Is underwater thermal noise useful?

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ABSTRACT
Traditionally thermal noise has been considered an inconvenient nuisance in underwater acoustic sound pressure measurements. Theory (e.g. Mellen, 1952) shows that the noise spectrum increases with frequency, and provides a lower limit for measurements above 50 kHz. However, Weaver and Lobkis (2001, 2003) showed that by using an ultrasonic detector to correlate thermal fluctuations in a metal block, they could extract details of the block's dimensions, in much the same way as if they had used the detector for active transmission and reception. This paper will examine the theory and previous measurements, and provide further measurements of thermal noise.

Keywords: Underwater noise   I-INCE Classification of Subjects Number(s): 54.3

1. INTRODUCTION

In underwater acoustics it is stated (1) that thermal noise provides a lower limit on the detection of underwater acoustic signals. The implication is that weaker noise sources cannot be measured, and the thermal noise itself is of no use. However, Weaver and Lobkis (2, 3) used an ultrasonic detector to correlate thermal fluctuations in a metal block, from which they could extract details of the block's dimensions, in much the same way as if they had used the detector for active transmission and reception. Although the correlation method has since been used for various other non-thermal ambient noise sources, very little reported work has used thermal noise itself. This paper reviews the theory and experimental use of thermal noise, and provides some further measurements with a sensitive hydrophone.

2. THERMAL NOISE THEORY

Theoretical calculations of thermal noise have been made for air media as well as for water, for which the theory is the same. The spectrum of the mean square of the thermal noise pressure in air was first calculated by Sivian and White (4). They considered a rigid mass-less piston free to vibrate in an infinite rigid wall and exposed to the atmosphere on one side. By analogy with Nyquist's (5) theory of the Johnson effect (6), they calculated the square of the thermal acoustic pressure in the frequency interval *df*, centred at frequency *f* and averaged over the surface of the piston, to be

$$\langle P^2 \rangle df = 4k_B TR/(\pi a^2)^2 \;\; df \qquad (1)$$

$$= \frac{4k_B T\rho c}{\pi a^2}\left[1 - \frac{J_1(2ka)}{ka}\right] df \qquad (2)$$

where k_B is Boltzmann's constant, T is the absolute temperature, R is the acoustic radiation resistance at frequency *f*, k is the wavenumber, ρ is the air density, c is the speed of sound in air, a is the radius of the piston, and *J* are Bessel functions of the first kind.

Using an extension of Nyquist's theory and quantum mechanics, Callen and Welton (7) derived what has become known as the fluctuation-dissipation theorem, relating the mean square value of the spontaneous fluctuating force to the mean energy at temperature T of an oscillator of frequency *f*. At high temperatures, where $k_B T \gg hf$, the mean energy takes its equipartition value $k_B T$. h is Planck's constant. Using expressions for the acoustic radiation resistance, they derived the square of the fluctuating force on a small sphere immersed in a gas at any temperature. This equated to a squared fluctuating pressure averaged over the surface of the sphere as given by

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$$\langle P^2 \rangle df = 4\pi k_B T \rho f^2 / c [1 + (ka)^2] \ \ df. \tag{3}$$

They gave the fluctuating pressure as the compressive force per unit area on a vanishingly small sphere, leading to

$$\langle P^2 \rangle df = 4\pi k_B T \rho f^2 / c \ \ df \tag{4}$$

for frequencies and temperatures where $k_B T \gg hf$. This result was confirmed using another theoretical approach: by counting the number of acoustic modes in the frequency interval, computing the acoustic energy density and relating this to the mean square excess pressure. Near a planar wall they found the mean square fluctuating pressure to be double that of Equation 4, since the pressure waves in the gas required velocity nodes at the wall. For a circular piston set in the wall, they obtained the fluctuating force, which for $k_B T \gg hf$ equates to the same square of the fluctuating pressure averaged over the surface of the piston as obtained by Sivian and White, namely Equation 2.

For water Mellen (1) also counted acoustic modes, and calculated the acoustic energy density to arrive at the same expression for the mean square noise pressure in a band one Hertz wide as given by Equation 4. It is this expression and reference which are usually cited for the thermal noise spectrum in texts dealing with underwater sound and hydrophones (e.g. 8-14). When considering the mean square voltage response of the hydrophone, Mellen introduced the acoustic radiation resistance, which would have led, although not explicitly stated, to the square of the pressure fluctuations averaged over the hydrophone as in Equation 1. For a spherical hydrophone or a piston hydrophone set in a wall this would give Equations 2 and 3, respectively. This work of Mellen was later reviewed by Sullivan and Kemp (15).

The above theories are the most widely accepted, however a number of others have been proposed. Space precludes a complete description here, but the interested reader might pursue these in publications by Baerwald (16), Weymann (17; reproduced by Van der Ziel, 18 p22-23), Weber (19), De Vries (20, 21), Boyarsky (22), Becking and Rademakers (23), Hunt (24, 25), Stephens and Bates (26), Tucker and Gazey (27), Van Burik and Alkemade (28), Skudrzyk (29), Van Burik (30), Lifshitz and Pitaevskii (31), Tarnow (32), and Barabanenkov and Passechnick (33).

The only attempt at experimental verification of the thermal noise spectrum of water was made by Ezrow (34). He measured the electrical radiation resistance of thin-disk hydrophone transducers loaded by water, and pulsed with rf signals. He confirmed the formula for the acoustic radiation resistance of a rigid piston set in a baffle and considered this a verification of the thermal noise spectrum. As this was not a direct measurement of thermal noise, the claim of verification relied upon the validity of the relationship between pressure fluctuations and radiation resistance, and the radiation resistance of a pulsed piston equalling that of a piston subject to pressure fluctuations.

3. PREVIOUS MEASUREMENTS USING THERMAL NOISE

The first use of thermal noise came in papers by Weaver and Lobkis (2, 3). Using an oil couplant, they attached a sensitive piezoelectric acoustic emission transducer to the centre of one face of a cylindrical block of aluminium. The transducer was sensitive up to 1 MHz, and was connected to a battery-powered preamplifier, low pass filter, digital oscilloscope and computer. The sample, transducer and preamplifier were all housed in a Faraday shield. The digitised data were divided into 32,000 byte waveforms, each of which was Fourier transformed, absolute-value squared, inverse Fourier transformed, and time differentiated. This process was repeated many times, and the result averaged, with good convergence obtained after some 100 ms of thermal noise had been collected. In effect the time derivative of the autocorrelation of the noise from the transducer had been obtained. They plotted this correlated signal as a function of time and demonstrated that the signal trace was the same as if they had transmitted a transient pulse from the transducer, and recorded the echo from the other end and sides of the block with the same transducer. And so by using thermal noise they could deduce the size of the aluminium block.

In a separate experiment they attached two transducers to opposite sides of the aluminium block, and this time the noise from each transducer was recorded simultaneously. Each 32,000 byte waveform was Fourier transformed, the cross-spectral density was formed, and the result averaged over many captures amounting to 32 s of thermal noise data. An inverse Fourier transform and time derivative were then applied. A plot of this correlated signal as a function of time was almost the same as if they had transmitted a transient pulse from one transducer and recorded its reception on the other transducer.

This correlation method sparked a flurry of activity in a variety of fields, most notably in seismic work, but also in underwater acoustics, although in almost all cases using ambient noise which wasn't thermal in origin. There have been just a handful of papers which report experiments with thermal noise.

Loyau and Feuillard (35) glued a transducer, sensitive between 1 and 5 MHz, to a nylon 6/6 plate. Its output was fed to an amplifier, and thence to a spectrum analyser. The sample, transducer and preamplifier were all housed in a Faraday shield. From the recorded spectral power density, an echo waveform was calculated, which was compared with that obtained directly with a pulser/receiver. Like the work of Weaver and Lobkis, a good match was achieved between the waveform from a transmitted transient, and that calculated from thermal noise.

Lani *et al*. (36) examined the cross-correlation method using a capacitive micromachined ultrasonic transducer (CMUT) ring array monolithically integrated with complementary metal oxide semiconductor (CMOS) electronics. It consisted of two concentric ring arrays: an outer ring of 32 receive elements, and an inner ring of 24 transmit elements, the whole being less than 1 mm in diameter. The array was placed in a small tray of water of 2 mm depth, and 1 s of thermal noise was recorded between 5 and 25 MHz. The broadband data from two receive elements was cross-correlated and showed echoes corresponding to arrivals from the water surface. The measurements were repeated for 1.5 mm water depth. Time-frequency analysis showed that the echoes were primarily in a 12-22 MHz band.

In a second experiment the ring array was placed in oil and a 500 µm diameter scatterer was placed 1.3 mm above it. Thermal noise data was recorded on all 32 receive elements, collected 4 elements at a time, then filtered between 16.5 and 30 MHz, cross-correlated, and an image formed in range and cross range. Using all 768 transmit-receive element pairs, active pulse echo waveforms were also recorded, from which a conventional ultrasonic image was formed between 16.5 and 30 MHz. The two images showed the scatterer, but differed in sidelobe structure. As far as this author is aware, this experiment constitutes what might be termed ‘thermal noise imaging’.

Gurun *et al*. (37) used the same ring array to measure the noise spectrum of an element from DC to 30 MHz. This showed good agreement with the noise calculated via a finite difference / boundary element matrix model, which incorporated the mutual impedance between the elements.

Davy *et al*. (38) took the correlation method of Weaver and Lobkis and applied it to thermal noise in electromagnetic radiation. Using two antennae in an anechoic chamber, they were able to calculate the cross spectral density from the cross correlation of the two thermal noise signals, compare the correlation signal with the transient response obtained with a pulse generator, and examine the effect of temperature on the thermal noise. Finally they created a synthetic array by moving one antenna over ten positions, and by beamforming the ten sets of cross-correlated data, produced a thermal noise image of an aluminium cylinder within the chamber.

4. CORRELATION MEASUREMENTS

As part of a series of experiments undertaken from 1999 to 2002, thermal noise correlation measurements were performed with an International Transducer Corporation ITC8257 hydrophone. This hydrophone has a solid cylindrical PZT-4 crystal of 10.67 mm diameter and 9.65 mm height epoxied into an aluminium well and covered with a 4.78 mm thick polyurethane layer, as shown in Figure 1. It has an integral moderately low noise preamplifier providing 60 dB of gain.

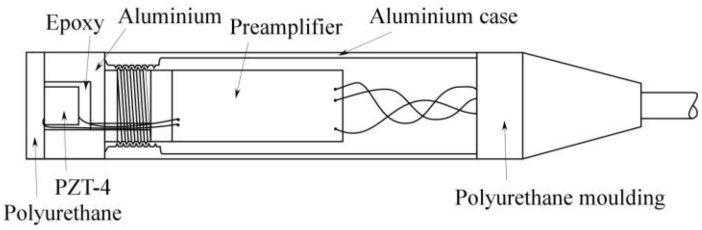

Figure 1 – Construction of an ITC8257 hydrophone.

The hydrophone was mounted in an oblong plastic tub of water, close to one 6.4 mm thick wall, but far from other walls. Data was recorded with the hydrophone’s polyurethane matching layer touching the wall and moved progressively away in 1 mm steps, out to 15 mm. All measurements were undertaken in a screened room acting as a Faraday shield. A second set of data was collected with two ITC8257 hydrophones facing each other with both polyurethane matching layers touching, and then progressively moved away from each other in 1 mm steps, out to a 30 mm separation.

The data from each hydrophone was digitised at a rate of 1 MHz using a National Instruments PCI-6110E simultaneous sampling card inserted into an IBM-compatible AMD Athlon XP computer. The first set of data was successively filtered into 2 kHz wide bands by passage through a windowed linear-phase finite impulse response digital filter, centred on frequencies between 50 and 350 kHz, and the normalised auto-correlation taken. Figure 2 plots the correlation corresponding to the time lag which matches the travel time from the transducer crystal surface, through the polyurethane layer and water to the wall, and back again, after correcting for the phase change at the wall. The strong correlations occur when the two waves (outgoing and returning waves) are an integral number of wavelengths apart and hence in phase. The second set of data was likewise filtered, but this time the normalised cross-correlation was taken. Figure 3 plots the correlation corresponding to the time lag which matches the travel time from one transducer surface to the other. Again the strong correlations occur when the two waveforms are an integral number of wavelengths apart and hence in phase.

Apart from demonstrating that the ITC8257 hydrophone is capable of measuring thermal noise in the medium over and above any in the electronics at all frequencies between 50 and 350 kHz, both sets of data indicate that the source of the noise is close to the crystal face, either in the polyurethane layer or on the other side of the plating. The temporal resolution of the measurements is not sufficiently fine to localise the origin more precisely.

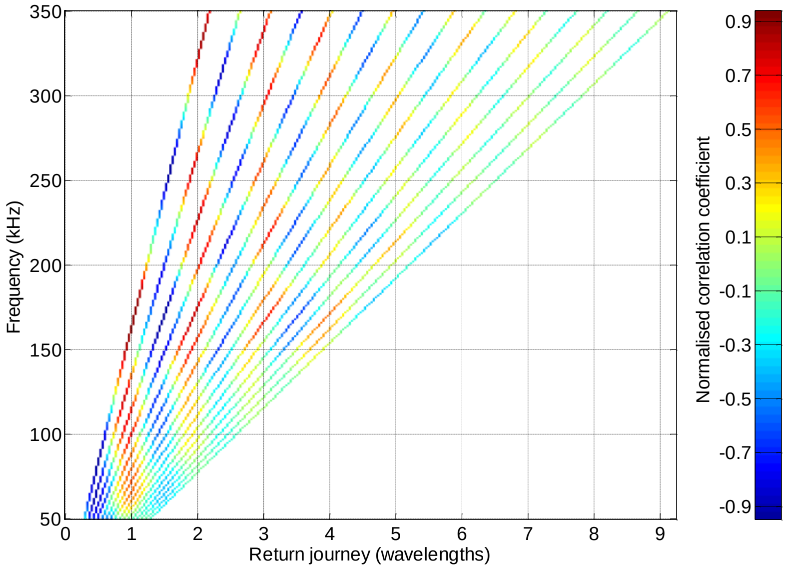

Figure 2 – The auto-correlation of thermal noise travelling through water between a hydrophone and wall boundary.

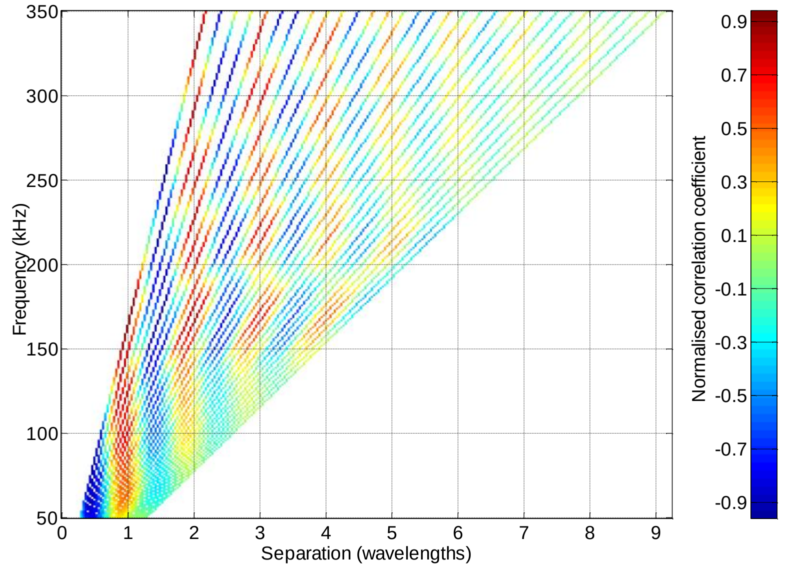

Figure 3 – The correlation of thermal noise travelling through water between two hydrophones.

5. SUMMARY

A brief review has been provided of thermal noise theory as applied to acoustics in fluids. The most widely accepted theory is based on the seminal paper of Nyquist (5), and states that the square of the pressure fluctuations as averaged over, and thus recorded by a hydrophone, is proportional to the product of Boltzmann's constant, the absolute temperature and the radiation resistance of the hydrophone, although this has never been verified experimentally.

Only a few experiments have been performed using thermal noise, and have revolved around using correlations of the noise to reproduce the waveform which would have been obtained by transmitting a pulse and recording the echo. However one interesting experiment used thermal noise to image a target in water.

Data has been presented showing noise emanating from near a hydrophone crystal's front surface can be picked up by another hydrophone, or can be reflected from a planar surface back to the hydrophone.

These various experimental results demonstrate that thermal noise has the potential to detect targets underwater, at least over short ranges, when no other ambient noise (such as from breaking waves or snapping shrimp) is present, which will often be the case at high frequencies (i.e. tens of kiloHertz). There is also the possibility that it can be used for imaging in a manner not too dissimilar to acoustic daylight (38-40), which uses ambient noise for imaging. Therefore the answer to the question posed in the title to this paper, is probably "yes", but has yet to be definitely proven.

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