An efficient model for prediction of underwater noise due to pile driving at large ranges

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ABSTRACT

Modelling the sound levels in the water column due to pile driving operations nearby and out to large distances from the pile is crucial in assessing the likely impact on marine life. Standard numerical techniques for modelling the sound radiation from mechanical structures such as the finite element (FE) and boundary element method are not well suited to predict the sound field efficiently at large ranges. Models better suited for prediction of sound propagation in waveguides over large distances, such as wavenumber integration and ray tracing, require careful attention in order to capture the source characteristics of a complex source such as a pile radiating from both water and sediment. To circumvent these issues, a new hybrid model is proposed using a local FE model that accurately captures the source characteristics of the pile which is coupled to a normal mode based model for efficient evaluation of the sound propagation over large distances in a range dependent environment. The model is validated using the well-known solution for a point source in a Pekeris wave guide. Results are shown for a generic pile driving scenario that was used in the international benchmarking workshop COMPILE for underwater pile driving models.

Keywords: Underwater Pile Driving Noise, Normal Mode, Source Characterization I-INCE Classification of Subjects Number(s): 54.3

1. INTRODUCTION

Acoustic models play an essential role in predicting the likely impact on marine life due to pile driving operations. Depending on the level of he produced noise during pile driving, such models should be capable to accurately predict the sound levels in the water column nearby and/or out to large distances from the pile. Below, an efficient model for predicting impact pile driving noise at both close and long ranges for range dependent (shallow) water waveguides is presented.

Standard numerical techniques for modelling the sound radiation from mechanical structures in the vicinity of the vibrational source such as the finite element (FE) and boundary element method are not well suited to predict the sound field efficiently at large ranges. On the other hand, models better suited for prediction of sound propagation in waveguides over large distances, such as wavenumber integration and ray tracing, require careful attention in order capturing the source characteristics of a complex source such as a pile radiating from both water and sediment. To circumvent these issues, so-called hybrid models can be used which combine a source model, capturing the source characteristics of the pile, with a propagation model which offers efficient evaluation of the sound propagation over large distances. Previously, hybrid models consisting of various combinations of source and propagation models were presented for instance by Zampolli et al. (1), Reinhall and Dahl (2), Lippert et al. (3), and Tsouvalis and Metrikine (4).

The present study is a direct continuation of a study that was carried out by TNO during 2010-2011 which led to the development of a so called hybrid model, consisting of a one-way coupled FE based source model and Helmholtz-Kirchhoff-Integral (HKI) based propagation model (1). A number of issues were identified that needed to be resolved in order to use the model for efficient systematic quantitative prediction of impact pile driving noise under real operational conditions. These issues include the need for a significant improvement in efficiency of calculating Sound Exposure Level (SEL) far from the source (required for the generation of noise maps), and extending the model capabilities to include range dependent waveguide properties (e.g., bathymetry). The

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previously presented hybrid model used a Helmholtz-Kirchhoff integral (HKI) based propagation model which is not efficient when evaluating the pressure field for many points at larger distances and is not capable of dealing with range dependent environments in a straightforward manner. A good candidate to replace the HKI model is the propagation model used in AQUARIUS, a noise mapping framework that was previously developed at TNO (5), which is both efficient and capable of dealing with range dependent environments. AQUARIUS was originally designed to work with (monopole) point sources. In this paper an extension for arbitrary sources is presented. AQUARIUS is based on flux theory, but can be rewritten in terms of normal modes. The proposed method requires a source characterization in terms of normal modes as input. Note that in the current work, results are shown for an intermediate step where the FE model is coupled to a normal mode propagation model to validate the required coupling. Ultimately, the FE model will be coupled to the more efficient flux based propagation model used in AQUARIUS or, alternatively, the recently developed flux based model SOPRANO (6).

In the next section, the Hybrid model is described in more detail. Subsequently, results obtained with the new hybrid model are presented, which are predominantly aimed at validation of the model.

2. HYBRID FE/normal mode model

The hybrid model consist of an FE based source model which is coupled (one-way) to a propagation model based on normal modes. The FE model predicts the sound field at a range r_0. From this result the contribution of each normal mode is determined such that the superposition of normal modes matches the result at r_0. The normal mode model is also used to predict the propagation loss of individual modes due to propagation to the receiver location (r, z). Combining the contribution of each mode and the associated propagation loss leads to a prediction of the sound field due to the modeled source. The situation is sketched in Figure 1 for a point source located at depth z_s.

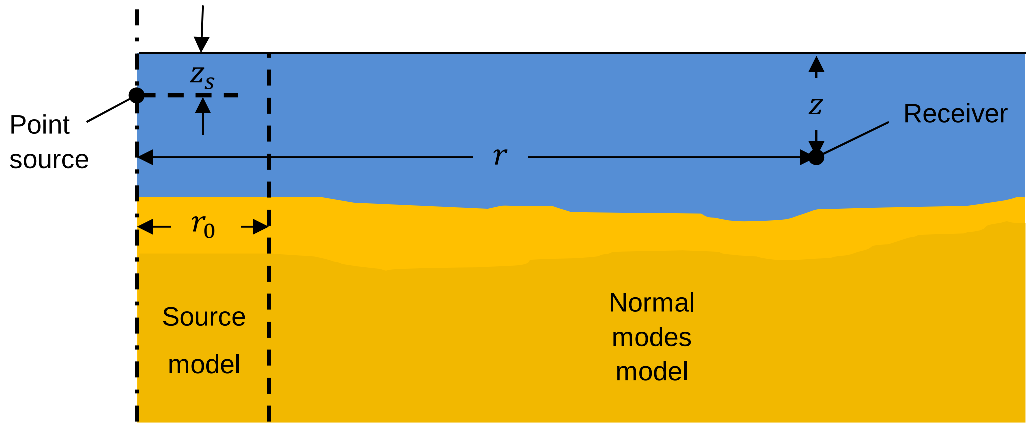

Figure 1 – Schematic geometry of a range dependent water waveguide including a (point) source and (point) receiver. The waveguide is modelled by a source and normal mode model that are coupled at r_0.

The normal mode model is extended to the case of range dependent waveguides using the adiabatic assumption (for instance described in reference (7)). Using this method, the waveguides range is discretized by cutting it into pieces over which waveguide properties can be assumed to be constant by approximation. If waveguide properties between two adjacent pieces only differ by a relatively small amount the energy carried by a certain mode can be assumed to transfer to the associated mode of the next waveguide piece. Using this assumption allows a straightforward one way coupling between waveguide pieces.

The waveguide geometry and source that are modeled are axially symmetric with respect to the origin of the source. Using an N×2D approach, where the individual axially symmetric 2D models represent a 2D slice of the actual 3D waveguide, the model can be used to generate noise maps for areas of arbitrary 2D bathymetry. This approach is depicted schematically in Figure 2.

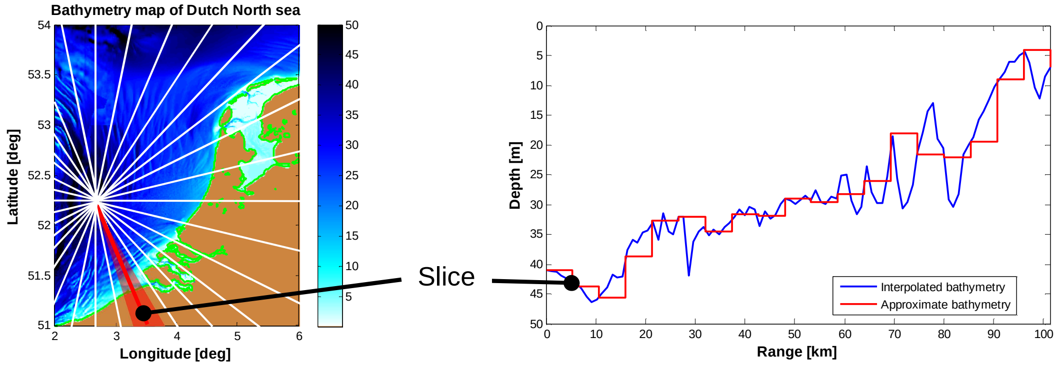

Figure 2 – Bathymetry map of the Dutch North sea divided into slices (left) and the interpolated and approximated bathymetry for a 2D bathymetry slice (right).

Note that the results presented below are based on models for a Pekeris waveguide; In both models the sediment is represented by an equivalent homogenous fluid. In addition, the water/sediment interface is assumed to be flat. Although the theory described below is valid for (or can be extended to) more arbitrary stratified water waveguides including elastic layers, the various results shown in section 3 are valid for the Pekeris waveguide consisting of two homogeneous fluids schematically represented in Figure 3.

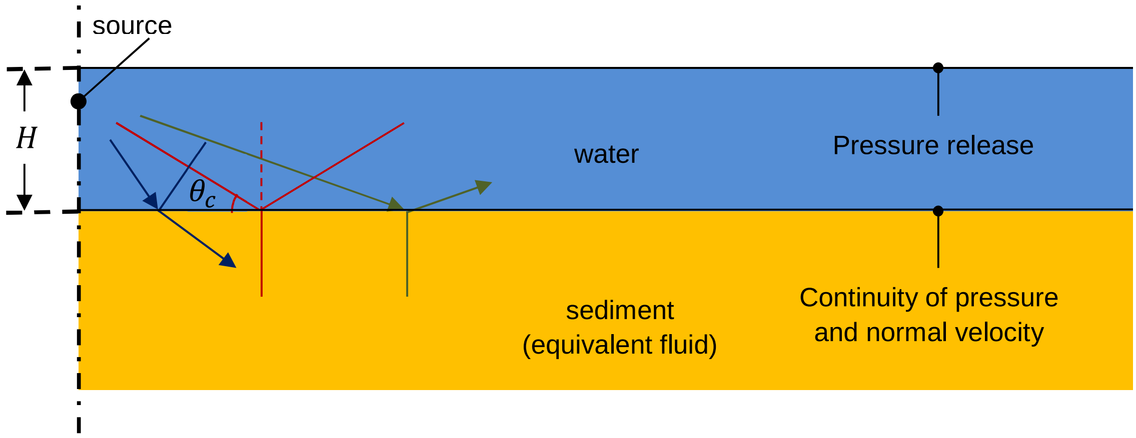

Figure 3 – Schematic geometry of a Pekeris waveguide with a point source. The green arrow represents a wave incident at angle θ below the critical angle θ_c (indicated by the red lines) that is fully reflected back into the waveguide (associated with propagating modes). The blue arrow represents a wave incident above the critical angle that partially transmits its energy to the sediment (associated with leaky modes).

The schematic representation of the behavior of the waves in the waveguide depending on incident angle shown in Figure 3 are valid for the case where the sound speed in the sediment is larger than that in the water. This is common to the cases for which the model is intended to be used and holds for the results presented in section 3. Although the theory presented below is also valid for cases where the sediment wave speed is lower than that of the water, the type of modes that occur and their characteristics differ from what is described below.

2.1 FE model

The source model consists of a linear, axial-symmetric frequency domain FE model in COMSOL. A schematic overview of the FE setup is given in Figure 4. The infinite extent of the depth and range dimensions of the Pekeris waveguide are modelled by lining the bottom and outer range of the domain with so-called Perfectly Matched Layers (PML). The FE model is excited by applying a unit force to the area of the hammer impact (see Figure 4). Damping due to friction between pile and sediment is represented by applying damping to the p- and s-waves in the pile section that is imbedded in the sediment. For the source model, damping associated with wave propagation in the water and sediment is neglected. Stress release boundary conditions are applied between the structure and the air, and pressure release conditions are applied at the water surface. The boundary conditions at the fluid/solid interfaces between water and pile and sediment and pile consist of enforcing continuity of normal force and normal velocity, whereas a slip condition is applied in the direction tangential to the interface.

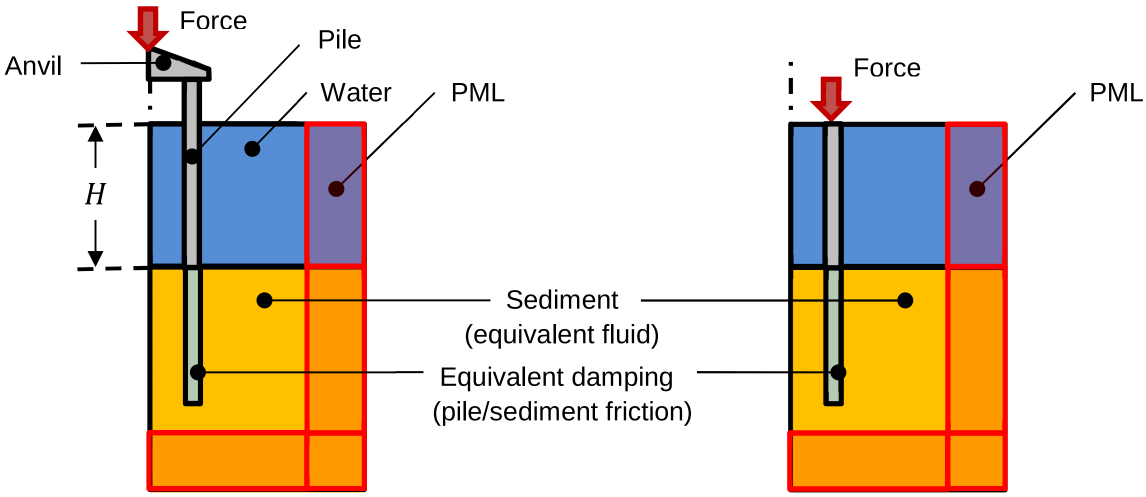

Figure 4 – Schematic overview of an FE model for a generic pile driving case (left) and the COMPILE benchmark case (right) as defined in section 3.2.

2.2 Normal mode model

An in-depth description of the theory underlying normal mode models can be found for instance in the book of Boyels (8). The normal mode implementation developed at TNO was based on the theory described in reference (7). First the expansion in normal modes of the field produced by a point source in a Pekeris waveguide is given. Next the approach is expanded to deal with arbitrary sources. Lastly, some comments on the required approximations and the root-finding algorithms that are used are given.

2.2.1 Normal modes expansion for a point source in a Pekeris waveguide

Following the theory presented in (7), the (complex) sound field p due to a point source at depth z_s in an range independent waveguide can be expanded in terms of the normal modes Ψ_m as

$$p(r,z) \approx \frac{i}{4\rho(z_s)} \sum_{m=1}^{\infty} \Psi_m(z_s)\Psi_m(z)\, H_0^{(1)}(k_{rm} r) \tag{1}$$

With i the imaginary unit, ρ the fluid density, $H_0^{(1)}$ the Hankel function of the first kind of order zero, and k_{rm} the wave number in r-direction for mode m. For small damping values in the sediment and water, decay can be included explicitly using the theory of Kornhauser-Raney (9), and the expression in Equation (1) can be written as:

$$p(r,z) \approx \frac{i}{4\rho(z_s)} \sum_{m=1}^{\infty} \Psi_m(z_s)\Psi_m(z)\, H_0^{(1)}(k_{rm} r)\exp(-\beta_m r) \tag{2}$$

with β_m a positive real number known as the decay factor. Using the definition in reference (9), the decay factor can be expressed as:

$$\beta_m = m^2\pi^2 \cdot \epsilon \cdot \frac{(\rho_1/\rho_0)\cot^3(\theta_c)\sec(\theta_c)}{k_0 k_{mr} D^3 F_m} \tag{3}$$

Where ϵ, F_m and the effective depth D are defined, respectively as:

$$\epsilon = \frac{\alpha_b}{4\pi \cdot 10\log_{10}(e)} \tag{4}$$

$$F_m = \frac{H}{D}\left(\sigma_m^3 + \frac{\rho_1^2 k_{zm}^2}{\rho_0^2 k_0^2 \sin^2(\theta_c)}\sigma_m + \frac{(\rho_1/\rho_0)}{k_0 H sin(\theta_c)}\right) \tag{5}$$

$$D(r) = H + \frac{\rho_1/\rho_0}{k_0 \sin(\theta_c)} \tag{6}$$

where α_b the attenuation coefficient for wave propagation in the sediment in [dB/λ], H the water depth, ρ_0 is the water density, ρ_1 is the sediment density, θ_c is the critical angle, k_0 the wavenumber in water, and σ_m defined as:

$$\sigma_m = \sqrt{1 - \frac{k_{zm}^2}{k_0^2 \sin^2(\theta_c)}} \tag{7}$$

Note that any loss mechanism that can be described as exponential decay can be included in the model by adding the appropriate decay factor to β_m. Using this approach, the mode shapes are calculated for real valued sound speeds/wavenumbers in both sediment and water column. The resulting mode shapes form an orthogonal set of functions (with respect to integration over the depth) satisfying the ortho-normality relation:

$$\int_0^\infty \frac{\Psi_m(z)\overline{\Psi}_n(z)}{\rho(z)}\, dz = \delta_{nm} \tag{8}$$

2.2.2 Extension to arbitrary sources

In the equations above, which hold for the case of a point source located at depth z_s, the contribution of each individual mode m to the mode sum is given by the factors $\Psi_m(z_s)$. For an arbitrary source these factors are replaced by the unknown contribution factors C_m, and Equation (1) becomes:

$$p(r,z) \approx \frac{i}{4\rho(z_s)} \sum_{m=1}^{\infty} C_m \Psi_m(z)\, H_0^{(1)}(k_{rm} r) \tag{9}$$

The next step is to find the values for C_m for the source that is represented by the FE source model. The FE source model and normal mode propagation model are coupled at range r_0 by enforcing equality of both solutions at that range. With the solution of the FE model at range r_0 is denoted as $\Psi_{FE}(r_0,z)$ the solution of both models yields:

$$\Psi_{FE}(r_0,z) = \frac{i}{4\rho(z_s)} \sum_{m=1}^{\infty} C_m \Psi_m(z)\, H_0^{(1)}(k_{rm} r_0) \tag{10}$$

In order to determine the contribution of the discrete set of contribution factors C_m it is sufficient that the equation only holds in a weighted sense. Therefore, Equation (10) is re-written in to its so-called 'weak form' by multiplying both sides with mode shape $\Psi_n(z)$, devinding through $\rho(z)$, and integrating over the waveguide depth:

$$\int_0^\infty \frac{\Psi_{FE}(r_0,z)\overline{\Psi}_n(z)}{\rho(z)} dz = \int_0^\infty \frac{i}{4\rho(z_s)} \sum_{m=1}^{\infty} C_m \frac{\Psi_m(z)\overline{\Psi}_n(z)}{\rho(z)} H_0^{(1)}(k_{rm} r_0) dz \tag{11}$$

Using the ortho-normality relation in Equation (8) and the fact that C_m, $\rho(z_s)$, and $H_0^{(1)}(k_{rm} r_0)$ are independent of z, the following expression for C_m can be obtained by rewriting Equation (11):

$$C_m = \frac{4\rho(z_s)}{i H_0^{(1)}(k_{rm} r_0)} \int_0^\infty \frac{\Psi_{FE}(r_0,z)\overline{\Psi}_m(z)}{\rho(z)} dz \tag{12}$$

2.2.3 Approximations

In order to perform a numerical evaluation of the field at arbitrary range and depth using Equation (2) it is necessary to make an additional approximation; The normal mode sum in Equation (2) must be truncated to arrive at a finite number of evaluations. It is assumed that the modes are ordered according to the value of the wavenumber in r-direction (denoted by k_{mr}), starting from the largest real valued wave numbers and progressing to wave numbers having an increasingly smaller real part. If the mode sum is truncated after all real valued wavenumbers, associated with propagating modes, are taken into account, the influence of neglecting higher order (leaky) modes is only significant at relatively close ranges. The smallest value of the imaginary part of k_{mr} of all neglected modes determines the range at which truncation becomes acceptable.

In order to perform a numerical evaluation of the decomposition of the FE solution in terms of normal modes as described by Equation (12) an additional approximation is needed. The semi-infinite integral over depth in the equation must be truncated at a certain maximum depth. In Figure (5) the first four modes for an arbitrary Pekeris waveguide are shown.

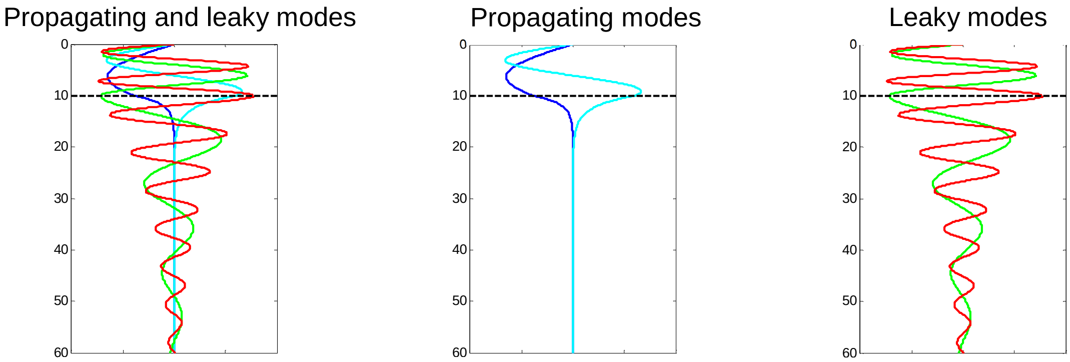

Figure 5 – Example of the first four mode shapes for an arbitrary Pekeris waveguide (left), and the same modes split into propagating modes (middle) and leaky modes (right).

The first two modes are propagating modes and having a real valued wavenumbers k_{mr} and consequently a pure imaginary wave number in z-derection in the sediment. As a result the modes exhibit exponential decay with depth in the sediment. The remaining two (leaky) modes that are shown have a complex valued wavenumber k_{mr} and consequently also a complex valued wave number in z-direction in the sediment. As a result the associated mode shape exhibits damped oscillatory behavior in the sediment.

The characteristics of the different mode types as described above are important when it comes to the effect of the depth at which the semi-infinite integral in Equation (12) is truncated. For both mode types, the amplitude of the mode shape decays exponentially with depth, implying that the influence of the truncation on the accuracy of the determined mode contribution can be controlled by the truncation depth. The amplitude of the propagating modes vanishes much faster with depth than that of the leaky modes. Together with the fact that the modes are orthogonal, this implies that for a given truncation depth the contribution factor C_m of the propagating modes will be determined more accurately then that of the leaky modes. So, to get accurate predictions at closer ranges to the source more modes are required and the truncation depth in Equation (12) needs to be chosen at a greater depth. At larger ranges only including the propagating modes suffices and the truncation depth can be kept relatively small while retaining good accuracy.

Note that for a Pekeris waveguide, the modal sum in Equation (1) does not provide the full solution (see (7)). Using a normal mode approach as described above, the evanescent modes and contributions due to the branch-cut integral are neglected. The result of neglecting the leaky modes and branch-cut integral is shortly discussed in section 3.1 and 3.3, respectively.

2.2.4 **Root-finding algorithm**

An important step in obtaining a normal mode solution is obtaining the mode wavenumbers k_{mr} using a so called root-finding algorithm. The root-finding algorithm that was implemented allows to find all propagating modes with real valued wave numbers and an arbitrary predefined number of leaky modes with complex valued wave numbers.

Alternatively, normal modes were obtained using the software tool Kraken (10). The propagating modes obtained by both codes led to very similar results. However, calculation of the leaky modes proved to be more cumbersome with the version of Kraken that was used (some of the leaky modes are ‘skipped’ by the root finding algorithm). The results of the Hybrid FE/normal mode model presented below were all obtained using the implementation by TNO.

3. VALIDATION

3.1 Point source

The hybrid model was extensively validated using the well-known case of a point source in a Pekeris waveguide. An FE model extending to a range of four times the water depth (80m) was used as a reference solution. Some example results of the hybrid FE/normal mode approach and the reference (full) FE solution are presented in Figure 6. The presented results were generated without inclusion of leaky modes.

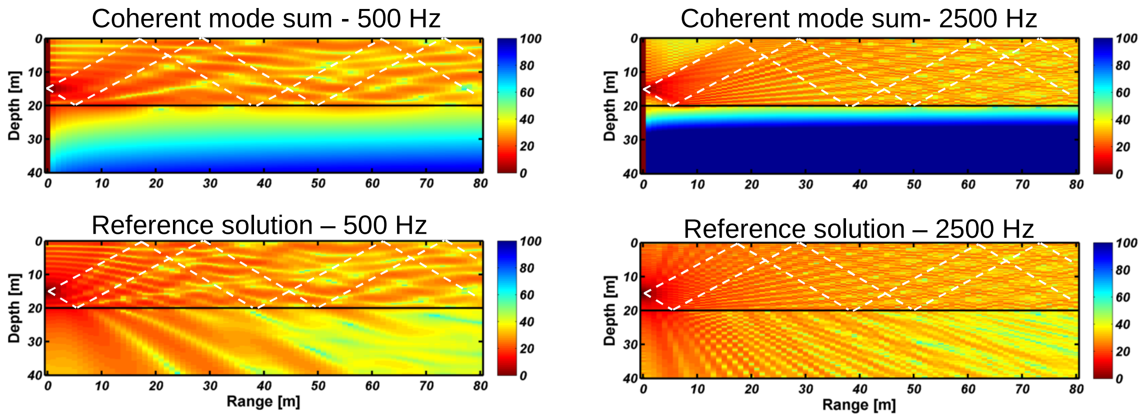

Figure 6 – Example of validation of the hybrid model results including only propagating modes against a reference FE solution for a point source in a Pekeris waveguide of depth 20 m at 500 Hz and 2500Hz. The fluid sound speed in the water and sediment are 1500 and 2000 m·s^{-1} respectively, and the density in the water and sediment are 1000 and 2000 kg·m^{-3} respectively. Sound rays leaving the source at the critical angle are indicated by white dashed lines.

Note that at larger ranges ($r > 50$ m) the hybrid solution matches well with the references FE model in the water column for frequencies above the waveguide cut-off frequency. At closer ranges the omission of leaky modes prevents convergence. The sound field in the areas directly above and below the source are dominated by the contribution of leaky modes which are not included in the coherent sum. Therefore, in the results of the coherent mode sum a large difference in amplitude can be observed at close range ($r < 20$ m) between the area enclosed by lines leaving the source at the critical angle and the areas above and below these lines. This artifact of the normal mode solution is more pronounced for higher frequencies.

3.2 COMPILE

The Hybrid FE/normal mode model presented above was benchmarked against other models for prediction of underwater pile driving noise in the international COMPILE workshop held in Hamburg on 18 and 19 June 2014. The compile case included a Pekeris waveguide of 10m depth with a 0.05m thick, 2m diameter pile of 25m length penetrating 15m into the sediment. The fluid sound speed in the water and sediment are 1500 and 1800 m·s^{-1} respectively, and the density in the water and sediment are 1025 and 2000 kg·m^{-3} respectively. The steel pile has a density of 7850 kg·m^{-3}, a Young’s modulus of 210 GPa, and Poisson ratio of 0.3. The damping of the p-wave in the sediment is 3·10^{-5} Np·m^{-1}Hz^{-1}, and the equivalent damping (accounting for the friction between pile and sediment) of the p- and s-wave in the section of the pile penetrating in the sediment are 3·10^{-5} and 11·10^{-5} Np·m^{-1}Hz^{-1}. The forcing applied to the top of the pile as a function of time linearly increases from zero to the maximum value during a short rise time, followed by exponential decay (resembling the exponential pulse described by Reinhall and Dahl (2)). The maximum force exerted is 20 MN, the rise time of the pulse is 0.2 ms and the decay time is 1.6 ms.

Seven institutes contributed results obtained with their own Hybrid models: Curtin University, German Federal Armed Forces - WTD 71, Hamburg University of Technology (TUHH), JASCO, Seoul National University, University of Southampton and TNO. The close range and far range models that were used varied from FE models in frequency and time domain, Normal mode, Finite Difference, wave number integration, Parabolic Equation, equivalent point source arrays to empirical models. The results for each model performed at two depths at ranges 1 m, 11 m, 31 m, 750 m, 1500 m, 10 km, 20 km, and 50 km are presented in Figure 7. The results at a depth of 9 m which are not shown here are very similar to those at 5m depth. The similarity between the results of the hybrid FE/normal mode model presented above (shown in red) and the other models (shown in gray) builds confidence that the hybrid model is well suited for this type of problem and is implemented correctly.

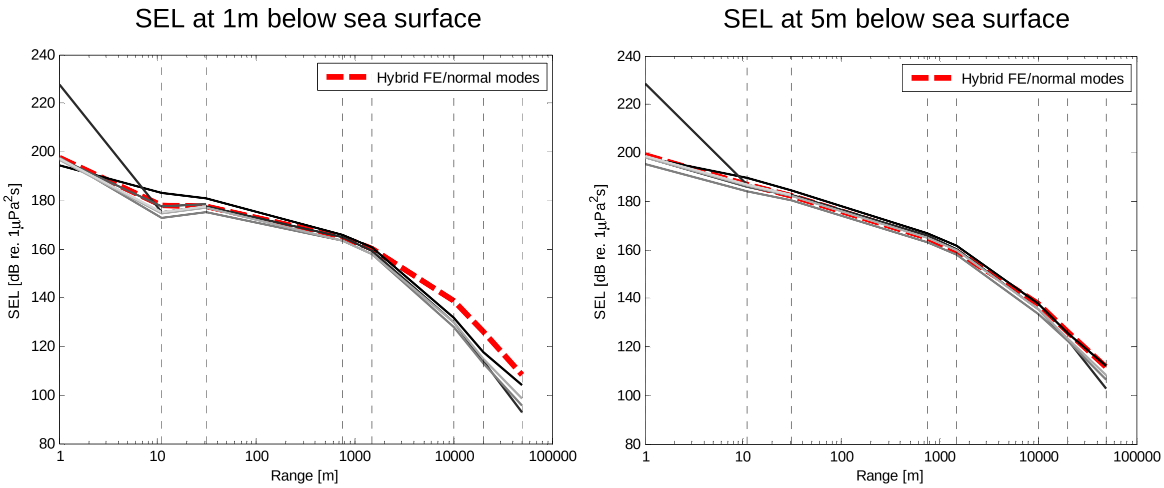

Figure 7 – Sound exposure level as a function of range for the seven models developed by participants of the COMPILE workshop benchmark case. The results of the TNO hybrid/FE normal mode model are in red, the other models are in different shades of gray. The distances for which calculations are performed are 1m, 11m, 31m, 750m, 1500m, 10km, 20km, and 50km indicated by the vertical dashed black lines. Overall, the spread between the model results is remarkably small, especially at 5m below the sea surface.

Note that leaky modes were taken into account only for the smallest three ranges. For these ranges, all leaky modes having a wavenumber with a real part at least half as large as its imaginary part were taken into account.

3.3 Influence of leaky modes

The influence of including leaky modes on the calculated sound field is illustrated in Figure 8. The predicted sound as radiated by a pile in a Pekeris waveguide at 500 Hz for the COMPILE case described in section 3.2 is shown for three different models. The results of the FE model serve as a reference solution. The other two solutions shown are obtained with the hybrid FE/normal mode model described above with and without including leaky modes.

For the frequency that is considered (500 Hz) the number of leaky modes taken into account is sufficient for convergence of the sound field in the water column and sediment at ranges larger than 10m. However, while the solution in the water column corresponds relatively well with the reference solution for those ranges, the solution in the sediment deviates dramatically from the reference solution for all considered ranges. As mentioned in section 2.2.3 the presented normal mode sum does not provide the full solution to the Pekeris waveguide problem. One of the possible causes for the observed differences is the omission of the branch-cut integral. Another possibility is that the normal modes for a Pekeris waveguide do not form a full basis to expand the source that is modeled here correctly. In most problems involving a Pekeris waveguide considered in literature, the sources are contained in the water column, while in this case, the source extents into the sediment.

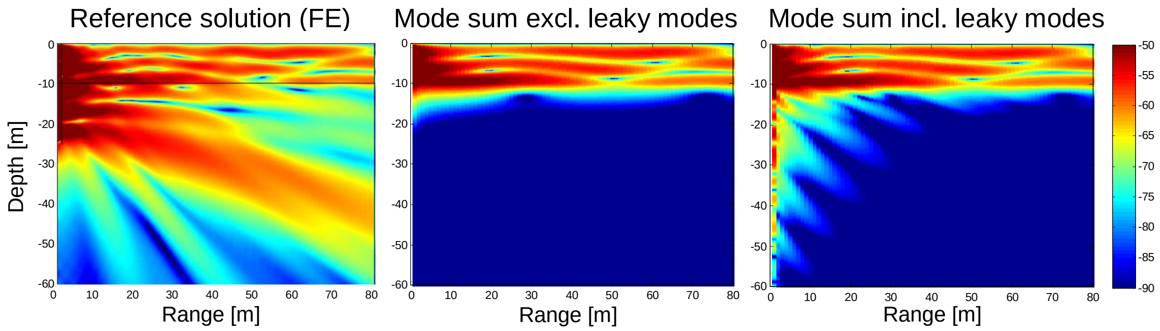

Figure 8 – The sound radiated by a pile in a Pekeris waveguide at 500 Hz for the case described in section 3.2; reference FE solution (left), coherent mode sum excluding leaky modes (middle), coherent mode sum including leaky modes (right). The number of leaky modes is such that convergence has been reached for r > 10 m. In the sediment both solutions with and without leaky modes show large differences with the reference solution.

Preliminary investigations of the reconstruction of the field using normal modes at the location where the decomposition using Equation (12) is performed, shows that the field in the sediment is not reconstructed properly while the field in the water column is reconstructed fairly accurately. This supports the theory that the mode base used for decomposition is insufficient to describe sources in the sediment.

An important question is to what extend the currently not represented part of the solution is orthogonal to the normal modes that are used. If the additional solutions are orthogonal to the normal modes, the calculated contributions of the normal modes using Equation (12) are correct. If this is not the case, Equation (12) will only yield an approximation of the contribution of the different normal modes. Another important question is to what extend the missing part of the solution involves energy entering the waveguide from the sediment. If no additional energy enters the waveguide through these non-modeled mechanisms, the calculated SEL based on normal modes is correct. If this is not the case the calculated SEL levels are an approximation. The fact that the normal mode solution closely resembles the reference solution at larger ranges suggest that at these ranges there is no unaccounted mechanism that inserts (or extracts) energy into the waveguide.

4. CONCLUSIONS

The presented results show that a Hybrid FE/normal mode model for the prediction of sound in a Pekeris waveguide due arbitrary sources was successfully implemented. It was demonstrated that the hybrid model yields a good prediction of the sound field in the water column (which is the intended region of use for the model).

there are larger unexplained differences observed for the sound field in the sediment. A preliminary investigation suggests that these differences can be associated with the fact that a normal mode approach does not yield a full solution for sources in a Pekeris waveguide that extend into the sediment.

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