm{d}t}+\frac{\partial p}{\partial y}=0; \tag{3.8b}$$

substitution in (3.7) gives

$$\frac{1}{{c_0}^2}\frac{\mathrm{d}^2p}{\mathrm{d}t^2}-\frac{\partial^2p}{\partial x^2}-\frac{\partial^2p}{\partial y^2}=2\rho_0\frac{\mathrm{d}U_0}{\mathrm{d}y}\frac{\partial v}{\partial x}+V_0\frac{\mathrm{d}U_0}{\mathrm{d}y}\frac{\partial\rho}{\partial x}. \tag{3.9}$$

The l.h.s of (3.9) is the convected wave equation for pressure which is valid when the r.h.s is zero, i.e. in the absence of shear flow. In the presence of shear flow in order to eliminate (v,ρ) from the r.h.s of (3.9) and obtain a wave equation for the pressure alone, d/dt is applied once more leading to

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{{c_0}^2}\frac{\mathrm{d}^2p}{\mathrm{d}t^2}-\nabla^2p\right)=2\rho_0\frac{\mathrm{d}U_0}{\mathrm{d}y}\frac{\partial}{\partial x}\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)+2\rho_0V_0\frac{\mathrm{d}^2U_0}{\mathrm{d}y^2}\frac{\partial v}{\partial x}+V_0\frac{\mathrm{d}U_0}{\mathrm{d}y}\frac{\partial}{\partial x}\left(\frac{\mathrm{d}\rho}{\mathrm{d}t}\right)+{V_0}^2\frac{d^2U_0}{\mathrm{d}y^2}\frac{\partial\rho}{\partial x}. \tag{3.10}$$

In the case (2.9a) of a linear shear flow (3.11a) two terms on the r.h.s of (3.10) vanish, and the linearized adiabatic equation (3.11b):

$$\frac{\mathrm{d}^2U_0}{\mathrm{d}y^2}=0 \tag{3.11a}$$

and

$$\frac{\mathrm{d}p}{\mathrm{d}t} = {c_0}^2 \frac{\mathrm{d}\rho}{\mathrm{d}t}, \tag{3.11b}$$

together with (3.8b) substituted in (3.10) give

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{{c_0}^2}\frac{\mathrm{d}^2 p}{\mathrm{d}t^2} - \nabla^2 p\right) + \frac{\mathrm{d}U_0}{\mathrm{d}y}\left[2\frac{\partial^2 p}{\partial x \partial y} - \frac{V_0}{{c_0}^2}\frac{\partial}{\partial x}\left(\frac{\mathrm{d}p}{\mathrm{d}t}\right)\right] = 0. \tag{3.12}$$

The first three terms on the l.h.s of (3.12) specify the acoustic-vortical wave equation in a unidirectional shear flow [2–5,18,19] in the absence of cross-flow. The generalization to include a uniform cross-flow adds the fourth term on the l.h.s of (3.12) and restricts the shear flow to the linear velocity profile (3.11a).

4. PRESSURE PERTURBATION INSIDE THE BOUNDARY LAYER

Because the mean flow is steady and uniform in the wall direction, a Fourier integral representation exists

$$p(x, y, t) = P(y; k, \omega)\, \mathrm{e}^{\mathrm{i}(kx - \omega t)}, \tag{4.1}$$

where P is the pressure perturbation spectrum for a wave of frequency ω and horizontal wavenumber k at the distance y from the wall. The linearized material derivative (3.3) leads to (4.2a)

$$\frac{\mathrm{d}}{\mathrm{d}t} = V_0 \frac{\mathrm{d}}{\mathrm{d}y} - \mathrm{i}\omega_*(y) \tag{4.2a}$$

and

$$\omega_*(y) = \omega - kU_0(y), \tag{4.2b}$$

where ω_* is the Doppler-shifted frequency calculated for the shear flow alone. Substitution of (4.1) and (4.2b) in the acoustic-vortical wave equation (3.12) leads to the dependence of the pressure perturbation spectrum on the distance from the wall:

$$\begin{aligned} &V_0({V_0}^2 - {c_0}^2)P''' + \mathrm{i}\omega_*({c_0}^2 - 3{V_0}^2)P'' + [2\mathrm{i}kU_0'({c_0}^2 + {V_0}^2) - 3V_0{\omega_*}^2 + V_0{c_0}^2k^2]P' \\ &+ \mathrm{i}\omega_*({\omega_*}^2 - k^2{c_0}^2 - 2\mathrm{i}kV_0U_0')P = 0. \end{aligned} \tag{4.3}$$

Because the acoustic-vortical wave equation (3.12) is of the third order, (4.3) is (i) a cubic dispersion relation in the frequency ω and horizontal wavenumber k, retaining and (ii) the dependence on the distance from the wall as a third-order differential equation. In the absence of the cross-flow (4.4a), the differential equation (3.12) reduces to the second order

$$V_0 = 0: \quad \omega_* P'' + 2kU_0'P' + \omega_*\left[\left(\frac{\omega_*}{c_0}\right)^2 - k^2\right]P = 0. \tag{4.4a, b}$$

Equation (4.4b) has a singularity when the coefficient of P'', that is the Doppler-shifted frequency (2.27b) vanishes. To interpret this physically, consider (figure 2a) a wave of frequency

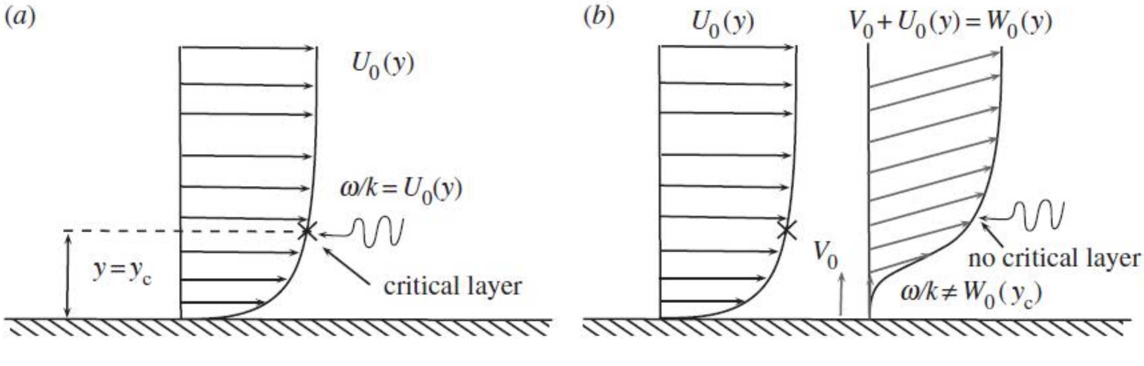

Figure 2. Graphical explanation of the existence of the critical layer. (*a*) Acoustic propagation in pure shear flow. (*b*) Acoustic propagation in shear and cross-flow.

ω and horizontal wavenumber k, hence horizontal phase speed $w = \omega/k$ propagating against a unidirectional shear flow. At the critical layer when the horizontal phase speed equals the mean flow velocity (4.5a), the Doppler-shifted frequency vanishes (4.5b)

$$\frac{\omega}{k} = U(y_c) \quad \Longleftrightarrow \quad \omega_*(y_c) = \omega - kU_0(y_c) = 0. \tag{4.5a, b}$$

The wave can propagate no further, and the wave equation (4.4*b*) has a singularity, implying one of the following possibilities: (i) the wave becomes evanescent beyond the critical layer that acts as a total reflector; and (ii) the wave is partly absorbed, partly reflected and partly transmitted as another mode able to propagate beyond the critical layer. In all cases, the critical layer occurs at the point where the wave is 'stopped' by the mean flow. The sonic condition in a potential flow is also a singularity of the acoustic wave equation. The difference is that sound waves in a potential flow are non-dispersive and the critical layer occurs at the sonic condition that is the same for all wavenumbers. In the shear flow, the critical layer occurs at different positions for fixed frequency ω and varying wavenumber k (or vice versa) so that the forbidden values form a continuous spectrum. Applying a cross-flow (figure 2*b*), this convects the wave away from the critical layer and removes the singularity. Thus, the acoustic-vortical wave equation with cross-flow (3.12) has no singularity, that is the coefficient of the third-order derivative does not vanish, except if the cross-flow reaches the sonic condition. The acoustic-vortical wave equation in a unidirectional shear flow is simpler in the presence of cross-flow, in the sense that it has no singularity, so it has solution as Taylor series valid in the whole flow region.

The acoustic-vortical wave equation (3.12) is applied inside the boundary layer (2.6a), and the distance from the wall is normalized to its thickness (4.6a) leading for the pressure perturbation spectrum (4.4b):

$$z = \frac{y}{L}, \tag{4.6a}$$

and

$$P(y; k, \omega) = Q(z), \tag{4.6b}$$

to the third-order differential equation:

$$M_0(M_0{}^2 - 1)Q''' + \mathrm{i}\bar{\omega}_*(1 - 3M_0{}^2)Q'' + [2\mathrm{i}\varepsilon M_\infty(1 + M_0{}^2) - 3M_0\bar{\omega}_*{}^2 + M_0\varepsilon^2]Q' + \mathrm{i}\bar{\omega}_*(\bar{\omega}_*{}^2 - \varepsilon^2 - 2\mathrm{i}\varepsilon M_0 M_\infty)Q = 0, \tag{4.7}$$

that involves three independent dimensionless parameters, namely (i) the cross-flow Mach number (4.8a); (ii) the free-stream Mach number (4.8b); (iii) the dimensionless frequency or Helmholtz number (4.8c) comparing the thickness of the boundary layer L to the wavelength λ0 in an homogeneous medium at the rest:

$$M_0 \equiv \frac{V_0}{c_0}, \tag{4.8a}$$

$$M_\infty \equiv \frac{U_\infty}{c_0} \tag{4.8b}$$

and

$$\Omega \equiv \frac{\omega L}{c_0} = \frac{2\pi L}{\lambda_0}. \tag{4.8c}$$

Thus, $\Omega \gg 1$ for sound rays in the boundary layer, $\Omega \ll 1$ for acoustically thin boundary layer and $\Omega \sim 1$ in the more interesting case of wavelength compared with the thickness of the boundary layer. In (4.7) appear another two dimensionless coefficients, namely (i) the horizontal compactness defined as the product of the horizontal wavenumber (4.9a) by the thickness of the boundary layer (4.9b) that depends on the angle of incidence.

$$k = \frac{\omega}{c_0}\cos\theta : \quad \varepsilon \equiv kL = \frac{\omega L}{c_0}\cos\theta = \Omega\cos\theta, \tag{4.9a,b}$$

(ii) the dimensionless (4.7c) Doppler-shifted frequency (4.2b):

$$\bar{\omega}_* \equiv \frac{\omega_* L}{c_0} = \frac{L}{c_0}[\omega - kU_0(y)] = \frac{\omega L}{c_0} - \frac{kU_\infty y}{c_0} = \Omega - \varepsilon M_\infty z. \tag{4.9c}$$

The angle θ is measured from the horizontal in the direction of the core flow. The word 'horizontal' means parallel to the wall or along the core flow (2.6a,b); in the presence of crossflow (2.5), the total velocity is oblique to the wall and streamwise is not 'horizontal'. Similarly, the vertical direction is perpendicular to the wall not perpendicular to the stream. Substituting (4.9c) in (4.7), it follows that the pressure perturbation spectrum in the boundary layer satisfies a third-order differential equation with polynomial coefficients:

$$\begin{aligned} &M_0({M_0}^2-1)Q''' + \mathrm{i}(1-3{M_0}^2)(\Omega - M_\infty \varepsilon z)Q'' \\ &\quad + [2\mathrm{i}\varepsilon M_\infty(1+{M_0}^2) + {M_0}^2\varepsilon - 3M_0\Omega^2 + 6\varepsilon M_0 M_\infty z - 3M_0\varepsilon^2 {M_\infty}^2 z^2]Q' \\ &\quad + \mathrm{i}(\Omega - \varepsilon M_\infty z)(\Omega^2 - \varepsilon^2 - 2\mathrm{i}\varepsilon M_0 M_\infty - 2\varepsilon\Omega M_\infty z + \varepsilon^2 {M_\infty}^2 z^2)Q = 0. \end{aligned} \tag{4.10}$$

The coefficient of the highest order derivative is a non-zero constant except for a bias flow at sonic speed; excluding this case, the differential equation (4.10) has no singularities, and the solution exists as a MacLaurin series (4.11b) with infinite radius of convergence (4.11a):

$$z < \infty: \quad Q(z) = \sum_{n=0}^{\infty} a_n z^n. \tag{4.11a, b}$$

Substitution of (4.11b) in (4.12) leads to the recurrence formula for the coefficients:

$$\begin{aligned} &M_0({M_0}^2-1)n(n-1)(n-2)a_n + \mathrm{i}\Omega(1-3{M_0}^2)(n-1)(n-2)a_{n-1} \\ &\quad - [\mathrm{i}(n-1)\varepsilon M_\infty(1-3{M_0}^2) - 2\mathrm{i}\varepsilon M_\infty(1+{M_0}^2) - {M_0}^2\varepsilon + 3M_0\Omega^2](n-2)a_{n-2} \\ &\quad + [6\epsilon M_0 M_\infty(n-3) + \mathrm{i}\Omega(\Omega^2 - \varepsilon^2 - 2\mathrm{i}\epsilon M_0 M_\infty)]a_{n-3} \\ &\quad - [3M_0\varepsilon^2 {M_\infty}^2(n-4) + \mathrm{i}\varepsilon M_\infty(3\Omega^2 - \varepsilon^2 - 2\mathrm{i}\varepsilon M_0 M_\infty)]a_{n-4} \\ &\quad + \mathrm{i}3\Omega\varepsilon^2 {M_\infty}^2 a_{n-5} - \mathrm{i}\Omega\varepsilon^3 {M_\infty}^3 a_{n-6} = 0. \end{aligned} \tag{4.12}$$

The MacLaurin series (4.11b) with coefficients satisfying (4.12) specifies the pressure perturbation spectrum in the boundary layer to be matched to the acoustic field in the free stream.

5. CONCLUSION

The pressure field inside the shear layer is matched to incident plane waves in the free stream outside. The effect of the angle of incidence θ = 30º, 60º, 90º, 120º, 150º is shown (Figure 2) on the modulus (left) and phase (right) of the pressure as a function of the distance from the wall (divided as the thickness of the boundary layer). The bias flow has a small mach number M_0 = 0.06 and the case of no bias flow M_0 = 0 is included for comparison at θ = 90º. It is seen that for normal incidence θ = 90º there is a large difference in pressure amplitude (left) between the presence (solid line) and absence (dotted line) of bias flow, with the pressure decreasing away from the wall in the presence of bias flowand instead increasing in the absence of bias flow; the effect on the phase (right) is smaller as it increases away from the wall both in the presence and absence of bias flow, but faster in the latter case. The phase variation (right) is larger for longer angles of incidence, in particular for downstream propagation θ > 90º. The amplitude of the pressure (left) decreases away from the wall, more slowly for upstream propagation θ > 90º and faster for downstream propagation θ > 90º although it can have inversion due to interaction with the shear and bias flows. The effect on sound pressure of other parameters, like frequency, wall impedance and free stream and bias flow Mach number can be analyzed similary [1]. The present paper extends to bias flow the propagation of sound in a shear flow with linear velocity profile [2], that has also been considered in the presence of temperature gradients [3]. Other shear velocity profiles have been considered, including an exponential boundary layer [4], hyperbolic tangent shear layer [5] and parabolic shear flow in a duct [6]. Besides the propagation of sound in shear flows the sound generation has been considered for sourced outside [7] or inside [8] boundary layer.

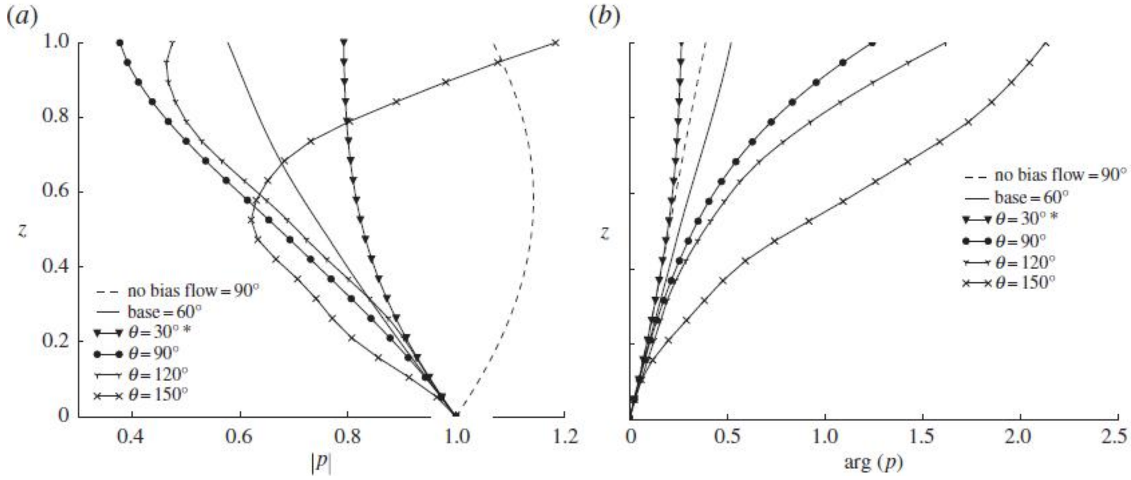

Figure 3. For five values of the angle of incidence is plotted the (a) modulus and (b) phase of acoustic pressure versus dimensionless distance from the wall (4.4a) the pressure is also normalized to the wall value. The free stream Mach number (4.8b) is M_∞= 1.2, the bias flow Mach number (4.8a) is M_0= 0.05 and the dimensionless frequency (4.8a) is Ω = 1. The wall specific impedance is Z = 1 + i.

REFERENCES

1. Campos LMBC, Legendre C. and Sambuc C. On the acoustics of an impedance liner with shear and cross flow. *Proc. Roy. Soc.* 2014, A20130732.
2. Campos LMBC, Oliveira JMGS, Kobayashi MH. 1999 On sound propagation in a linear shear flow. *J. Sound Vib.* **95**, 739–770.
3. Campos LMBC, Kobayashi MH. 2007 On sound propagation in a high-speed non-isothermal shear flow. *Inst. J. Aeroacoust.* **6**, 93–126.
4. Campos LMBC, Serrão PGTA. 1998 On the acoustics of an exponential boundary layer. *Phil. Trans. R. Lond. Soc. A* **356**, 2335–2379.
5. Campos LMBC, Kobayashi MH. 2000 On the reflection and transmission of sound in a thick shear layer. *J. Fluid Mech.* **424**, 303–326.
6. Campos LMBC, Oliveira JMGS. 2011 On the acoustic modes in a duct containing a parabolic shear flow. *J. Sound Vib.* **330–336**, 1166–1195.
7. Campos LMBC, Kobayashi MH. 2010 On sound transmission from a source outside a non-isothermal boundary layer. *AIAA J.* **48**, 878–892.
8. Campos, LMBC and Kobayashi, MH. 2014 On sound emission by sources in a shear flow. *Int. J. Aeroac. 2014*, **12**, 719–742.