Numerical Analysis of Sound Wave Propagation Using CIP-MOC Method with Non-Uniform Grid

Yuta Matsumura[1]; Kan Okubo[2]; Norio Tagawa[3]; Takao Tsuchiya[4]; Takashi Ishizuka[5]

[1] Tokyo Metropolitan University, Japan

[2] Tokyo Metropolitan University, Japan

[3] Tokyo Metropolitan University, Japan

[4] Doshisha University, Japan

[5] Shimizu Corporation, Japan

ABSTRACT

In recent years, time-domain numerical analysis for sound wave propagation has been investigated widely as a result of advances in computer technology. For sound field imaging and/or prediction, the development of accurate numerical schemes is an important issue. A method of characteristics (MOC) is used as a time domain numerical analysis method, examples of which are the constrained interpolation profile (CIP) method, the LAX method, and the QUICKEST method. We used the MOCs for numerical analyses of sound wave propagation in an earlier study. However, new grid systems are required for the CIP large-scale simulations of wave propagation. Additionally, for multidimensional analysis, the high-efficiency outer absorbing boundary is also required. To overcome these problems, we introduce the non-uniform grid system with perfectly matched layer (PML) technique. The purpose of this study is to evaluate these techniques for two-dimensional (2D) sound field numerical analysis using the MOCs. The present results indicate that these techniques for MOCs have advantages of small memory requirements and less calculation time.

Keywords: constrained interpolation profile (CIP) method, method of characteristics (MOC), non-uniform grid, perfectly matched layer (PML)

I-INCE Classification of Subjects Number(s): 01.4, 76.9

1. INTRODUCTION

To date, as a result of computer development, numerical analysis for sound wave propagation in time-domain has been investigated widely. The development of accurate numerical schemes is an important issue [1]. The finite-difference time-domain (FDTD) method [2, 3, 4] using the staggered grid is one of the most well-known schemes used in acoustics, although many numerical schemes have been proposed for time domain analysis. However, we know that, using Yee's leapfrog algorithm [2], finite difference approximation certainly causes error owing to numerical dispersion. This means that the scheme is not so suitable for the analysis including rapid change in sound pressure or large-scale modelling of wave propagation.

In this study, we examine the methods of characteristics (MOCs) [5] using the collocated grid as a numerical analysis method. These methods have an advantage that the treatment of the interface of different media is simpler than the staggered grid-based methods.

The constrained interpolation profile (CIP) method [6, 7, 8, 9, 10, 11], a method of characteristics (MOC), is a novel low-dispersive numerical scheme. In our past sdudy, we have applied the CIP method to numerical analyses of sound wave propagation. The feature of the CIP method is that it uses the values of acoustic field and their spatial derivatives at grid points to solve the problem of wave propagation. The family of this scheme is called "multi-moment Scheme". By these treatment of fields, the CIP method has high accuracy in numerical phase velocity for very wide frequency bands [7, 8, 9].

On the other hand, new grid systems are required for the CIP large-scale simulations of wave propagation.

[1] ymattsun28@gmail.com

[2] kanne@tmu.ac.jp

[5] ishiduka@shimz.co.jp

In the previous study, sub-grid techniques [12] are proposed for the CIP method to reduce the calculation time and memory usage. However, handling the derivatives of the perpendicular directions at the interface between different sizes of grid is complicated in this technique.

Therefore, we introduced the non-uniform grid for the CIP method. This technique as well as sub-grid has an advantage of using a small amount of memory. Additionally the acoustic numerical analysis by MOCs, including CIP method, requires to set the absorbing-boundary condition (ABC), because the so-called automatic absorbing boundary (without additional outer boundary treatment) does not exhibit high-efficiency absorbing performance for multidimensional analysis. Consequently, we introduce the perfectly matched layer (PML) [14] technique into the non-uniform grid system for MOC simulations of wave propagation. In this study, we evaluate the non-uniform grid technique with PML for two-dimensional (2D) sound field numerical analysis.

2. NON-UNIFORM GRID SYSTEM IN CIP-MOC METHOD

2.1 CIP method

The governing equations for linear acoustic fields are given as

$$\nabla \cdot \vec{u} = -\frac{1}{K}\frac{\partial p}{\partial t}, \tag{1}$$

$$\rho \frac{\partial \vec{u}}{\partial t} = -\nabla p. \tag{2}$$

Therein, ρ denotes the density of the medium, K represents the bulk modulus, p is the sound pressure, and $\vec{u}$ is the particle velocity. We assume that the calculation is for a lossless medium. Here, for simplicity, assuming $\vec{u} = (u_x, 0, 0)$ in order to analyze one-dimensional (1-D) acoustic field propagation of the x-direction, we can obtain the following equations:

$$\frac{\partial}{\partial t}p + c\frac{\partial}{\partial x}Zu_x = 0, \tag{3}$$

$$\frac{\partial}{\partial t}Zu_x + c\frac{\partial}{\partial x}p = 0. \tag{4}$$

In those equations, Z indicates the characteristic impedance (i.e. $Z = \sqrt{\rho K}$) and c represents the sound velocity in the medium (i.e. $c = \sqrt{K/\rho}$).

Then, in CIP analysis, by addition and subtraction of these two equations, we obtain

$$\frac{\partial}{\partial t}(p \pm Zu_x) \pm c\frac{\partial}{\partial x}(p \pm Zu_x) = 0. \tag{5}$$

In addition, through simple spatial differentiation of the equations, the equations of the derivatives are given as

$$\frac{\partial}{\partial t}(\partial_x p \pm Z\partial_x u_x) \pm c\frac{\partial}{\partial x}(\partial_x p \pm Z\partial_x u_x) = 0. \tag{6}$$

Therein, $\partial_x = \frac{\partial}{\partial x}$.

We show the procedure to calculate the fields of the $(n+1)$ time step from the fields of n time step, applying the MOCs (inc. the CIP method) to discretized acoustic field components. The calculation method of the $\pm x$-direction propagation is described below according to Fig. 1. We define $F_{x\pm}$, and $G_{x\pm}$ as follows: $F_{x\pm} = p \pm Zu_x$, and $G_{x\pm} = \partial_x p \pm Z\partial_x u_x$. Consequently, the field components defined on grid points ($x = i\Delta x$) at the time step n give F_{x+}, F_{x-}, G_{x+} and G_{x-} as

$$\begin{aligned} F^n_{x\pm}(i) &= p^n(i) \pm Zu^n_x(i), & (7)\\ G^n_{x\pm}(i) &= \partial_x p^n(i) \pm Z\partial_x {u_x}^n(i). & (8) \end{aligned}$$

Applying the interpolation operators (H and H') to $F^n_{x\pm}(i)$ and $G^n_{x\pm}(i)$ yields the following equations related to propagation to the $\pm x$-direction.

$$\begin{aligned} F^{n+1}_{x\pm}(i) &\leftarrow \mathsf{H}(F^n_{x\pm}, G^n_{x\pm}), & (9)\\ G^{n+1}_{x\pm}(i) &\leftarrow \mathsf{H}'(F^n_{x\pm}, G^n_{x\pm}). & (10) \end{aligned}$$

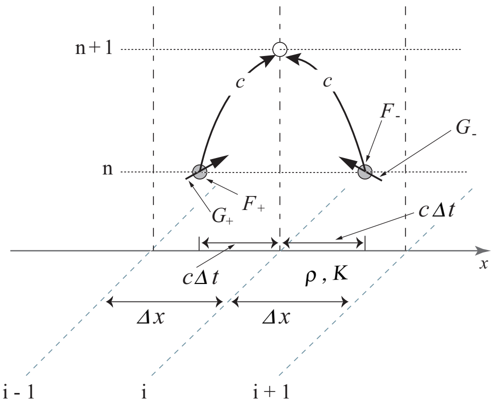


Figure 1 – The calculation method of $F_{x+}(i)$, $F_{x-}(i)$, $G_{x+}(i)$ and $G_{x-}(i)$ to $\pm x$ direction using the CIP method.

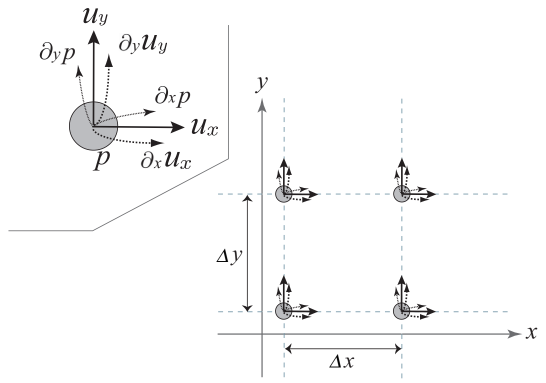


Figure 2 – Collocated grid of the acoustic field in MOC /CIP analysis.

Moreover, using the following eqs. (11), (12), (13) and (14), one can obtain acoustic field components (p and u_x) of time step $(n+1)$.

$$p^{n+1}(i) \leftarrow \frac{F_+^{n+1}(i)+F_-^{n+1}(i)}{2} \tag{11}$$

$$u_x^{n+1}(i) \leftarrow \frac{F_+^{n+1}(i)-F_-^{n+1}(i)}{2Z} \tag{12}$$

$$\partial_x p^{n+1}(i) \leftarrow \frac{G_+^{n+1}(i)+G_-^{n+1}(i)}{2} \tag{13}$$

$$\partial_x u_x^{n+1}(i) \leftarrow \frac{G_+^{n+1}(i)-G_-^{n+1}(i)}{2Z} \tag{14}$$

It is noteworthy that we can calculate sound wave propagation of the y-direction in a similar manner to that of calculation of the x-direction. Fig. 2 depicts the 2-D grid model used in this analysis, in which both acoustic field components (p, u_x and u_y) and the derivatives of fields ($\partial_x p$, $\partial_y p$, $\partial_x u_x$ and $\partial_y u_y$) are located on the same grid(i.e., collocated grid). Acoustic field propagation is solvable using these discretized components.

2.2 Non-uniform grids

Figures 3(a) and 3(b) show the schematic of a non-uniform grid and sub-grid system. Memory required for the non-uniform grid system is slightly larger than that of the sub-grid as shown in these figures. However, handling the interface between different sizes of grid is more complicated in the sub-grid system. Here, Δx_c and Δy_c represent the course grid size, while Δx_f and Δy_f are fine grid size, respectively.

Figure 4 shows the aspect of the discretized sub-grid for the CIP method. The difference between the type-M and type-C CIP methods is the handling of a second-order special derivative [11]. The type-M CIP method is a simple technique with smaller memory use and less calculation time required than the type-C CIP method.

2.3 PML in non-uniform grid

Next, we show the PML [14] formulation for MOCs. In the PML region, the advection equations are given as

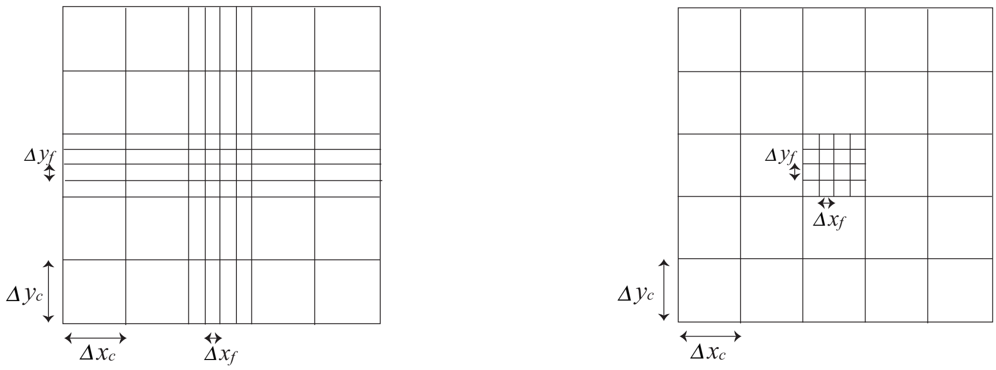
$$\frac{\partial}{\partial t}F_{x\pm} \pm c\frac{\partial}{\partial x}F_{x\pm} = -rF_{x\pm}, \qquad F_{x\pm} = p \pm Zu_x, \tag{15}$$

$$\frac{\partial}{\partial t}G_{x\pm} \pm c\frac{\partial}{\partial x}G_{x\pm} = -\left(\frac{\partial}{\partial x}r\right)F_{x\pm} - rG_{x\pm}, \qquad G_{x\pm} = \partial_x p \pm Z\partial_x u_x. \tag{16}$$

These equations are for calculations in the x-direction. We treat the advection and non-advection phases separately in the PML region [13]. The non-advection phase is solved by the simple finite difference method.

If we let here results of the advection calculations at a time step n be $F_{x\pm}^{n*}$ and $G_{x\pm}^{n*}$, in the non-advection phase, the solution of Eqs. (15) and (16) gives $F_{x\pm}^{n+1}$ as

$$F_{x\pm}^{n+1} = (1-r)F_{x\pm}^{n*} \tag{17}$$



(a) Non-uniform grids (b) Sub-grid

Figure 3 – Non-uniformity grid and sub-grid model

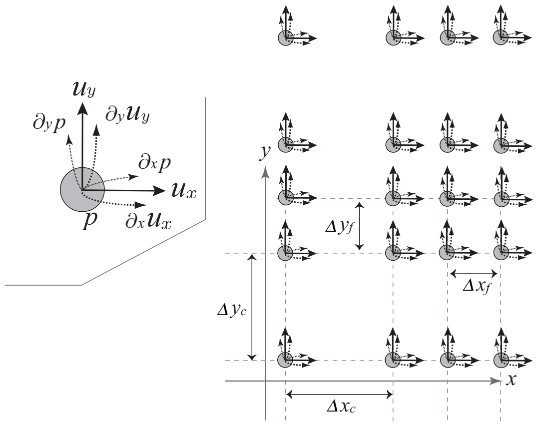


Figure 4 – Discretized acoustic field components in Non-uniform grid model

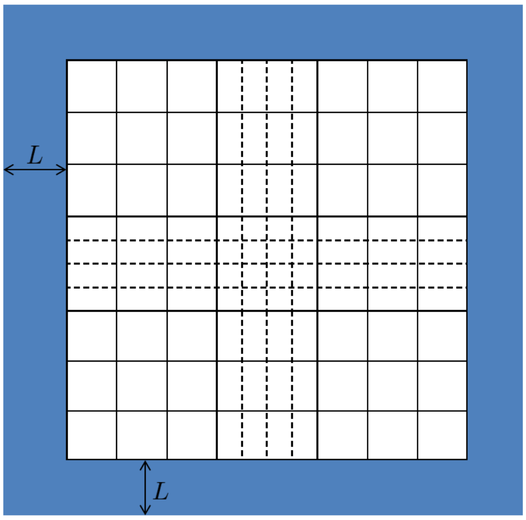


Figure 5 – Non-uniform grids model with PML

$$G_{x\pm}^{n+1} = G_{x\pm}^{n*} - \left(\frac{\partial}{\partial x} r\right) F_{x\pm}^{n*} - r G_{x\pm}^{n*} \tag{18}$$

where r is the attenuation parameter in the PML region. Here, note that we can also calculate the propagation in the y-direction as well as in the x-direction. Figure 5 depicts the schematic of a non-uniform grid model with PML, where L is the number of layers in the PML region.

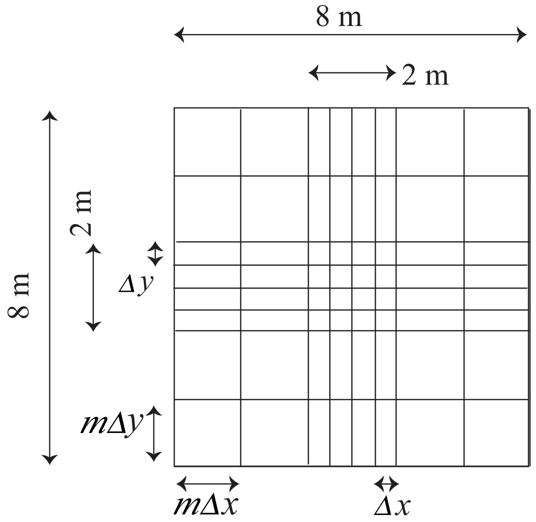


Figure 6 – Calculation model

Table 1 – The value of the constant used in the analysis

Δx	x-direction's grid width	0.01 m
Δy	y-direction's grid width	0.01 m
Δt	The discrete time width	2.0×10^{-2} ms
c	Sound velocity	330 m/s
L	the number of PML layer	32
Z	the characterisitic impedance	415.03 Pa· kg/m^3
	analysis domain	8 m $\times$ 8 m
	non-uniform grid domain	2 m $\times$ 2 m

3. SIMULATION RESULTS AND DISCUSSIONS

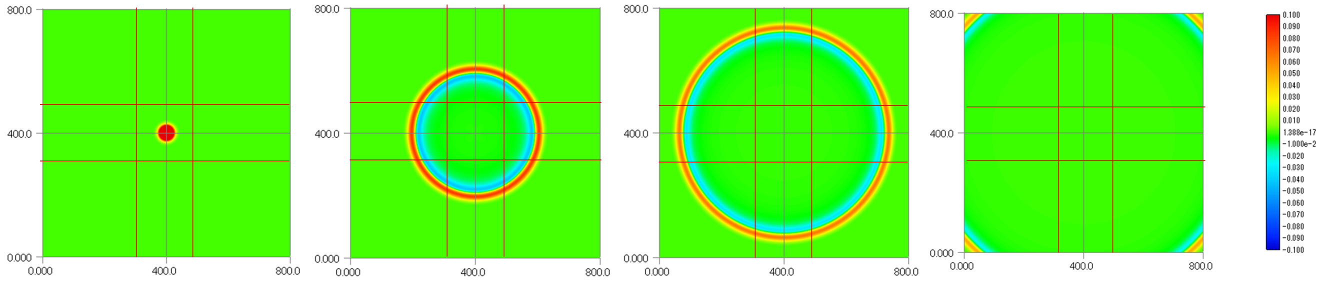
Figure 6 shows the geometry of the calculation model. The calculation parameters are as follows: Direction of acoustic field propagation, $\pm x$ and $\pm y$ (2-D analysis); fine grid size, $\Delta x_f = \Delta x = 0.01$m, $\Delta y_f = \Delta y = 0.01$m; course grid size, $\Delta x_c = m\Delta x$, $\Delta y_c = m\Delta y$, where m is a ratio of course grid size and fine one. Other calculation parameters used in the calculations are summarized in table 1.

We present numerical results obtained using the non-uniform grid technique for type-M / type-C CIP analysis. Figure 7 shows the sound pressure distribution obtained by type-M CIP analysis with non-uniform grids at $t = 10\Delta t$, $t = 300\Delta t$, $t = 500\Delta t$, and $t = 800\Delta t$. The input pressure is driven from region of the non-uniform grids. We can ascertain the propagation behaviour including the non-uniform grid region and find little reflection waves from PML boundaries. Figure 4 evaluates the error using non-uniform grid by means of comparison of the absolute pressure value at some points $((x,y) = (4.5[\mathrm{m}], 4.5[\mathrm{m}]), (4.0[\mathrm{m}], 1.8[\mathrm{m}]), (1.8[\mathrm{m}], 1.8[\mathrm{m}]))$.

The blue solid line indicates the sound pressure using non-uniform ($m = 2$) grid (|P|), and the red dashed line shows the difference between P and P_e ($|P - P_e|$), where P_e is sound pressure using uniform grid (i.e., $m = 1$) as a reference. As a result, we also find the boundary in the non-uniform grids has good permeability characteristics with an extremely low reflection.

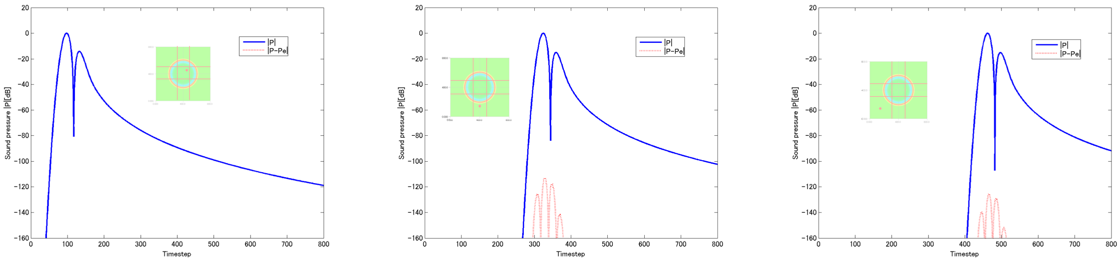
We also investigated the calculation time required for some non-uniform grid models. Here, we used a PC with Intel Core i7- 980X Extreme Edition 3.33GHz. This processor has 6 cores and 12 hyperthreaded cores, or effectively scales 12 threads. For all analyses, parallel computation using OpenMP was applied.

Tables 2 and 3 show a comparison of the calculation times using type-C and type-M CIP method, respectively, where calculations are divided into 500 time steps. These results illustrate that the non-uniform grid ($m \neq 1$) system requires less calculation time and uses less memory than the fine grid ($m = 1$). The calculation time of the type-M CIP method is about 0.7 times smaller than that of the type-C CIP method, whereas the memory use of the type-M CIP method is about 0.58 times smaller.



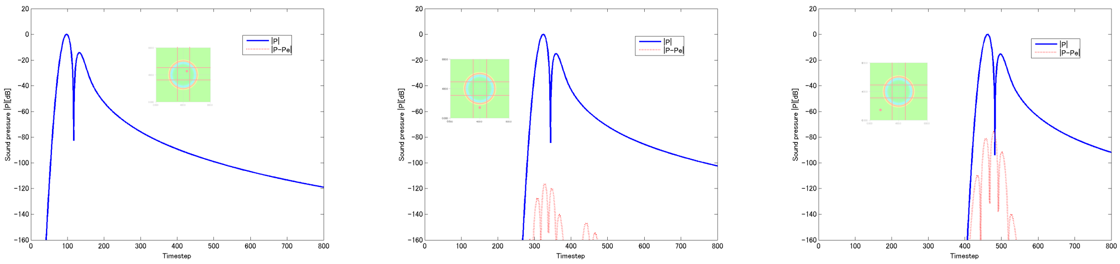
(a) $t = 10\Delta t$ (b) $t = 300\Delta t$ (c) $t = 500\Delta t$ (d) $t = 800\Delta t$

Figure 7 – Analysis of type-M CIP method using non-uniform grid



(a) $(4.5[\mathrm{m}], 4.5[\mathrm{m}])$ (b) $(4.0[\mathrm{m}], 1.8[\mathrm{m}])$ (c) $(1.8[\mathrm{m}], 1.8[\mathrm{m}])$

Figure 8 – The numerical error at each point of type-C CIP method using non-uniform grid



(a) $(4.5[\mathrm{m}], 4.5[\mathrm{m}])$ (b) $(4.0[\mathrm{m}], 1.8[\mathrm{m}])$ (c) $(1.8[\mathrm{m}], 1.8[\mathrm{m}])$

Figure 9 – The numerical error at each point of type-M CIP method

4. CONCLUSIONS

Using the type-C and type-M CIP-MOC methods, we assessed non-uniform grid systems for the numerical simulation of sound wave propagation. The numerical results obtained by the type-C and type-M CIP methods with non-uniform grid techniques were compared for a two-dimensional acoustic field. Examinations reveal that the correct treatment of the interface between the course grids and non-uniform grids causes extremely low reflection from the boundaries. The use of a suitable non-uniform grid reduces the time and memory necessary for calculation.

we also examine the PML absorbing boundary condition for the CIP-MOC 2D simulation using non-uniform grid system in this study. From the numerical results, PML implementation can be an effective method for non-uniform grid system.

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Table 2 – Calculation time of type-C

ratio of grids	calculation time [s] (relative)	maximum Error[dB]
$m = 1$	343.619 (1)	0
$m = 2$	138.547 (0.40)	-112.8
$m = 3$	96.281 (0.28)	-84.39
$m = 4$	76.492 (0.22)	-66.53
$m = 5$	66.271 (0.19)	-54.12
$m = 6$	60.577 (0.17)	-45.06
$m = 10$	48.339 (0.14)	-25.22

Table 3 – Calculation time of type-M

ratio of grids	calculation time [s] (relative)	maximum Error[dB]
$m = 1$	238.886 (1)	0
$m = 2$	99.422 (0.41)	-75.7
$m = 3$	67.533 (0.28)	-55.59
$m = 4$	54.682 (0.22)	-43.77
$m = 5$	48.440 (0.20)	-35.87
$m = 6$	42.953 (0.17)	-30.38
$m = 10$	34.058 (0.14)	-19.45

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