



# Acoustic radiation response prediction of thin-walled box with particle dampers using multiphase flow theory of gas-particle

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## ABSTRACT

In this paper, acoustic radiation response of thin-walled box with particle dampers is predicted. First, the effect of the collisions and friction between the granular inter-particles is interpreted as an equivalent nonlinear viscous damping based on multiphase flow theory of gas-particle. Then the contribution of particle damper is estimated as an equivalent mass-damper system. And the acoustic radiation response of thin-walled box with particle dampers is predicted in the finite element method. Finally, an originality and novelty simulation method is developed to evaluate acoustic radiation response characteristics of particle damping composite structure by COMSOL. With this as a base, detailed numerical studies using the originality simulation method are also carried out to analyze the acoustic radiation response characteristics of the particle-damping thin-walled box. An experimental verification is conducted, and a good agreement between the theoretical results and the experimental data shows that the theoretical work in this paper is valid

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I-INCE Classification of

## 1. INTRODUCTION

Shock and acoustic environments can have significant impact in the overall design of the precision instrument, machinery and machine tools, auto industry and aeronautic equipment such as a type spacecraft. Active or passive control techniques are developed to suppress vibrations in mechanical and structural systems or the civil engineering. General approach to reduce vibration and noise is to provide attenuation by increasing damping. In practice, passive control is preferred due to its simplicity and ease to implement and control, with no need for any auxiliary power equipment (1). Currently in engineering field, widely used damping devices include viscoelastic materials, frictional devices, tuned absorbers, isolators, single unit impact damper (1-DOF or 2-DOF), multi-unit impact damper, bean bag impact damper, resilient impact damper and buffered impact damper, etc. Often, operating environment and temperature requirement and other restrictions pose a considerable challenge in the design of an effect passive control device to suppress vibrations and acoustic radiation that can survive these environments.

One attractive alternative is particle damping which has been shown to be an effective technique for vibration suppression, and has been successfully utilized in numerous aerospace, automotive and industrial applications. Particle damping is a derivative of single-mass impact damper, which can perform well even in severe environments where traditional passive damping methods such as the use of viscoelastic materials are ineffective. Granular particle damping is a promising technique of providing damping with granular particles placed in an enclosure attached to or embedded in the holes drilled in the vibrating structure(2, 3). Additional benefits of using granular materials instead of a single mass include the elimination of excessive noise and potential damage to the interior wall of the containing hole. The dynamic response of the primary structure is improved without adding considerable mass or requiring hardware redesign. It offers several advantages due to its conceptual simplicity, potential effectiveness over broad frequency range, temperature and degradation

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insensitivity, and very low cost(4-9). And it has been implemented extensively on several aerospace applications to reduce shock and acoustic radiation. In typical applications, the use of particle dampers has resulted in significant reductions in resonant and broad band vibration response(10, 11).

Particle damping has been thoroughly studied over the years with a large volume of books and papers in the published literature. Most particle damping researches have focused on experimental rather than analytical results, but recently there have been attempts at developing analytical models to simulate the energy dissipation performance of particle dampers(8, 9, 12).

Although at this stage comprehensive constitutive relationships for modeling have not been fully established, several approaches which have proved useful are available for predicting the particle damping behavior, particularly for engineering application purposes. Many authors using the lumped mass approach by numerical analysis modeled a bed of particles as a single particle, estimating the performance of the particles damper based upon this equivalent particle(6, 9, 13-17). There have been considerable researches in the field of particle damping, and some analytical models have been developed by Discrete Element Method(DEM)(8, 18-26). It is very regrettable that the above application fields are only limited to the Single-Degree-of-Freedom (SDOF) system or the equivalent SDOF. One of the principal reasons is that particle damping is a very complicated phenomenon and remarkable nonlinear behavior. Several parameters have been shown to affect damping performance, including particle size, material, density, particle interface friction, cavity geometry, cavity fill ratio, and damper orientation with respect to gravity, etc(27). Although there have been some limited numerical and experimental studies and applications of the particle damping technology, the theory studies of the continuous particle damping structure to predict acoustic radiation response are relatively scarce.

Therefore, it is desirable to develop a feasible modeling method for the continuous particle damping structure to predict acoustic radiation response. The primary objective of this paper is to develop a new simulation method based on the multiphase flow theory of gas-particle, which is capable of rapidly predicting acoustic radiation response for the complex continuous structure with particle dampers by the COMSOL. It is worth mentioning that the proposed modeling approach is studied here for the first time in the existing literature for particle damping technology.

## 2. MODEL DEVELOPMENT

More recently, Wu et al. (28)explored a theoretical model based on multiphase flow theory of gas-particle to evaluate the granular particle damping characteristics. Fang and Tang (29)further validated the multiphase flow theory of gas-particle approach based on previous work of Wu et al. (28), and carried out detailed studies where the damping effect due to different energy dissipation mechanisms was quantitatively analyzed. Wu et al. (30, 31) further carried out detailed studies to the energy dissipation in particle damping. An improved analytical model for particle damping was developed based on multiphase flow theory of gas-particle.

As mentioned in Ref.(30, 31) the effect of the collisions and friction between the particles is interpreted as an equivalent nonlinear viscous damping based on multiphase flow theory of gas-particle as the following forms:

$$C_{eq} = c_1 |\dot{x}|^{1/2} + c_2 |\dot{x}| - c_3 |\dot{x}|^{2/3} + c_{11} |\dot{x}| + c_{21} |\dot{x}|^2 - c_{31} |\dot{x}|^3 \quad (1)$$

where

$$c_1 = 4\bar{c}\alpha^{1/2} f^{1/2}, \quad c_2 = 4\bar{c}\alpha, \quad c_3 = \bar{c}\alpha^{3/2} f^{-1/2} \quad (2)$$

$$c_{11} = 4\bar{c}\alpha_1^{1/2} f^{1/2}, \quad c_{21} = 4\bar{c}\alpha_1, \quad c_{31} = \bar{c}\alpha_1^{3/2} f^{-1/2} \quad (3)$$

with

$$\bar{c} = (3/16)\pi^3 d^2 h \rho_m, \quad \alpha = K_1 / K_3 \quad (4)$$

$$\alpha_1 = K_2 / K_3, \quad K_3 = \pi d^2 \rho_m \quad (5)$$

$$K_1 = \frac{1}{5} \sqrt{\frac{6}{\pi}} (1 + e_p) \alpha_p^2 g_p \rho_p d_p \quad (6)$$

$$K_2 = \frac{(\alpha_p \rho_p + 2\rho_p (1 + e_p) g_p \alpha_p^2) \sin \phi}{12\sqrt{I_{2D}}} \quad (7)$$

where  $e_p$  is the restitution coefficient of the particle,  $\alpha_p$  is the packing ratio defined as the volume of particles to the total volume of the cavity,  $g_p$  is the radial distribution function,  $\phi$  is the angle of

internal friction,  $I_{2D}$  is the second invariant of the deviatoric stress tensor,  $\rho_m$  is the equivalent volume density of the mixture flow related to the densities of the gas,  $|x^*|$  is the amplitude of vibration velocity,  $f$  is the frequency,  $d$  is the diameter of the cavity and  $h$  is the height of the cavity.  $\rho_p$  and  $d_p$  denote the density and the mean diameter of particles respectively, The derivation process of the formulas and the description of parameters can be found in our previous work(30, 31).

For the continuous particle damping structure, the contribution of particle damper is estimated as an equivalent spring mass system; however the system does not exhibit any stiffness, i.e., mass damping system. The Figure 1 represents a schematic of the particle damper and adopted model. The damping coefficient of the spring mass system is to respond to the equivalent nonlinear viscous damping coefficient  $C_{eq}$  (see the equation(1)) determined for different levels of excitation and depending on the excitation velocity amplitude.  $M_{eq}$  represents the mass of the particle damper with the particles. So the complicated continuous structures treated with the particle dampers are conducted in finite element method by COMSOL.

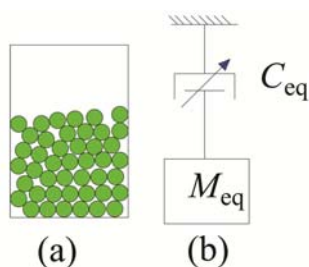


Figure 1 – (a) Particle damper and (b) model

Next, for the sake of brevity, here a thin-walled box with particle dampers is considered as an attempt to valid this method, which is zoomed to a scale of 1:10 to a car body-in-white size. Figure 2 presents a schematic of the apparatus and the particle dampers exerted position. The car is fixed at both ends as shown in the Figure 2(a). The material of the thin-walled box is steel with a mass density  $\rho = 7850 \text{ kg/m}^3$ , Young’s modulus  $E = 205 \text{ GPa}$ , and Poisson ratio  $\nu = 0.3$ . The geometry dimensions of the car model are length  $\times$  height  $\times$  depth =  $505 \text{ mm} \times 192 \text{ mm} \times 190 \text{ mm}$ . And the wall thickness of the car model is 2mm. The particle dampers exerted position is illustrated in the Figure 2(b). The particle dampers are evenly exerted on the circumference of a circle with the diameter 100mm, and another one particle damper is exerted on the center of the bottom. The five particle dampers used in this experiment have the same design parameters. The mass of the enclosure is 14.52 g and its interior diameter and height are 16 mm and 20mm, respectively. The excitation position is at 55mm distance from the center of the bottom and the microphones are at the xy plane, which is 96mm above the bottom (see in the Figure 2(a)). The microphones are evenly arranged on the circumference of a circle with the diameter 1000mm with the interval  $60^\circ$  (as shown in the Figure 3(b)).

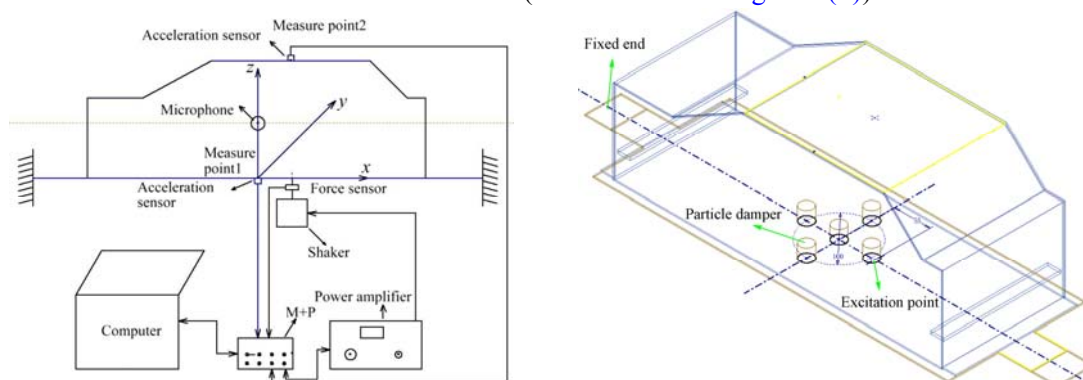


Figure 2 – (a) A schematic of the apparatus (b) particle dampers exerted position

The particle is made of tungsten powder whose density is  $17000 \text{ kg/m}^3$ , and the mean diameter of particles is 0.3 mm. The restitution coefficient of particles is 0.6 on the basis of testing. The kinetic friction coefficients between the individual particle is 0.3, from experimental results. In addition, the kinematic viscosity and density of air are  $1.51 \times 10^{-5} \text{ m}^2/\text{s}$  and  $1.21 \text{ kg/m}^3$ , respectively. This test is

made with the same mass of particles ( $\alpha_{mp} = 40\%$ ). The particles mass of each particle damper filled is  $11.60 \times 10^{-3}$  kg. The mass packing ratio  $\alpha_{mp}$ , which is defined as the actual packing mass of particles to the maximum permissive packing mass of particles in a cavity, is also introduced to describe the packing condition of the damper.

Consider the intrinsic structure damping and particle damping, the motion of the global system is governed by

$$[M] \cdot \{\ddot{X}\} + [C] \cdot \{\dot{X}\} + [K] \cdot \{X\} = \{F\} \tag{8}$$

where  $\{X\}$  is the nodal displacement of the plate,  $\{F\}$  is the external force applied to the system.  $[K]$  and  $[M]$  represent, respectively, the stiffness and mass matrix of both the model car and the particle dampers.

For the acoustic-solid interaction, this boundary condition includes the following interaction from fluid (gas) to solid and vice versa: to couple the acoustic pressure wave to the solid wall, the boundary load  $\mathbf{F}$  (force/unit area) on the wall is set to

$$\mathbf{F} = -\mathbf{n}_s p \tag{9}$$

where  $\mathbf{n}_s$  is the outward-pointing unit normal vector seen from inside the solid domain,  $p$  is the pressure ( $\text{N/m}^2$ ).

The structural acceleration acting on the boundaries between the solid and the fluid(gas) makes the normal acceleration for the acoustic pressure on the boundary equal to the acceleration based on the second derivatives of the structural displacements  $\mathbf{u}$  with respect to time:  $a_n = \mathbf{n}_a \cdot \omega^2 \mathbf{u}$ .

To couple back the frequency response of the solid to the acoustics problem, use a normal acceleration boundary condition

$$-\mathbf{n}_a \cdot \left( -\frac{1}{p_o} \nabla p + \mathbf{q} \right) = a_n \tag{10}$$

where  $\mathbf{n}_a$  is the outward-pointing unit normal vector seen from inside the acoustics domain,  $\mathbf{q}$  is an optional dipole source ( $\text{m/s}^2$ ). In the frequency domain,  $p_o$  is the amplitude of a harmonic pressure source. Also set the normal acceleration  $a_n$  to  $(\mathbf{n}_a \cdot \mathbf{u}) \cdot \omega^2$ , where  $\mathbf{u}$  is the calculated harmonic-displacement vector of the solid structure.

Figure 3 shows a sketch of the modeled system. The ellipsoid is located inside a computational domain of total radius  $R_i + R_{pml}$ , where  $R_i = 500\text{mm}$  and  $R_{pml} = 50\text{mm}$ . The layer of thickness  $R_{pml}$  is the absorbing Perfectly Matched Layer (PML); see Figure 3(a). The PML is used as a non-reflecting and absorbing boundary which mimics a domain stretching to infinity. The scattered field off the ellipsoid is denoted by  $p$ .

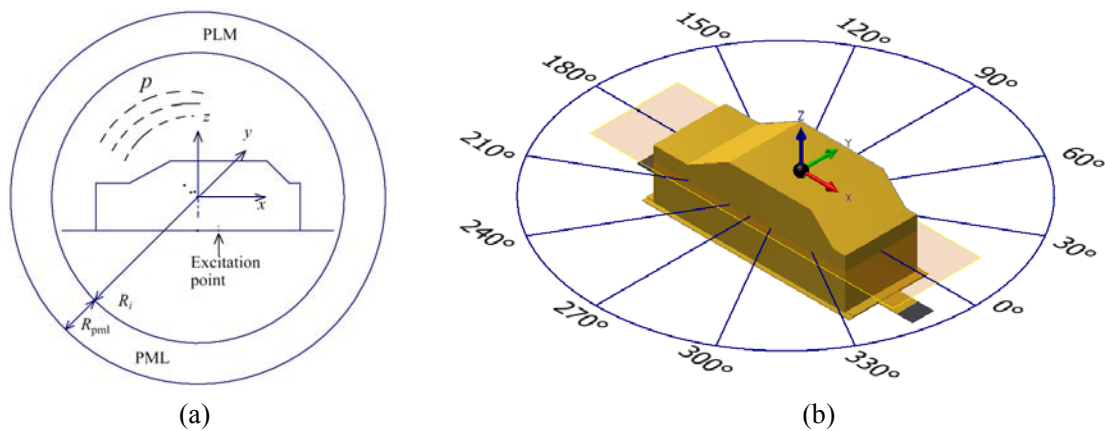


Figure 3– Sketch of the modeled system presenting the –scattered field  $p$ , geometric scales, and the PML layer

### 3. NUMERICAL SIMULATION

In order to identify the damping of the particle damper on acoustic radiation response effect of the system versus the frequency of excitation, polar plot of the far-field sound pressure level in the xy-plane is measured. An excitation frequency is used with 700Hz under forced vibration.

Using the far-field calculation feature, the scattered field is determined at a given distance outside the computational domain. The result is presented as 2D plot groups and as polar plots of the scattered far-field sound pressure level.

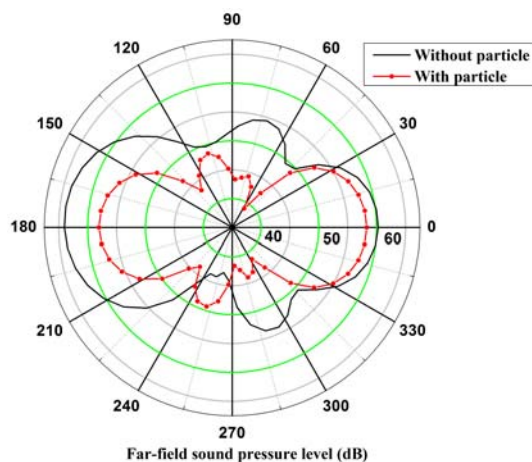


Figure 4 – Polar plot of the far-field sound pressure level in the xy-plane

Figure 4 plots the sound pressure level in the far field at the distance  $R_{far} = 1000\text{mm}$ . The data is retrieved in the xy-plane and presented as a polar plot, with  $0^\circ$  corresponding to the positive x direction (see the [Figure 3\(b\)](#)). This can be demonstrated by the fact that the scattered acoustic performance pressure may be effectively improved by added particle dampers. The result also indicates that the sound field has obvious symmetry, the sound pressure level in the head and tail of the model car is bigger than that in two sides of the model car. It is worth noting that the far-field sound pressure level of the system with particle damper is even bigger than that of the system without particle dampers within a certain azimuth range. The far-field sound pressure level of the system with particle damper leads to the increase within the range of angles from  $225^\circ$  to  $270^\circ$  as shown in [Figure 4](#). It is possible that the mode shapes of the system could be changed due to the added particle dampers. This result confirms that the damper efficiency is related to the mode shapes. This also reveals that total particle mass and the particle dampers arrangements appear to have a fairly significant effect on damping for the particle damping structure.

### 4. EXPERIMENTAL VERIFICATION

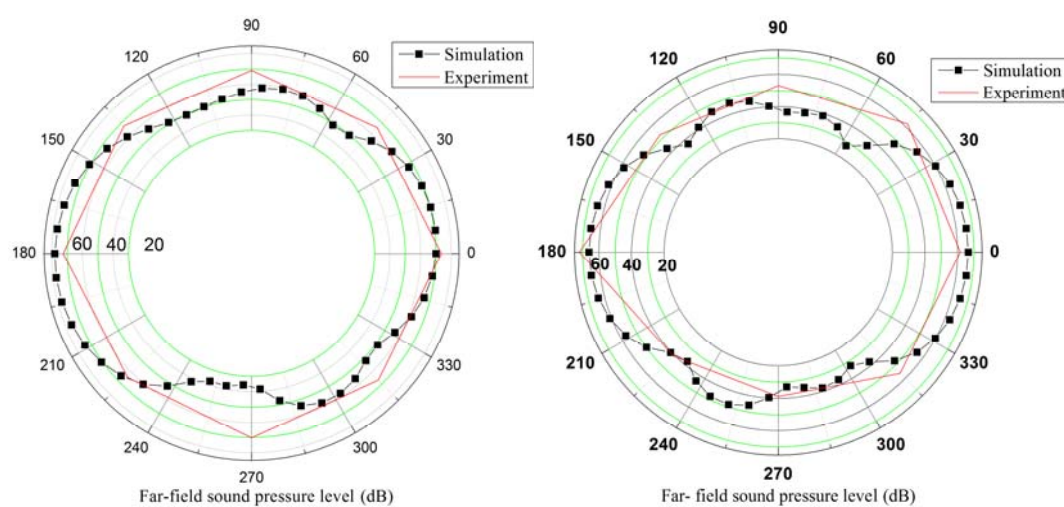
To verify the theoretical model developed in this study, an experiment for the thin-walled box with granular particle dampers is set up. The specifications for the experiment are the same as those shown in the simulation for the purpose of comparison. A schematic of the experimental apparatus used is shown in [Figure 5](#).



Figure 5 – A schematic of the experimental apparatus used

The experimental model consists of the primary structure (thin-walled box) and five aluminium enclosures containing tungsten particles. The enclosures that are partially filled with tungsten particles are attached to the bottom of the model car which is itself attached to an electromagnetic shaker (M B MODAL 50A) providing an excitation force. The signal of the harmonic excitation is amplified by Power Amplifier (M B500VI) to the shaker. The microphone (Dytran 40AE/26CA) is measure the far field sound pressure. A Dynamic Signal Analyzer (M+P SO Analyzer) made in Germany is used to collect and process the data. The whole test process is made in the semi-anechoic room.

Figure 6 shows the polar plot of the sound pressure level of the simulation and experiment result when the system without particle dampers and with the particle dampers. It can be seen from the compared results that good correlations are obtained between the analytical results and the experimental data. It is noted that the simulation results in COMSOL differ slightly from the experimental results. There are also differences within a certain azimuth ranges between the numerical and experimental acoustic radiation response for the system without particle dampers and with particle dampers. These differences stem from the hypothesis considered when modeling the system. Nevertheless, the analysis of simulation results of the structure with particle dampers show the ability of the model developed in this work to predict acoustic radiation response of the continuous structure taking into account the effect of particle damping.



(a) Without particle dampers

(b) with particle dampers

Figure 6 – Polar plot of the sound pressure level at distance  $R_{far} = 1000\text{mm}$

## 5. CONCLUSION

In this paper, in view of predicting the acoustic radiation responses of particle-damping composite structures rapidly and effectively, the combined studies of the theoretical derivation, numerical simulation and experimental research are conducted. The acoustic radiation response of thin-walled box, which is zoomed to a scale of 1:10 to a car body-in-white size, are studied, and experimental verifications are performed. An analytical model based on the two-phase flow theory of gas-particle is developed, where an equivalent viscous damping for the inter-particle collisions and friction is introduced. It is convenient to investigate the performance of particle damping in terms of the effective viscosity due to inter-particle collisions and friction. This makes the entire model provide convenience for further studies in depth. The numerical and experimental studies show that the particles are helpful to structures for adding damping to attenuate the far-field sound pressure of the host structures. In general, good correlation is obtained between the measured responses and the theoretical model predictions for thin-walled box with particle dampers. It shows that the new simulation method in this paper is valid which can predict acoustic radiation response of a continuous structure treated with the particle dampers. In a word, the research results show that the analytical model based on the two-phase flow theory of gas-particle are simple and effective, and they not only lay an important theoretical foundation for the further study of the vibration and acoustic radiation of particle damping composite structure, but also provide an important theoretical guidance for low noise optimization design of particle-damping structure, and have important reference value for noise control of this kind of structures.

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