



Analysis on propulsion shafting coupled torsional-longitudinal vibration under different applied loads

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ABSTRACT

With the improvement of the requirements of large ships to ship economy, the coupling dynamics problem of the ship propulsion shafting is particularly prominent. Theoretical method for propulsion shafting coupled vibration needs to be solved to improve the reliability of the propulsion shafting and raise the sailing performance of the ship. This paper mainly studies on the coupled torsional-longitudinal dynamics analytical theory and numerical simulation of the propulsion shafting. Different kinds of incentives can cause different kinds of coupling responses. The changes of displacement and rotation angle caused by coupled torsional-longitudinal vibration of the propulsion shafting is studied by the finite element method. Through comprehensive analysis for the coupling response of the propulsion shafting under the condition of different applied loads individually and simultaneously, the research reveals the dynamics mechanism of coupled torsional-longitudinal vibration and puts forward the theoretical guidance for coupled torsional-longitudinal vibration of the propulsion shafting.

Keywords: Dynamic analysis, Coupled vibration, Propulsion shafting

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1. INTRODUCTION

In the process of the development of shipping industry, the broken shaft accidents caused by propulsion shafting constant torsion prompted the research of shafting vibration^[1]. Holzer put forward the application on calculation of the free torsional vibration and promoted the development of the torsional vibration numerical calculation in 1907. Morley proposed a reduced order model method to control the longitudinal vibration of shafting system in 1910. Dunkerley used modal parameter identification method and modal synthesis to analyze the whirl vibration of shafting system in 1894. Although these scholars studied the vibration of shafting propulsion without considering the influence of the coupling interaction, these theory and methods provided the basis for the study of propulsion shafting coupled vibration later.

The coupling vibration of propulsion shafting is a frontier topic in the study of ship research in recent years. The marine main engine acts as the heart of the ship and plays an important role in the safety and reliability of vessel operating. The propulsion shafting which is the connection between marine main engine and propeller, it transmits the torque generated by the marine main engine to the propeller to promote the operation of the ship. The pulsation of the cycle work in marine main engine and the non-uniformity of propeller working area lead to the exciting force on propulsion shafting is extremely complex. Shafting vibration will lead to the vibration of the hull girder and superstructure, which directly affects the performance of the safety of navigation. The compatibility of hull and propulsion shafting needs to be considered on the basis of the analysis of propulsion shafting structure characteristics^[2].

The vibration forms of propulsion shafting contains torsional vibration, longitudinal vibration, transverse vibration, whirling vibration and their coupled vibration forms^[3-5]. Shafting system suffers the effect by all kinds of different shock and cyclic exciting force in actual operation. Due to the main exciting force on vibration of propulsion shafting are various, the coupling vibration types of

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propulsion shafting caused by the excitation force are also different. Deformation and vibration will be produced under the action of excitation. Each point on the shafting generates a response not only in the direction of the disturbing force but also in the other directions. The mechanics basic for coupled torsional-longitudinal vibration of the propulsion shafting is that linear displacement coupled with angular distortion at the same time.

In the study of coupled torsional-longitudinal vibration for propulsion shafting, M G Parsons [6] started the research early, the research analyzed the coupled torsional and axial vibration caused by propeller. They concluded that the coupled vibration of the propulsion shafting contains inertia coupled vibration and damping coupled vibration based on the research of the propeller added with mass and damping. S M Han [7] studied the longitudinal and transverse coupling vibration of beam structure. They deduced the equations of motion and boundary conditions of nonlinear coupled equations by Hamilton’s variational principle and solved the non-linear coupled partial differential equations through finite difference approach. Y Wang [8] analyzed the response of coupled torsional-axial vibration of propulsion shafting combined with seven measured curve. The research showed that the coupling response trend to increase when the critical speed of the torsional and axial vibration close or coincide. H T Zhang [9] considered the diesel engine by equivalent coupling stiffness, the propeller by coupled coefficient of equivalent acceleration with water weight and coupled coefficient of equivalent velocity with water damping. The research established the system matrix method for propulsion shafting longitudinal and torsional coupled vibration based on discrete system model.

In this paper, the dynamic equations of coupled torsional-longitudinal vibration are presented respectively. Then the finite element harmonic and transient analysis of a coupled propulsion shafting system is carried out in ANSYS. The result shows different kinds of coupled vibration performance of the propulsion shafting with different kinds of exciting force.

2. DESCRIPTION OF THE SYSTEM

The propulsion shafting can equivalent to a cantilever beam modeled as a Timoshenko beam with a mass payload that its mass center need not be coincident with the centerline of the beam [10-12]. Then the motion equations and corresponding boundary conditions of the system are derived using Hamilton’s principle. The length of the shafting is denoted by L , A denotes the cross-sectional area, I denotes the second moment area about the bending axis. The Young’s modulus of the material from which the propulsion shafting is constructed are respectively denoted by E . The schematic of propulsion shafting with a mass point is depicted in Figure 1. The main physical dimensions of propulsion shafting are shown in Table 1.

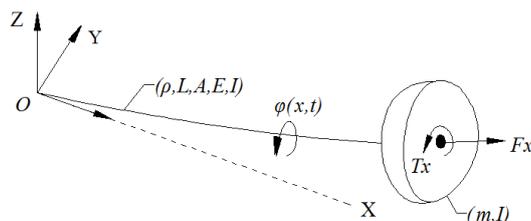


Figure.1 –The schematic of propulsion shafting

Table 1 – The values of structure parameters

Structure parameters	Values
Elasticity modulus (GPa)	2.1
Poisson ratio	0.3
Density (Kg/m3)	7850
Length (m)	54.88
Rotation speed (rpm)	110

This paper establishes the finite element model of the propulsion shafting with solid unit through ANSYS. The propeller and its surrounding waters are simulated by mass units. The displacement and rotation constraints are imposed to the bearing of this model. The coupled constraint is applied to the propeller of this model. The rotation speed of the shaft system is defined as 110rpm. The finite element model of propulsion shafting is shown in Figure 2.

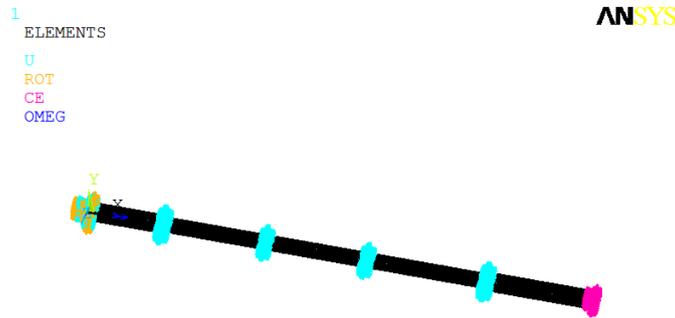


Figure.2 – The finite element model of propulsion shafting

3. EQUATION OF MOTION

The coupled torsional-longitudinal vibration for propulsion shafting has not yet form a kind of calculation method. Some scholars followed the law to force the disposal methods for longitudinal vibration and constructed an equivalent longitudinal force. According to the equal principle of the longitudinal displacement, the amplitude of equivalent virtual force given by the type F_e [13]:

$$F_e = K_x \cdot \Delta_x = K_x K_{XT} \varphi_T$$

where, Δ_x is the axial shrinkage deformation produced by torque T ; φ_T is the relative crank angle produced by torque T ; K_{XT} is the torsional-longitudinal coupling stiffness; K_x is the axial stiffness.

For the coupled axial force caused by torsional vibration, the torsional deformation will cause the axial shrinkage either forward or reverse. Changes of axial deformation does not vary with the changes of the distortion direction. The deformation of axial direction changes over the direction of the distortion. The coupled restoring force is $K_{XT}\theta$ when $\theta > 0$. It changes to $-K_{XT}\theta$ when $\theta < 0$. And it is zero when $\theta = 0$. The coupled axial restoring force F_{XT} caused by torsional vibration is expressed as:

$$F_{XT} = Sgn(\theta) K_{XT} \cdot \theta$$

where, $Sgn(\theta)$ is a symbol function. The mathematic model of the coupled free vibration can be expressed as:

$$\begin{cases} I\ddot{\theta} + K_T\theta - Sgn(\theta) K_{TX} \cdot X = 0 \\ m\ddot{X} + K_X X - Sgn(\theta) K_{XT}\theta = 0 \end{cases}$$

According to Maxwell's theory, the equation can be written as matrix form. There appeared a non-linear stiffness. Two natural frequencies ω_{r1} , ω_{r2} and the modal matrix ϕ of the coupled system can be obtained according to the basic theory of free vibration.

$$\omega_{r1,2}^2 = \frac{1}{2} \left(\frac{K_T}{I} + \frac{K_X}{m} \pm \sqrt{\left(\frac{K_T}{I} - \frac{K_X}{m} \right)^2 + \frac{4k^2}{mI}} \right)$$

$$[\phi] = \begin{bmatrix} 1 & 1 \\ DI / Sgn(\theta)k & -Sgn(\theta)k / Dm \end{bmatrix}$$

where, $D = \frac{1}{2} \left(\frac{K_T}{I} - \frac{K_X}{m} - \sqrt{\left(\frac{K_T}{I} - \frac{K_X}{m} \right)^2 + \frac{4k^2}{mI}} \right)$

The frequency $\omega_{r1}^2 = K_T / I$ and $\omega_{r2}^2 = K_X / m$ when the $k = 0$. These represent the natural frequency of torsional vibration and longitudinal vibration without coupling respectively.

3.1 Longitudinal force is zero, only the exciting torque

Supposing exciting torque is $T_x \sin \omega t$ and longitudinal excitation is zero, the motion equation is:

$$\begin{bmatrix} I & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{\theta} \\ \ddot{X} \end{Bmatrix} + \begin{bmatrix} K_T & -Sgn(\theta)k \\ -Sgn(\theta)k & K_x \end{bmatrix} \begin{Bmatrix} \theta \\ X \end{Bmatrix} = \begin{Bmatrix} T_x \sin \omega t \\ 0 \end{Bmatrix}$$

The steady state solutions caused by exciting torque is:

$$\begin{Bmatrix} \theta \\ X \end{Bmatrix} = \begin{Bmatrix} \frac{T_x \sin \omega t}{Ik^2 + mL^2 D^2} \left[\frac{k^2}{\omega_{01}^2 - \omega^2} + \frac{mID^2}{\omega_{02}^2 - \omega^2} \right] \\ Sgn(\theta) \frac{kDT_x \sin \omega t}{mID^2 + k^2} \left[\frac{1}{\omega_{01}^2 - \omega^2} - \frac{1}{\omega_{02}^2 - \omega^2} \right] \end{Bmatrix}.$$

3.2 Exciting torque is zero, only the longitudinal force

Supposing longitudinal force is $F_x \sin \omega t$ and exciting torque is zero, the motion equation is:

$$\begin{bmatrix} I & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{\theta} \\ \ddot{X} \end{Bmatrix} + \begin{bmatrix} K_T & -Sgn(\theta)k \\ -Sgn(\theta)k & K_x \end{bmatrix} \begin{Bmatrix} \theta \\ X \end{Bmatrix} = \begin{Bmatrix} 0 \\ F_x \sin \omega t \end{Bmatrix}$$

The steady state solutions caused by longitudinal force is:

$$\begin{Bmatrix} \theta \\ X \end{Bmatrix} = \begin{Bmatrix} Sgn(\theta) \frac{kDF_x \sin \omega t}{mID^2 + k^2} \left[\frac{1}{\omega_{01}^2 - \omega^2} - \frac{1}{\omega_{02}^2 - \omega^2} \right] \\ \frac{F_x \sin \omega t}{mk^2 + Im^2 D^2} \left[\frac{mID^2}{\omega_{01}^2 - \omega^2} + \frac{k^2}{\omega_{02}^2 - \omega^2} \right] \end{Bmatrix}$$

3.3 Longitudinal force and exciting torque at the same time

Supposing exciting torque is $T_x \sin \omega_2 t$ and longitudinal force is $F_x \sin \omega_1 t$, the motion equation is:

$$\begin{bmatrix} I & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{\theta} \\ \ddot{X} \end{Bmatrix} + \begin{bmatrix} K_T & -Sgn(\theta)k \\ -Sgn(\theta)k & K_x \end{bmatrix} \begin{Bmatrix} \theta \\ X \end{Bmatrix} = \begin{Bmatrix} T_x \sin \omega_2 t \\ F_x \sin \omega_1 t \end{Bmatrix}$$

According to the equation above, its steady state solution can be obtained:

$$\begin{Bmatrix} \theta \\ X \end{Bmatrix} = \begin{Bmatrix} \frac{T_x \sin \omega_2 t}{Ik^2 + mL^2 D^2} \left[\frac{k^2}{\omega_{01}^2 - \omega_2^2} + \frac{mID^2}{\omega_{02}^2 - \omega_2^2} \right] + Sgn(\theta) \frac{kDF_x \sin \omega_1 t}{mID^2 + k^2} \left[\frac{1}{\omega_{01}^2 - \omega_1^2} - \frac{1}{\omega_{02}^2 - \omega_1^2} \right] \\ Sgn(\theta) \frac{kDT_x \sin \omega_2 t}{mID^2 + k^2} \left[\frac{1}{\omega_{01}^2 - \omega_2^2} - \frac{1}{\omega_{02}^2 - \omega_2^2} \right] + \frac{F_x \sin \omega_1 t}{mk^2 + Im^2 D^2} \left[\frac{mID^2}{\omega_{01}^2 - \omega_1^2} + \frac{k^2}{\omega_{02}^2 - \omega_1^2} \right] \end{Bmatrix}$$

It can be seen that the coupled torsional-longitudinal response is the linear superposition of torsional response and longitudinal response at the same time.

4. DYNAMIC ANALYSIS

The dynamic analysis of coupled torsional-longitudinal vibration for propulsion shafting contains harmonic response and transient response. Harmonic response analysis is used to determine the steady-state response of linear structure under the load changing with time. It aims to calculate the frequency curve of structures under several frequencies. So that designers can predict the dynamic characteristics of structures and fatigue harmful effects caused by forced vibration. Transient response analysis is used to determine the displacement or stress response of each point on structure under the load changing with time. It aims to calculate the displacement or stress curve of structures under several times. So that designers can predict the ultimate displacement or stress of structures and avoid failure destruction.

This paper study the coupling response of the propulsion shafting under the condition of different applied loads individually and simultaneously. First, the torsional response and coupled longitudinal response are analyzed under exciting torque. Then, the longitudinal response and coupled torsional response are studied under longitudinal force. Finally, the torsional-longitudinal response under exciting torque and longitudinal force are analyzed.

4.1 Response to exciting torque

Supposing the exciting torque $T_x \sin \omega t = 1E4 \cdot \sin 100t \text{ N} \cdot \text{m}$. The results in figure 3 is the torsional response under exciting torque. The critical frequency is 6.9Hz and the ultimate rotation angle is $1.2E-5\text{rad}$ in torsional vibration.

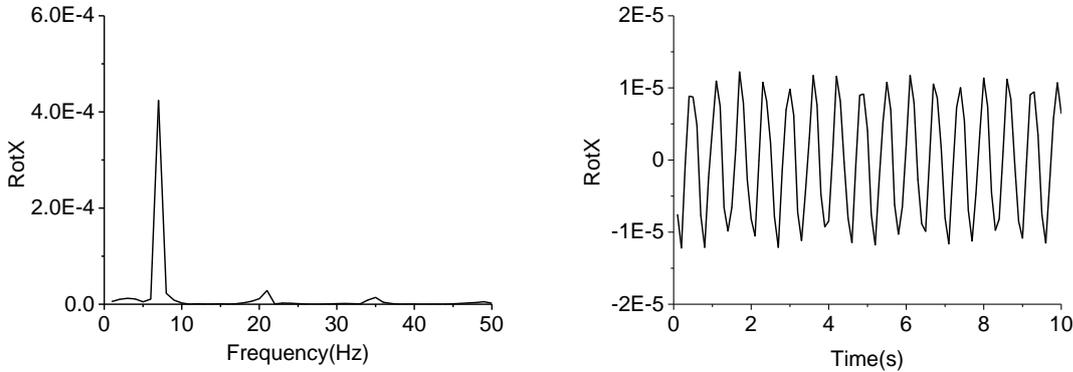


Figure.3 – Torsional harmonic and transient response under exciting torque

The results in figure 4 is the coupled longitudinal response under exciting torque. It shows exciting torque can excite coupled longitudinal displacement response. The critical frequency is 6.9Hz and the ultimate displacement is $5.5E-9\text{m}$ in coupled longitudinal vibration.

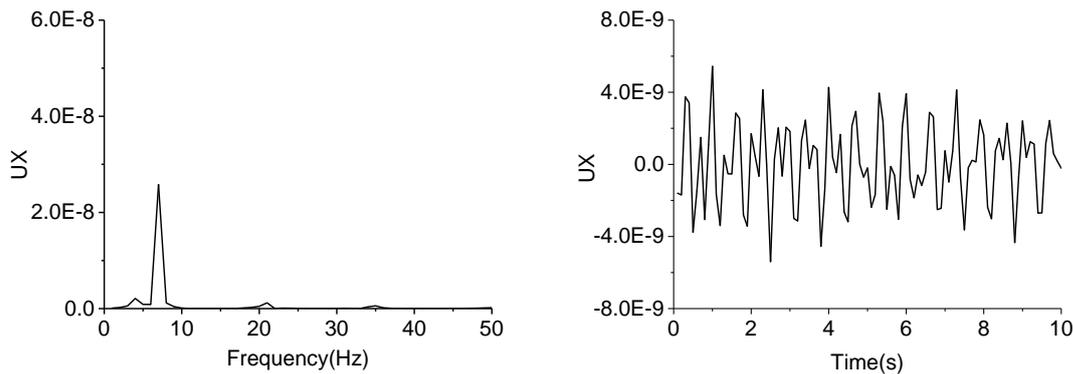


Figure.4 – Longitudinal harmonic and transient response under exciting torque

4.2 Response to longitudinal force

Supposing the longitudinal force $F_x \sin \omega t = 1E4 \cdot \sin 100t \text{ N}$. The results in figure 5 is the longitudinal response under longitudinal force. The critical frequency is 4.4Hz and the ultimate displacement is $2.0E-7\text{m}$ in longitudinal vibration.

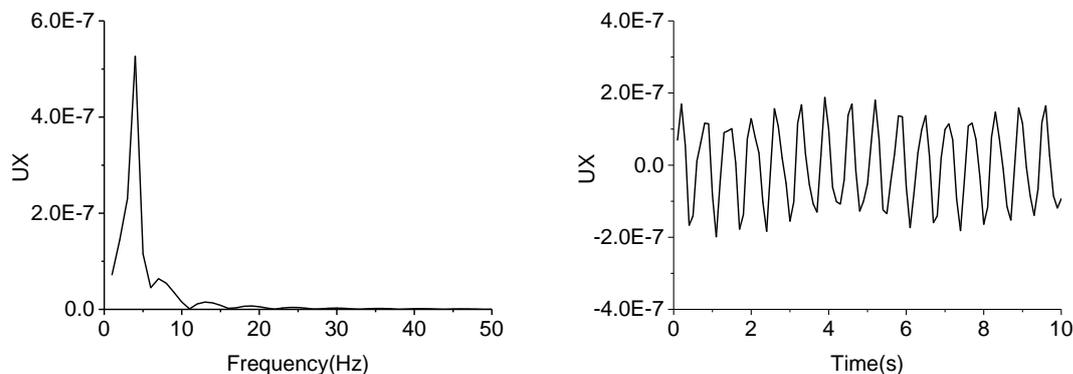


Figure.5 – Longitudinal harmonic and transient response under excitation force

The results in figure 6 is the coupled torsional response under longitudinal force. It shows longitudinal force can excite coupled rotation angle response. The critical frequency is 6.9Hz and the ultimate rotation angle is 5.4E-9rad in coupled torsional vibration.

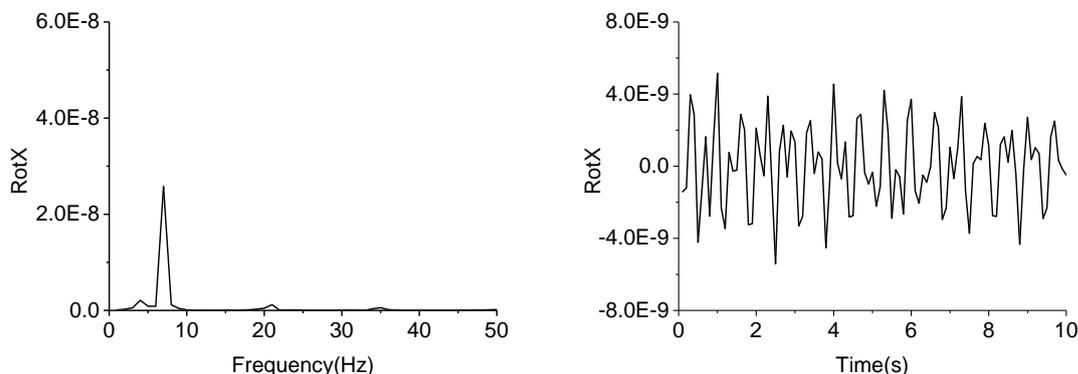


Figure.6 – Torsional harmonic and transient response under excitation force

4.3 Response to exciting torque and longitudinal force

Supposing the exciting torque $T_x \sin \omega_2 t = 1E4 \cdot \sin 100t \text{ N} \cdot \text{m}$ and axial force $F_x \sin \omega_1 t = 1E4 \cdot \sin 100t \text{ N}$. The results in figure 7 is the torsional response under exciting torque and longitudinal force. The critical frequency is 6.9Hz and the ultimate rotation angle is 1.2E-5rad.

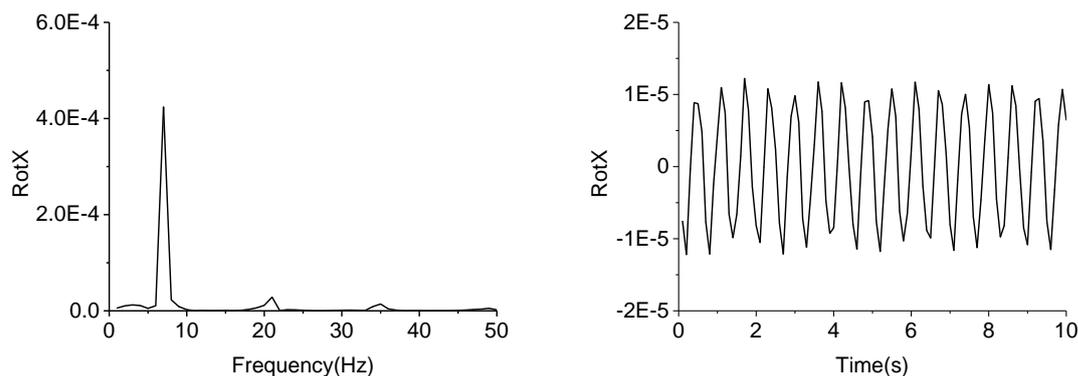


Figure.7 – Torsional harmonic and transient response under exciting torque and force

The results in figure is the longitudinal response under exciting torque and longitudinal force. The critical frequency is 4.4Hz and the ultimate displacement is 2.0E-7m.

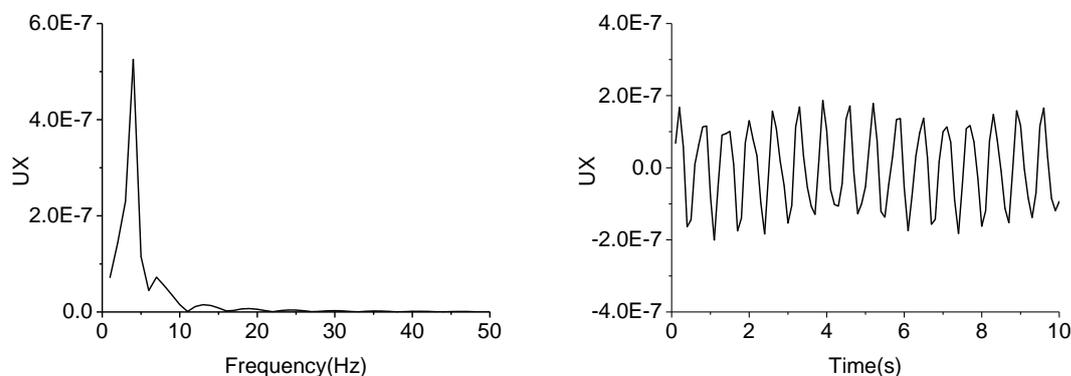


Figure.8 – Longitudinal harmonic and transient response under exciting torque and force

5. CONCLUSIONS

The dynamic behaviors of coupled torsional-longitudinal vibration for propulsion shafting subjected to exciting torque and longitudinal force are studied utilizing the finite element method. The dynamic response spectra orbits of displacement and frequency response for the propulsion shafting with individual and coupled incentives respectively are discussed in detail. Some conclusions are obtained as follows:

- 1) The simulation results show that exciting torque can excite coupled the longitudinal displacement response and longitudinal force can excite coupled the rotation angle response. It is necessary to consider the coupled response of the system for propulsion shafting.
- 2) The numerical calculation showed that the coupled torsional-longitudinal vibration is the sum of the torsional vibration and longitudinal vibration respectively. It makes good agreement with theoretical analysis.

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