



## Three dimensional quasi-periodic noise barriers

Samaneh M.B. FARD<sup>1</sup>; Herwig PETERS<sup>1</sup>; Nicole KESSISSOGLU<sup>1</sup>; Steffen MARBURG<sup>2</sup>

<sup>1</sup> School of Mechanical and Manufacturing Engineering, UNSW Australia, Sydney, NSW 2052, Australia

<sup>2</sup> LRT4-Institute of Mechanics, Universität der Bundeswehr München, D-85579 Neubiberg, Germany

### ABSTRACT

Two dimensional noise barrier studies investigate the insertion loss in the shadow zone of an infinite noise barrier for a coherent line source. Such models are often preferred to avoid the high computational cost of three dimensional models. The major limitation of two dimensional barrier models is the constant cross section of the barrier in the third direction. This paper presents a new solution to evaluate the performance of 3D noise barriers that do not have a constant cross section. A quasi-periodic noise barrier model is developed to reduce the size of a 3D infinite boundary element model and thereby reduce the computational time. Different quasi-periodic noise barrier designs are developed and their acoustic performance at different frequencies and receiver positions are compared.

Keywords: Noise barrier, Boundary element method, Insertion loss  
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### 1. INTRODUCTION

Barriers are used to prevent direct sound waves from road traffic noise to receiver locations. The geometrical configuration of a barrier, surface impedance of the barrier and the ground, locations of sources and receivers and meteorological factors such as wind and temperature inversions can affect the acoustic performance of the barrier.

There has been numerous studies on the geometry and top edge of a barrier to improve its acoustic performance. Changing the slope of the top part of a noise barrier to 120° was found to provide higher attenuation of low frequency sound (1). Ishizuka and Fujiwara (2) showed that the performance of a soft 3 m high T-shaped barrier was similar to that for a 10 m high plain barrier. A wave trapping barrier with multiple wedges on its surface to redirect reflected waves downward to the ground and reduce the scattered waves on the top surface of the barrier have been developed (3, 4). Compared to a straight barrier, most profiled barrier designs usually provide higher insertions loss at lower frequencies. However it is also possible to have reduced performance at low frequencies using a random-edge profile barrier design (5, 6).

A common numerical approach to model exterior acoustic problems such as noise barriers is the boundary element method. In two dimensional boundary element method (BEM) studies, a noise barrier is often assumed to be a coherent line extended to infinity which limits the variation of the barrier shape in the third direction. Noise barrier models for either a monopole source or an infinite line source using BEM have been developed (7, 8). Jean et al. (9) showed that the insertion loss of a barrier with an incoherent line source is lower than the insertion loss predicted using a coherent line source.

A three dimensional noise barrier model can predict the insertion loss at locations away from the barrier plane (10). The use of a 3D quasi-periodic structure instead of a 3D infinite structure can significantly reduce the size of the numerical domain and hence save computational cost. A periodic structure consists of a number of identical structures connected together in series and extended to infinity. The size of the numerical domain depends on the number of periodic sections associated with the convergence analysis. In this study, the insertion loss of different noise barrier designs using a quasi-periodic boundary element method are computed. Insertion loss results of a quasi-periodic noise barrier model developed using the boundary element method are initially validated by comparing results with those obtained from a finite element model.

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<sup>1</sup>fardsmb@gmail.com

## 2. BOUNDARY ELEMENT METHOD

Figure 1 shows the definition of the boundary  $\Gamma$ , its complement  $\Omega_c$ , the domain  $\Omega$  and the inward normal vector  $\mathbf{n}$  for an exterior acoustic problem. Assuming a time harmonic dependence of the form  $e^{-i\omega t}$ , the linear wave equation is transformed to the Helmholtz equation for the sound pressure  $p$  in domain  $\Omega$  (11)

$$\Delta p(\mathbf{x}) + k^2 p(\mathbf{x}) = 0 \quad (1)$$

where  $\Delta$  is the Laplacian operator,  $k = \omega/c$  is the acoustic wavenumber,  $\omega$  is the angular frequency and  $c$  is the speed of sound.

The normal fluid particle velocity  $v_f$  is related to the normal derivative of the sound pressure  $p$  by

$$v_f(\mathbf{x}) = \frac{1}{i\omega\rho_0} \frac{\partial p(\mathbf{x})}{\partial n(\mathbf{x})} \quad (2)$$

In equation (2),  $\rho_0$  is the average density of the fluid and  $i = \sqrt{-1}$  is the imaginary unit. The vector  $\mathbf{n}(\mathbf{x})$  represents the outward normal at the surface point  $\mathbf{x}$  and  $\partial/\partial n(\mathbf{x})$  is the normal derivative.

The Neumann boundary condition prescribes the particle velocity of the fluid is zero on the barrier and the ground surface, that is

$$v_f(\mathbf{x}) = 0, \quad \mathbf{x} \in \Gamma \quad (3)$$

Hence the ground and surface of the barrier are considered to be rigid.

In the case of a quasi-periodic boundary condition, the boundary  $\Gamma$  is divided into  $\Gamma_N + \Gamma_{-M} + 1$  sections as follows

$$\Gamma = \Gamma_{-M} \cup \Gamma_{-M+1} \cup \dots \cup \Gamma_{-1} \cup \Gamma_0 \cup \Gamma_1 \cup \dots \cup \Gamma_n \cup \dots \cup \Gamma_{N-1} \cup \Gamma_N \quad (4)$$

$N$  and  $M$  are the number of boundaries on each side of the initial boundary  $\Gamma_0$ . From the boundary condition given by equation (4), the total length of the barrier is given by

$$L_T = (N + M + 1)x_p \quad (5)$$

where  $x_p$  is the length of one section of the periodic barrier.

Using the quasi-periodic boundary element method, the acoustic pressure radiated from a monopole source in the presence of a ground surface only (in the absence of a barrier) and the acoustic pressure with the barrier in place are predicted. The sound pressure at the receiver due to monopole sources with and without the barrier in place is computed by adding the sound pressure from the individual sources  $p_n$  as follows

$$p = \sum_{n=-M_s}^{N_s} p_n \quad (6)$$

where  $N_s$  and  $M_s$  are the number of monopole sound sources on each side of the initial source.

In the quasi-periodic BEM barrier model, equal number of sections ( $N=M$ ) are used in the positive and negative  $x$ -directions starting from the initial barrier section  $\Gamma_0$ . Each periodic section is linked with one monopole source in the  $y$ -direction. The sources are located on the ground at a distance of 1 m from the mid-plane of the noise barrier section. The receivers are also on the ground at distances from 0.5 to 10 m from the mid-plane of the noise barrier section in steps of 0.5 m in the  $y$ -direction.

## 3. GEOMETRIC BARRIER DESIGNS

Figure 2(a) shows a schematic diagram of one section  $\Gamma_0$  of the periodic noise barrier design. Rectangular sections are used to periodically extend the noise barrier in the positive and negative  $x$ -directions. Using this rectangular shape it is possible to build an almost infinite periodic structure. The height of one section of the noise barrier is  $h$  and its thickness and length are denoted by  $t$  and  $x_p$ , respectively.

One disadvantage of the rectangular quasi-periodic model is the open domain  $\Omega_c$  at the end sections ( $\Gamma_{-M}$  and  $\Gamma_N$ ). To achieve a closed domain, a hexagonal shape is also considered. Figure 2(b) presents a honeycomb-like section of the quasi-periodic barrier. The dimensions (thickness, length and height) are the same as for the rectangular section.

## 4. FINITE ELEMENT METHOD

A finite element model of a noise barrier using COMSOL Multiphysics (4.3b) was developed as follows. The Helmholtz wave equation is solved by applying the time harmonic dependence of the acoustic pressure  $p$

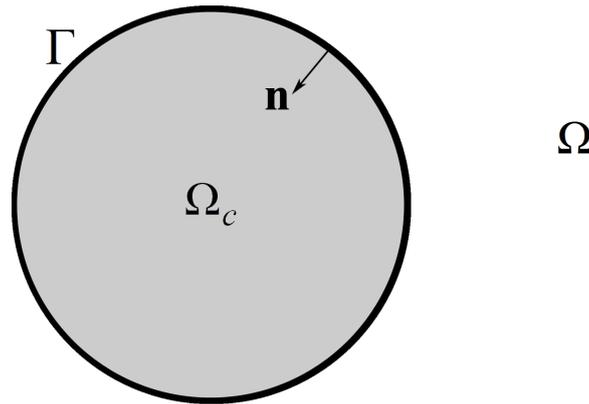


Figure 1 – Domain  $\Omega$  and boundary  $\Gamma$  of an exterior acoustic problem

at each frequency. The normal fluid particle velocity  $v_f$  is computed by the normal derivative of the acoustic pressure  $p$  in the frequency domain (11). The physical quantities (acoustic pressure and particle velocity) in a discretised domain are respectively given by

$$p(x) = \sum_{l=1}^{\hat{N}} \tilde{\Phi}_l(x) p_l \tag{7}$$

$$v_f(x) = \sum_{j=1}^{\hat{N}} \hat{\Phi}_j(x) v_j \tag{8}$$

where  $p_l$  and  $v_j$  corresponds to the discrete acoustic pressure and fluid particle velocity at point  $x$ .  $\Phi_l(x)$  and  $\hat{\Phi}_j(x)$  are basis functions (11). Substituting equations (7) and (8) into the weak formulation of the Helmholtz equation yields (11)

$$(\mathbf{K} - ik\mathbf{C} - k^2\mathbf{M})\mathbf{p} = \mathbf{f} \tag{9}$$

where  $\mathbf{K}$ ,  $\mathbf{C}$  and  $\mathbf{M}$  are the stiffness, damping and mass matrices, respectively. The excitation vector  $\mathbf{f}$  represents the monopole source. The point source can generate a time harmonic dependence sound pressure vector  $\mathbf{p}$  in the entire domain.

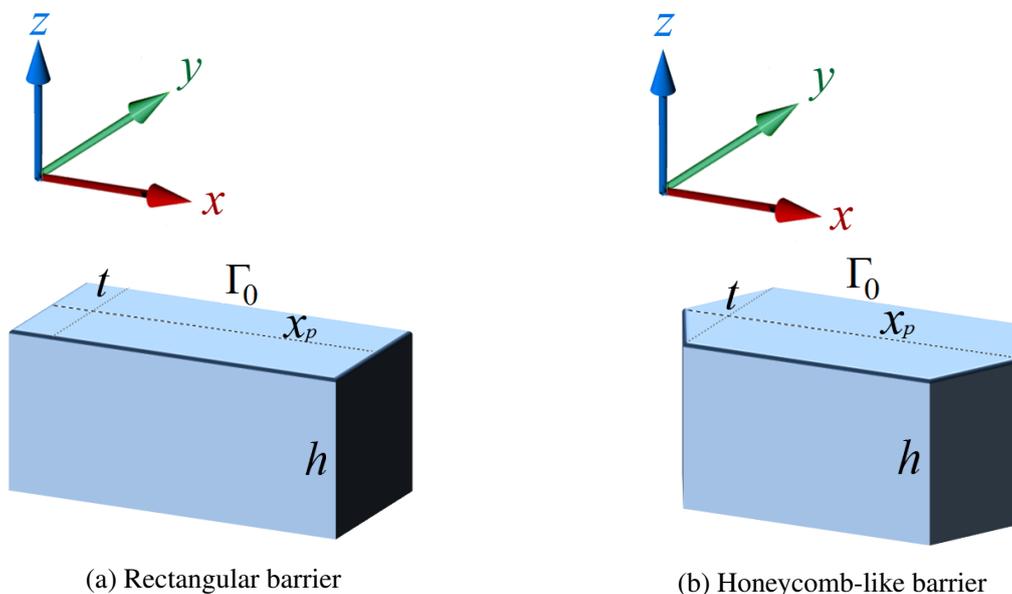


Figure 2 – Quasi-periodic section for rectangular and honeycomb-like noise barrier models

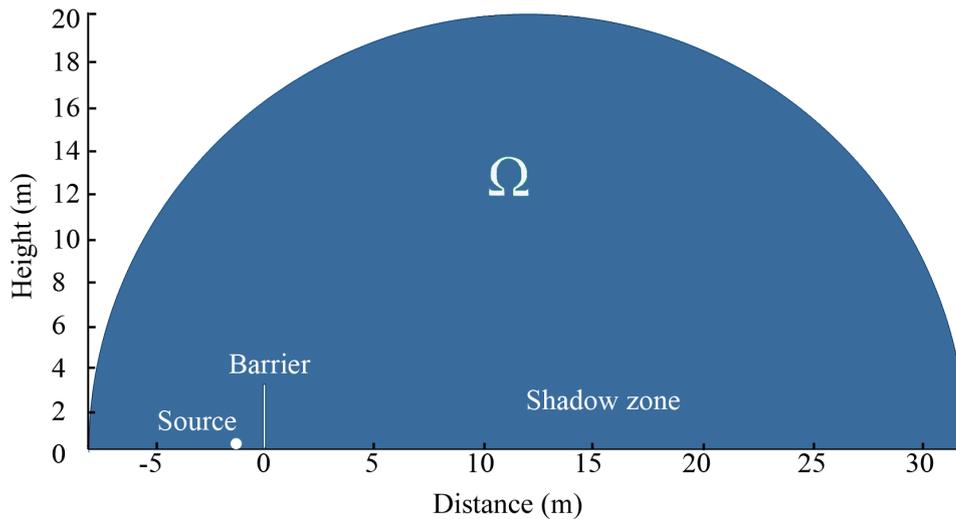


Figure 3 – Hemi-cylindrical acoustic domain  $\Omega$  with a 3 m high noise barrier and a monopole source located at -1 m

## 5. VALIDATION OF QUASI-PERIODIC BOUNDARY ELEMENT METHOD

In this section, the accuracy of the quasi-periodic noise barrier model is assessed by comparing insertion loss results obtained numerically using FEM and the quasi-periodic BEM. The quasi-periodic BEM model is developed in AKUSTA, a non-commercial computational code (12, 13), and the FEM model is developed using a commercially available software package, COMSOL (4.3b) (14). For the finite element model, a 2D noise barrier of infinite length is developed using a hemi-cylindrical domain  $\Omega$ , as shown in Figure 3. The fluid domains in AKUSTA and COMSOL are discretised using linear discontinuous boundary elements and quadratic Lagrange elements, respectively.

For the quasi-periodic boundary element model, the height of one section of the noise barrier is 3 m and its thickness and length are 0.5 m and 1.0 m, respectively. The quasi-periodic barrier model consists of one section of the barrier with a periodic length of 1 m connected to 402 quasi-periodic barrier sections on each side of the initial boundary  $\Gamma_0$ . Hence, the total number of periodic sections is  $M+N+1=805$ . The monopole source used for every quasi-periodic section is divided into several equi-spaced monopole sources to better approximate the line source used in the FEM model. It is assumed that the cross-sectional shape of the noise barrier does not vary across its length. That is, the geometrical variables are constant and are extended to infinity in the  $x$ -direction using the quasi-periodic boundary element method. The FEM model assumes an infinite line source extended parallel to the noise barrier. Similar to the sources in the quasi-periodic BEM model, in the FEM model the line source is located on the ground at a distance of 1 m from the mid-plane of the barrier.

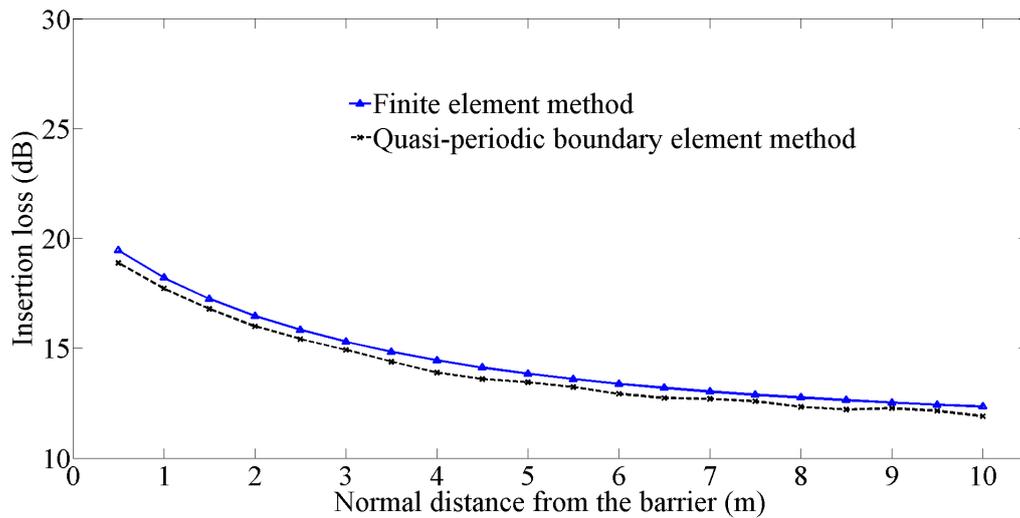
The insertion loss of the barrier is calculated using the following expression

$$IL = 20 \log \left| \frac{p_g}{p_b} \right| \quad (10)$$

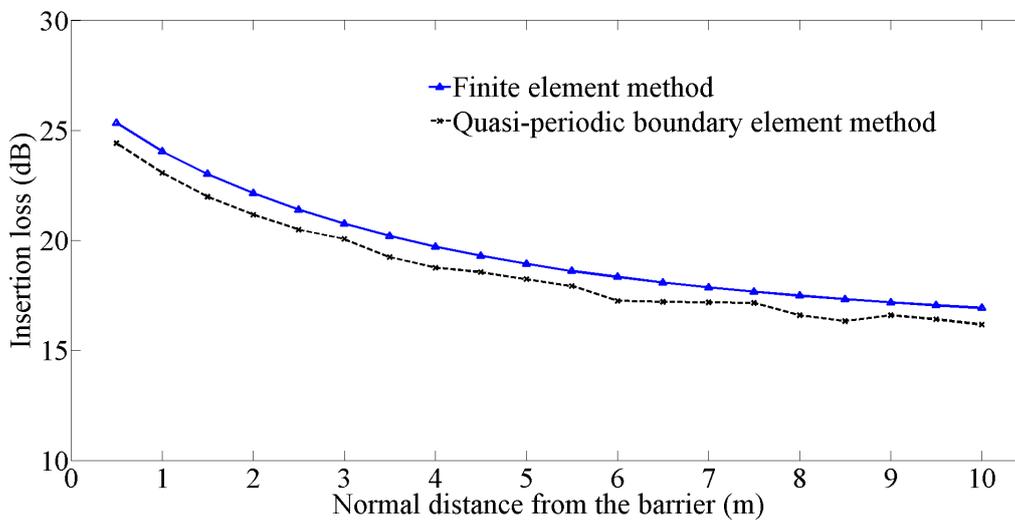
where  $p_g$  is the acoustic pressure without the barrier and  $p_b$  is the acoustic pressure with the barrier in place. In Figures 4(a) and 4(b), the insertion loss for the quasi-periodic rectangular noise barrier is compared to insertion loss results from the finite element model, at 200 Hz and 400 Hz, respectively. There is a gradual decrease in insertion loss with increasing normal distance from the noise barrier at both frequencies. The difference in the results between the two numerical methods is attributed to the type of source used in the 2D FEM and 3D BEM models and the limits of computational resources. The source in the BEM model is assumed to be a long array of equi-spaced monopole sources each radiating with spherical waves, but the source in the FEM model is a 2D line source extended to infinity.

## 6. RESULTS

To investigate the acoustic performance of the quasi-periodic noise barrier models, the insertion loss at a large area of receiver positions in the shadow zone of the barriers was examined. The shadow zone extends to a distance of 10 m from the barrier in the  $y$ -direction (normal to the barrier) and from -1.5 m to 1.5 m along the barrier length in the  $x$ -direction (parallel to the barrier).



(a) 200 Hz

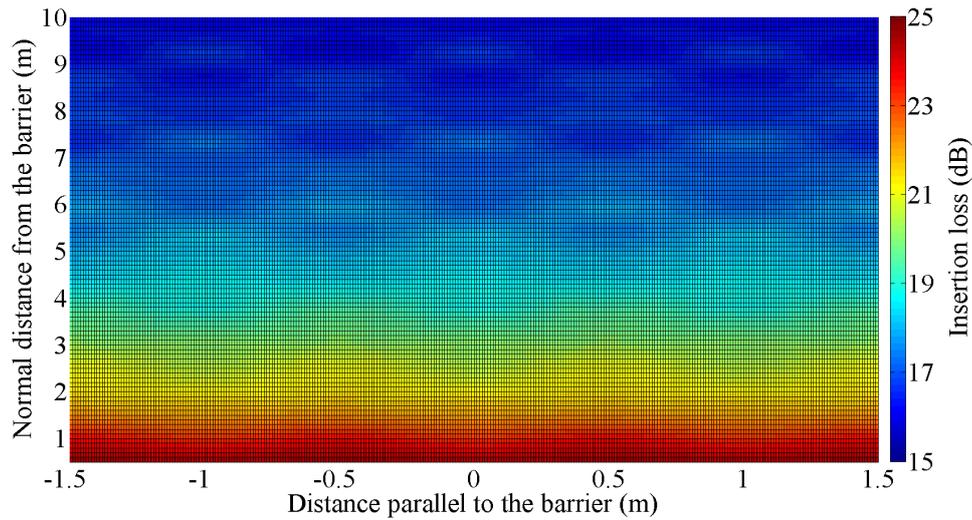


(b) 400 Hz

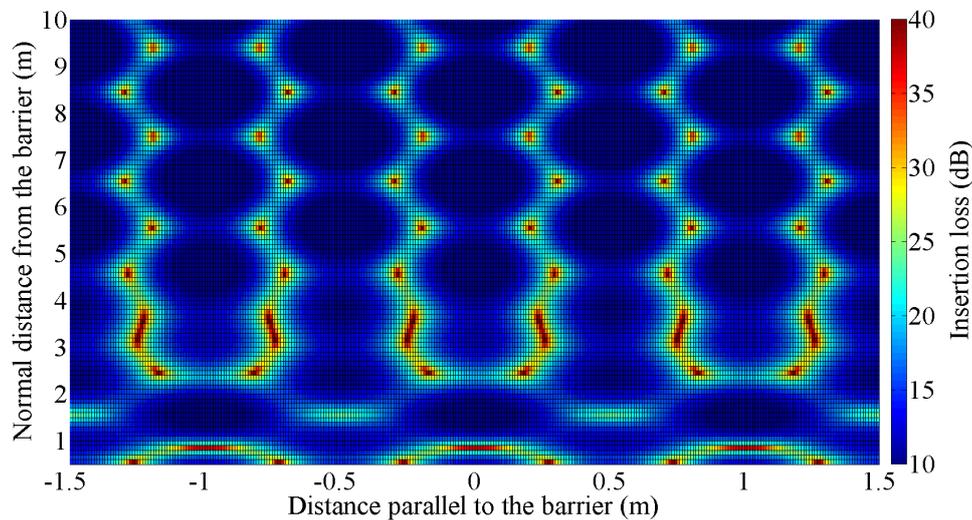
Figure 4 – Insertion loss with a rectangular noise barrier at 200 and 400 Hz

Figures 5(a) and 5(b) present the insertion loss at 400 Hz for rectangular and honeycomb-like quasi-periodic barrier models, respectively. In each figure, results for three quasi-periodic barrier sections are shown, as one barrier section has a length of 1 m in the  $x$ -direction, and is connected to a barrier section each of 1 m length in both the positive and negative  $x$ -directions. Figure 5(a) shows that there is a gradual reduction in insertion loss for the rectangular barrier shape with increasing normal distance from the barrier. However, for the case of the honeycomb barrier shape in Figure 5(b), interference effects due to the hexagonal barrier section shape generates an increase in insertion loss at receiver positions in a hexagonal pattern.

To further investigate the edge effects of the honeycomb shaped barrier, the insertion loss at distances normal to the barrier for several locations along the length of the barrier are compared. The locations along the length of the barrier are labelled as  $y_1, y_2, y_3$  and  $y_4$ .  $y_1=0$  corresponds to halfway along the length of the barrier section and  $y_4=0.5$  m corresponds to the edge of the barrier section in the  $x$ -direction.  $y_2, y_3$  respectively correspond to 0.3 m and 0.4 m. Figures 6(a) and 6(b) present the insertion loss at 400 Hz for the rectangular and honeycomb-like barrier shapes at the discrete locations along the length of the barrier section, for increasing distance normal to the barrier. As expected from Figure 5(a), similar results for the insertion loss are obtained for the rectangular barrier. However, significant differences can be observed in Figure 6(b) for honeycomb shaped barrier. The peaks and troughs in the insertion loss for different locations are attributed to interference effects from the geometry of the barrier sections at each end.



(a) Rectangular barrier

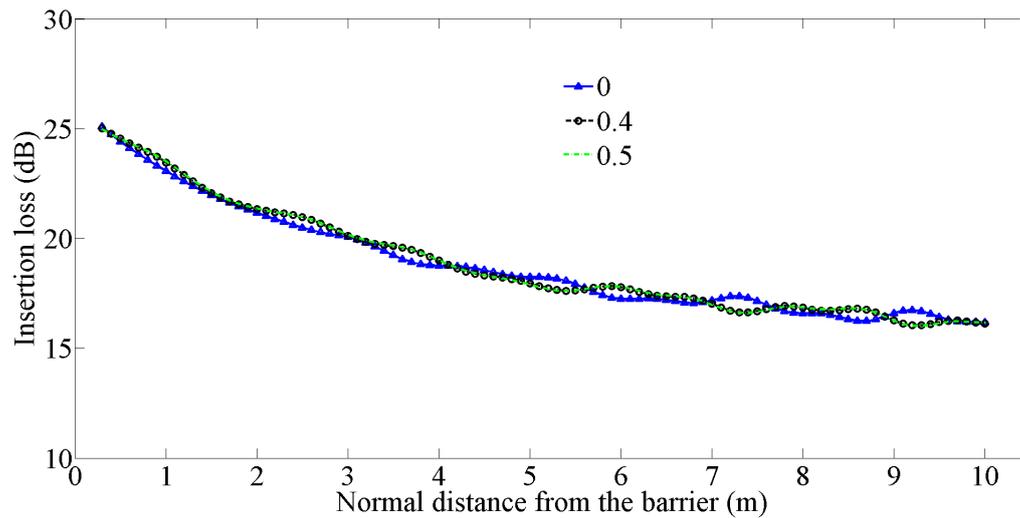


(b) Honeycomb barrier

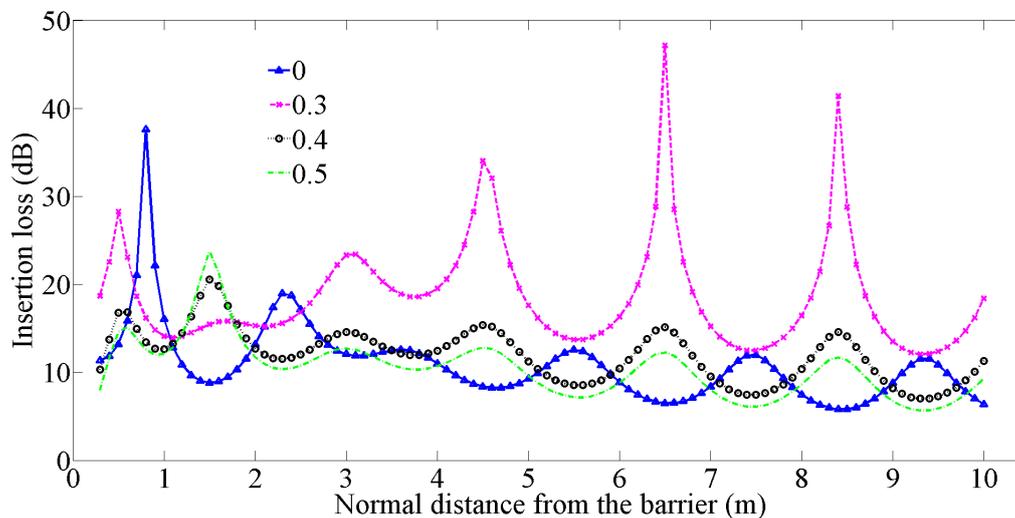
Figure 5 – Insertion loss of quasi-periodic rectangular and honeycomb noise barriers at 400 Hz

## 7. CONCLUSIONS

A quasi-periodic boundary element method has been presented for three dimensional noise barrier models. The benefit of the three dimensional quasi-periodic noise barrier model compared to an infinite noise barrier is the efficient computational processing time and its smaller domain size. Using a quasi-periodic boundary condition to periodically repeat the barrier sections in one dimension, the size of the BEM model is significantly reduced. The insertion loss of a quasi-periodic rectangular noise barrier model for different frequencies and different receiver positions was initially compared with results obtained from a finite element model. Good agreement between the results obtained using the quasi-periodic boundary element method and the finite element method was observed. Rectangular and honeycomb-shaped noise barriers were modelled using the quasi-periodic BEM. The particular benefit of the honeycomb-like shape is the closed domain at the end sections. Results for the honeycomb barrier showed higher insertion loss in a hexagonal pattern in the shadow zone attributed to interference effects due to the barrier edges. Further work using the quasi-periodic BEM technique will investigate novel barrier designs for low frequency noise reduction.



(a) Rectangular barrier



(b) Honeycomb barrier

Figure 6 – Insertion loss of rectangular and honeycomb noise barriers at 400 Hz

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## REFERENCES

1. Venckus Z, Grubliauskas R, Venslovas A. The research on the effectiveness of the inclined top type of a noise barrier. *J Environ Eng Landsc.* 2012;20(2):155–162.
2. Ishizuka T, Fujiwara K. Performance of noise barriers with various edge shapes and acoustical conditions. *Applied Acoustics.* 2004;65(2):125–141.
3. Bradley McGrath JP, Wang J. Wave trapping barriers and the effects of the tilting angle on scattered noise. *Proc 15th Int Cong Sound Vibration.* 2008;6-10 July, Daejeon, Korea.
4. Yang C, Pan J, Cheng L. A mechanism study of sound wave-trapping barriers. *J Acoust Soc Am.* 2013;134(3):1960–1969.
5. Ho SST, Busch-Vishniac IJ, Blackstock DT. Noise reduction by a barrier having a random edge profile. *J Acoust Soc Am.* 1997;101:2669–2676.

6. Shao W, Lee HP, Lim SP. Performance of noise barriers with random edge profiles. *Appl Acoust.* 2001;62(10):1157–1170.
7. Baulac M, Guillou A, Defrance J, Jean P. Calculations of low height noise barriers efficiency by using boundary element method and optimisation algorithms. *Proc Euronoise.* 2008;29 June-4 July, Paris, France.
8. Morgan PA, Hothersall DC, Chandler-Wilde SN. Influence of shape and absorbing surface-a numerical study of railway noise barriers. *J Sound Vib.* 1998;217(3):405 – 417.
9. Jean P, Defrance J, Gabillet Y. The importance of source type on the assessment of noise barriers. *J Sound Vib.* 1999;226(2):201–216.
10. Duhamel D. Efficient calculation of the three-dimensional sound pressure field around a noise barrier. *J Sound Vib.* 1996;197(5):547–571.
11. Marburg S, Nolte B. *Computational acoustics of noise propagation in fluids - Finite and boundary element methods.* Springer; 2008.
12. Marburg S, Schneider S. Influence of element types on numeric error for acoustic boundary elements. *J Comput Acoust.* 2003;11(3):363–386.
13. Marburg S, Amini S. Cat's eye radiation with boundary elements: Comparative study on treatment of irregular frequencies. *J Comput Acoust.* 2005;13(1):21–45.
14. COMSOL Multiphysics Release Notes. COMSOL Inc., USA, release 4.3b edition; 2013.