A shape classification for the acoustic radiator using its sound field

Koo-Hwan KIM[1]; Yang-Hann Kim[2]

[1] Korea Advanced Institute of Science and Technology, Korea

[2] Korea Advanced Institute of Science and Technology, Korea

ABSTRACT

A method for classifying the shapes of sound radiator is presented. It assumes that sound field can be measured by a linear array. A sound field, due to the radiator vibrating with uniform velocity, can be determined by its shape, size, and orientations. Measured data also can be varying from the array's position. To predict the shape of radiators from these measured data, mathematical relation between geometric parameter and measured information is needed. Assume that a radiator is cylinder, the magnitude and phase of measured pressure is related with the length and diameter of radiator, respectively. In this paper, the method for estimating length and shape of a finite cylinder by using its radiated pressure is proposed and verified through experiment.

Keywords: Shape estimation, cylindrical radiator

1. INTRODUCTION

As illustrated in figure (1), our objective is to classify the radiator based on limited measurement information; sound pressures on the discrete microphone positions. It is, of course, mathematically ill-posed problem, because the measure information is not sufficient to get unique solution that allows us to tell the kind of radiator.

To be practical, let's suppose that we have a line array of microphone and measure the radiated sound field. To start with accessing the problem, let's assume that a radiator is a finite cylinder. Assume that the normal surface velocity of a finite cylinder has uniform, the measured pressures are determined by the length, radius and orientation of the cylinder and distance between the array and radiator. Inversely, those parameters can be estimated from the measured pressures with appropriate approximation, priori information, or model. In this paper, we attempt to classify the shape of radiator: its radius and length of finite length cylinder by using the measured pressure information on a line array.

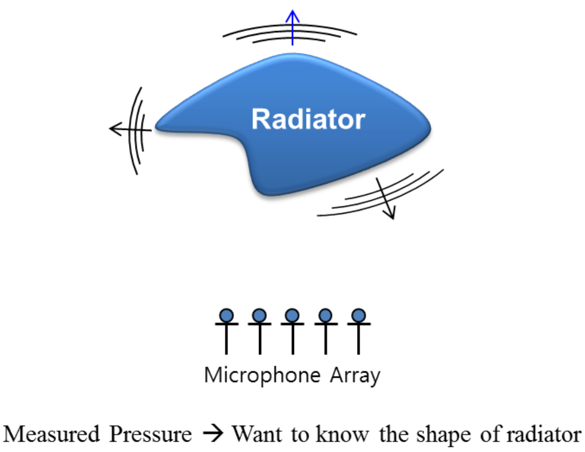

Figure 1 – Problem definition for classifying the shape of radiator

[1] kh-kim@kaist.ac.kr
[2] yanghannkim@kaist.ac.kr

2. Problem Statement

To estimate the parameter of the cylindrical radiator, understanding of the pressure field of cylindrical radiator is needed. Because a finite cylinder is non-separable geometry, the closed form solution of the pressure field from finite cylinder can’t be obtained [1-3]. Hence, to make a closed form solution of a pressure field from a finite cylinder, the pressure field results of the active band of a finite length on an otherwise rigid, infinitely long cylinder is used [4] (Figure 2).

The expression for an outgoing wave from a infinite cylinder, whose radius and length are a and $2b$, is given by

$$P(r,\theta)=\frac{2\rho_0 c U_0 b}{\pi}\frac{\mathrm{sinc}(kb\cos\theta)}{\sin\theta H_1(ka\sin\theta)}\frac{e^{jkr}}{r} \tag{1}$$

where ρ_0 is density of medium, c is the speed of sound, U_0 is normal velocity of radiated surface, $k=\omega/c$ is the wave number and where $H_1(z)$ is the first order Hankel function of the first kind. r is distance between the center of the cylinder and measurement position, θ represents an angle between the axis of cylinder and measurement position. In other to simplify and interpret equation (1), Hankel function is written by the asymptotic form, then equation (1) can be expressed as follow

$$P(r,\theta)\approx\frac{2\rho_0 c U_0 b}{\pi}\mathrm{sinc}(kb\cos\theta)\sqrt{\frac{\pi ka}{2\sin\theta}}\frac{e^{j\left(kr-ka\sin\theta-3\pi/4\right)}}{r} \tag{2}$$

Equation (2) implies that measured pressure is determined by two parameters. The first one is related with measurement position r and θ. The other is related with the geometric information of the finite cylinder a and b. Assume that if the measurement position r and θ are already given, Equation (2) can be considered as a pressure spectrum of measured location p by the dispersion relation $k=2\pi f/c$ and its frequency characteristics are determined by the length and radius of cylinder. Hence the geometric parameter (the length and diameter of a cylinder) can be estimated by measured pressure spectrum at known position.

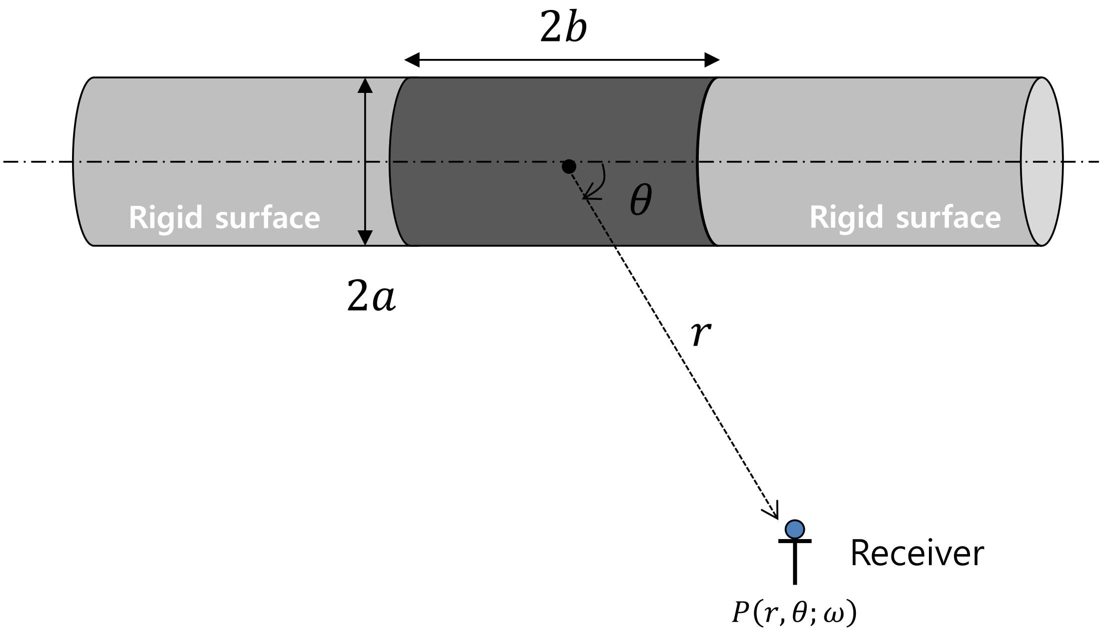

Figure 2 – Schematics of the active band of a finite length on an otherwise rigid, infinitely long cylinder

3. Estimation of the length and radius with cylinder

Equation (2) is constructed by three terms. The first term $2\rho_0 c U_0 b/\pi$ is constant with respect to the frequency change. And the last term is equal to the monopole radiaton. The magnitude change of pressure spectrum is deeply related with second term $\mathrm{sinc}(kb\cos\theta)\sqrt{\pi ka/2\sin\theta}$. This second term has important hint for estimating the length of cylinder b. In this term, the parameter b is in the argument of a sinc function. The sinc function has zero-crossing at integer multiples of π. Therefore, the zero-crossing frequencies of equation (2) can be expressed by

$$n\pi = kb\cos\theta = \frac{2\pi f_n}{c} b\cos\theta, \quad \therefore f_n = \frac{nc}{2b\cos\theta}. \tag{3}$$

And interval of zero-pressure is

$$\Delta f = f_{n+1} - f_n = \frac{c}{2b\cos\theta}. \tag{4}$$

Therefore the length of a finite cylinder can be obtained from the zero frequency intervals as

$$\hat{b} = \frac{c}{2\Delta f\cos\theta}. \tag{5}$$

The phase of the pressure spectrum is represented by

$$\arg\{P(r,\theta)\} = \frac{2\pi f}{c}\left(r - a\sin\theta\right) - \frac{3}{4}\pi + \left\{\mathrm{sgn}\left(\mathrm{sinc}(kb\cos\theta)\right) - 1\right\}\pi \tag{6}$$

where $\mathrm{sgn}(x)$ is the sign function of x and defined as follows:

$$\mathrm{sgn}(x) = \begin{bmatrix} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{bmatrix} \tag{7}$$

where $\mathrm{sgn}\left(\mathrm{sinc}(kb\cos\theta)\right)$ has discontinuity at zero-crossing frequencies. Equation (6) means the phase change is proportional to frequency and the derivative of phase change is constant between zero crossing frequencies. And this derivative value is determined by the measurement position and diameter of sphere. Hence, the radius of a finite cylinder can be estimated by follows:

$$\hat{a} = \frac{1}{\sin\theta}\left\{r - \frac{c}{2\pi}\frac{\Delta\{\arg(P)\}}{\Delta f}\right\} \tag{8}$$

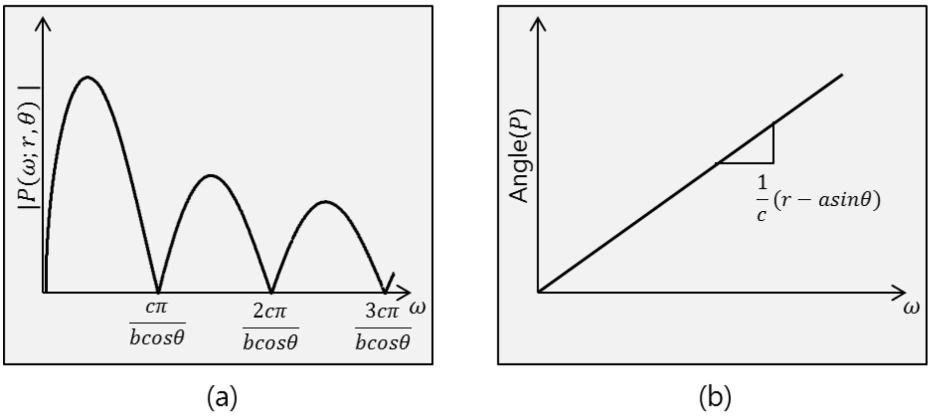

Figure 3 – (a) Magnitude of the equation (2), the zero-crossing frequencies are integer multiples of (b) Phase of the equation (2), the derivative of phase is related with not only measurement position but also the radius of cylinder.

4. Experiment

In order to demonstrate the feasibility for the shape estimation based on measured pressure, the experiment has been done in the water tank. The back scattered signal of a finite cylinder is used as the radiated sound signal from the cylinder. Figure (4) displays the geometry and configuration of the experiment. The transmitted signal is linear frequency modulated signal. The pulse duration is 0.2ms and its frequency modulates from 70kHz to 100kHz.

Figure (5) shows the measured pressure in time domain. Back scattered of finite cylinder can distinguish by using the geometry of experiment setup and time delay. The forth chirp signal is received from the finite cylinder directly. The spectrum of direct scattered signal is used for estimating the parameter of the target cylinder.

Figure (6-a) display the magnitude of the back scattered signal of the finite cylinder. The dashed line means the theoretical spectrum by equation (2), solid line means measured results. Dot and dashed lines represent the local minima frequencies of the magnitude spectrum. In measured spectrum, there are local minimum instead of zero-crossing. The measurement noise affects spectrum at zero-crossing frequencies, and pressure spectrum of a finite cylinder differ from the equation by having no zeros [4]. The estimated length by the equation (5) is 0.6834m and its relative error is 10%.

Figure (6-b) shows the phase change of the measured pressure spectrum. The dashed line expresses the phase of Eq. (2) and solid line shows the measurement data. This results shows the phase change rate with respect to frequency is almost constant during transmitted pulse frequency band and the estimated radius of the cylinder is 0.368m.

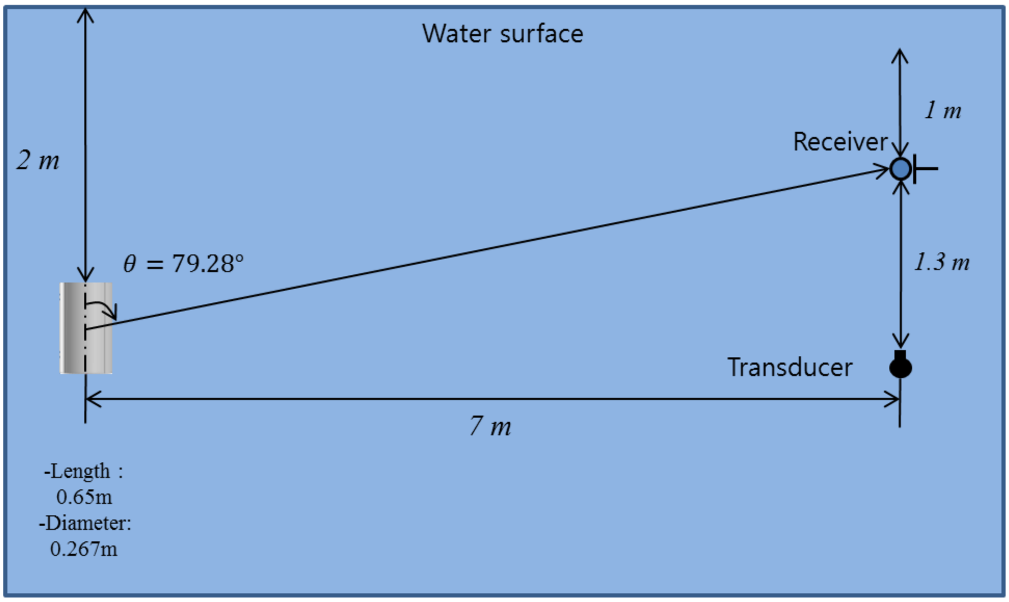

Figure 4 – Experiment configuration

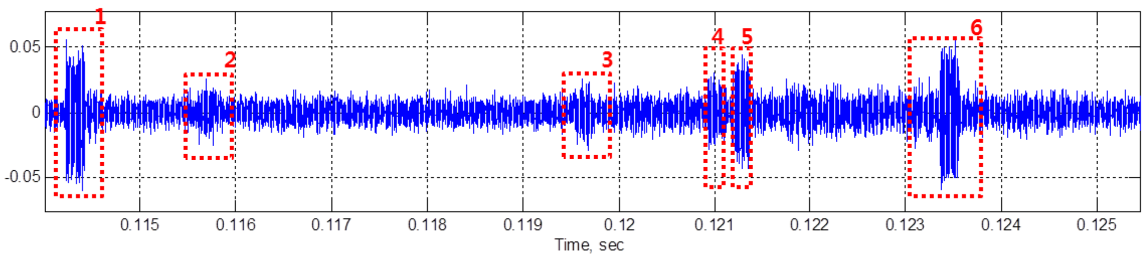

Figure 5 – Measured signal from the receiver; the path of the first pulse is from the transducer to the receiver directly. The second one is scattered by cylinder and surface. The third one is reflected by side-wall.

5. CONCLUSIONS

In this paper, we proposed a method to estimate the shape of finite cylinder by using measured pressure data. The magnitude of measured pressure at known position has periodic zero-crossing frequencies and these frequency intervals are determined by the length of a cylinder. The phase of measured pressure is proportional to a frequency. A radius of a cylinder can be estimated through the ratio between phase change of pressure and frequency. The experiment data showed that the estimated geometric properties of cylinder were over-estimated, because of the difference of the pressure fields of the infinite cylinder and finite ones.

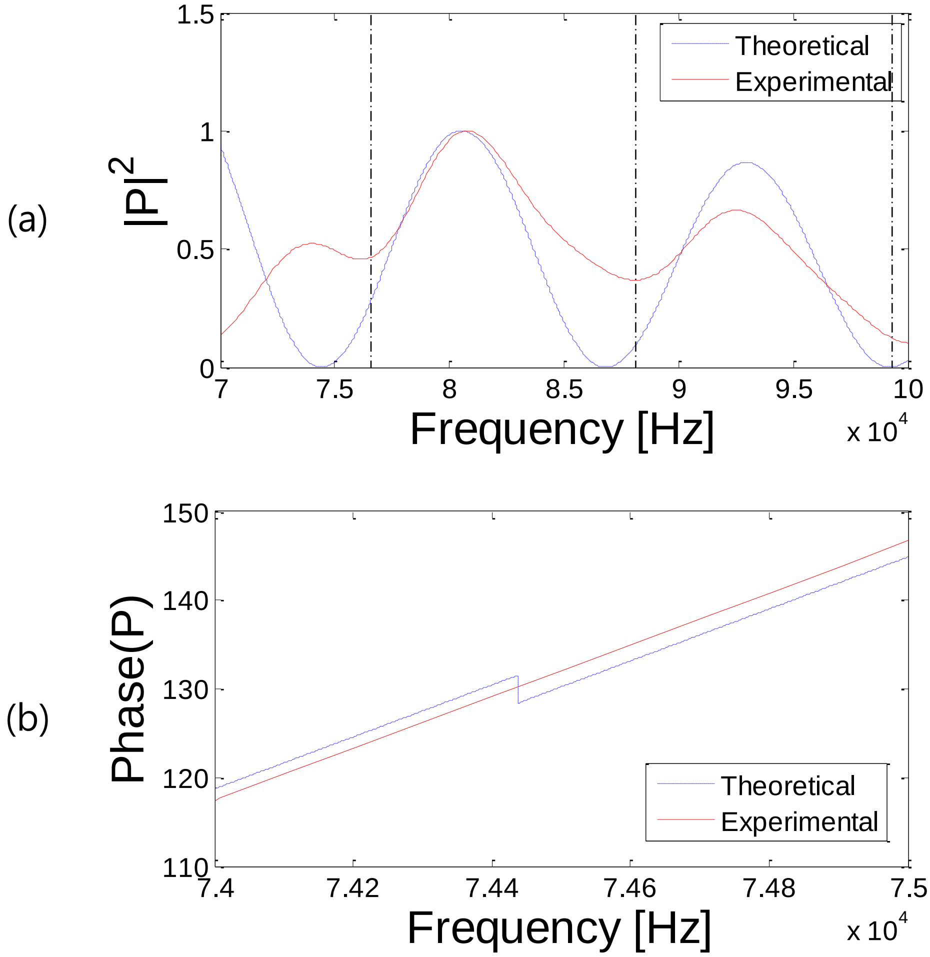

Figure 6 – The magnitude and phase spectrum of the measured datas

ACKNOWLEDGEMENTS

This work was supported by the Ministry of Trade, Industry and Energy (MOTIE) grant funded by the Korea government (No. 10037244), and the BK21 (Brain Korea 21) project initiated by the Ministry of Education, and Unmanned Technology Research Center (UTRC) at Korea Advanced Institute of Science and Technology (KAIST), originally funded by DAPA, ADD.

REFERENCES

1. W. Williams, N.G. Parke, D.A. Moran, and D.H. Sherman. Acoustic radiation from a finite cylinder. J Acoust Soc Am. 1996;100(1):31-48.
2. N.D. Perreira and D. Dawe. An analytical model for noise generated by axial oscillations of unbaffled cylindrical elements. J Acoust Soc Am. 1984;75 (1):80-87.
3. J.M. McCormick and M.L. Baron.Sound radiation from submerged cylindrical shells of finite length.Trans of ASME. 1965; Ser.B 87:393-405

4. D.T. Laird and H. Cohen. Directionality patterns for acoustic radiation from a source on a rigid cylinder. J Acoust Soc Am. 1952;24 (1):46-50.