inter.noise 2014
MELBOURNE AUSTRALIA
16-19 NOVEMBER

Design sensitivity analysis of the acoustic dispersion relations

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ABSTRACT

In general, the design approach using the frequency response (e.g., S-parameters) is widely used, in order to design the acoustic metamaterial (AMM). On the other hand, this approach is very sensitive to the frequency response. Also, the acoustic energy loss due to resonance characteristic of the sonic crystal always exists. Thus, we propose the acoustic dispersion relation-based design approach in order to complement the limitations of this conventional approach. As the first step towards this goal, this paper proposes the design sensitivity analysis methodology for the acoustic dispersion relation. The representative dynamic characteristics of acoustic dispersion relation are "a natural frequency" and "a group velocity". Any type of the acoustic dispersion relation can be described by these two dynamic characteristics. In other words, this means that the AMM satisfying the specified acoustic dispersion relation can be designed by adjusting these dynamic characteristics. The proposed design sensitivity analysis methodology is verified through the design of the sonic crystal tracking the target natural frequency and group velocity at a single wavevector. This proposed method will be used for design optimization of the acoustic zero-index metamaterial (AZIM) in the future.

Keywords: Acoustic metamaterial, acoustic dispersion relation, design sensitivity analysis

1. INTRODUCTION

Acoustic metamaterials mean artificial materials with peculiar characteristics (e.g., zero refractive index, etc.) which are originally never found in nature. These artificially engineered unit microstructure-types of acoustic metamaterials (e.g., sonic crystal) are a new material which can overcome the limitations of conventional acoustic system. With the help of these acoustic metamaterials, we can construct the new concept of acoustic system and the wave phenomenon that does not exist in nature. To date, there are many various application systems using these acoustic metamaterials. Representative systems are as followings: acoustic lens (1), acoustic cloaking (2), noise barrier (3), sonar, and omnidirectional radiation system, etc. As mentioned before, acoustic metamaterial system is generally composed of so many periodic unit microstructures. Thus, it is very desirable to design efficiently the periodic unit microstructure.

In general, most studies related to design of the acoustic metamaterial is based on the s-parameter retrieval method. In other words, this approach uses the frequency response. Here the s-parameter means scattering parameters (i.e., transmission and reflection coefficient). Many researches related to design of the acoustic metamaterial using s-parameter retrieval method have been performed so far. However, this frequency response-based method has several disadvantages: 1. it is highly sensitive with respect to the operating frequency (i.e., the design is not well converged near the resonant frequency of acoustic metamaterial), and 2. The acoustic energy loss due to resonance characteristic of the sonic crystal always exists.

Therefore, we propose an acoustic dispersion relation-based design method in order to

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overcome these limitations of the conventional frequency response-based design approach. This study carries out the design sensitivity analysis of acoustic dispersion relation as the first step towards this goal.

2. NUMERICAL MODELING OF THE PERIODIC ACOUSTIC METAMATERIAL

2.1 Governing equation and boundary conditions

We apply the Helmholtz equation as the governing equation, in order to model the acoustic metamaterial system in this study. The Helmholtz equation is represented as Eq. (1). Then, the Floquet-Bloch type of periodic boundary condition is considered based on expressions from Eq. (2). The conceptual domain and boundaries being considered for acoustic metamaterial system modeling appear in Figure 1.

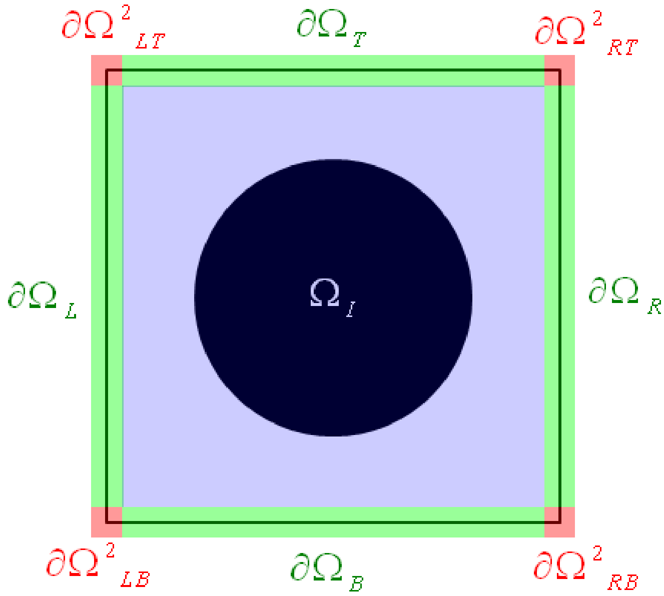

Figure 1 – Conceptual system domain and boundaries for acoustic metamaterial system modeling

Helmholtz equation

$$\nabla\cdot\left(\boldsymbol{\rho}^{-1}\cdot\nabla p\right)+\frac{\omega^2}{\kappa}p=Q \quad in\ \Omega_{Acous} \tag{1}$$

Periodic pressure boundary condition

$$\mathbf{P}(\mathbf{r}+\mathbf{a})=\mathbf{P}(\mathbf{r})e^{j\mathbf{k}\cdot\mathbf{a}} \quad on\ \partial\Omega_B,\ \partial\Omega_L,\ \&\ \partial\Omega^2{}_{LB} \tag{2}$$

where acoustic pressure is denoted by p. The $\boldsymbol{\rho}$, κ and ω are the mass density tensor, bulk modulus, and angular frequency of incident wave, respectively. Also, the $\mathbf{r}$ is position vector to describe inhomogeneous material characteristics. The $\mathbf{k}$ and $\mathbf{a}$ are the wave vector and lattice constant vector, respectively.

2.2 Finite element model

Helmholtz equation and periodic boundary conditions are discretized using the standard Galerkin method. After obtaining variational equations of the Helmholtz equation, algebraic equation in the form of a matrix is obtained. A standard discretization of the variational equation yields the following linear algebraic equation.

$$\hat{\mathbf{S}}_i(\mathbf{k})\hat{\mathbf{\Phi}}_i = \mathbf{0} \quad \textit{where,}\ \hat{\mathbf{S}}_i(\mathbf{k}) = -\omega_i^2\hat{\mathbf{M}}(\mathbf{k}) + \hat{\mathbf{K}}(\mathbf{k}) \tag{3}$$

where the pressure Bloch state for *i*-th band is denoted by $\hat{\mathbf{\Phi}}_i$. The stiffness matrix and mass matrix are denoted by $\hat{\mathbf{K}}(\mathbf{k})$, and $\hat{\mathbf{M}}(\mathbf{k})$, respectively. Fully combined system matrix is denoted by $\hat{\mathbf{S}}_i(\mathbf{k})$. The commercial software (COMSOL Multiphysics) and the Matlab-based programming are employed for all finite element procedures.

3. OPTIMIZATION FORMULATION FOR TAILORING THE ACOUSTIC DISPERSION RELATION

3.1 Optimization formulation for tailoring the acoustic dispersion

We explain the optimization formulation for tailoring the acoustic dispersion relation. The acoustic dispersion relation is classified into two types: (a) band structures, (b) iso-frequency contour (IFC). In this study, we concentrate on the design of acoustic metamaterial using IFC. IFC represents a set of propagating wavevector at the specific frequency for the periodic arranged microstructures (e.g., sonic crystal).

The numerical procedure for obtaining IFC is as following. The first step is to compute the propagation frequency (i.e., eigenvalue) at the certain wavevector (i.e., $\mathbf{k}^* = (\mathbf{k}_1,\ \mathbf{k}_2,\ \cdots,\ \mathbf{k}_j,\ \cdots,\ \mathbf{k}_N)$ and $\mathbf{k}_j = \begin{bmatrix} k_{xj} & k_{yj} \end{bmatrix}^T$) in the 1st Brillouin zone (i.e., Γ-X-M-Γ) using the algebraic eigenvalue problem (i.e., Eq. (3)). The second step is to obtain the contour line with the same propagation frequency. Thus, computational procedure of the propagation frequency at the certain wavevector plays the most important role in the IFC-based design optimization. In order to design acoustic metamaterial satisfying the specific IFC, we will now explain the formulation of the optimization problem, which takes the following Eq. (4). The objective function is to minimize the least-square error between the target IFC and the IFC obtained in every design iterations.

$$\begin{aligned}
&\min_{\gamma}\ J = \left(IFC\left(\omega_i\left(\mathbf{k}^*\right)\right)_{current} - IFC\left(\omega_i\left(\mathbf{k}^*\right)\right)_{target} \right)^2 \\
&\textit{subject to}\ \int_{\Omega_{Design}} \gamma d\Omega_{Design} \le \beta \int_{\Omega_{Design}} 1 d\Omega_{Design} \\
&\qquad \hat{\mathbf{S}}_i\left(\mathbf{k}^*\right)\hat{\mathbf{\Phi}}_i = \mathbf{0} \\
&\qquad \mathbf{k}^* = \left(\mathbf{k}_1,\ \mathbf{k}_2,\ \cdots,\ \mathbf{k}_j,\ \cdots,\ \mathbf{k}_N\right) \\
&\qquad \mathbf{k}_j = \begin{bmatrix} k_{xj} & k_{yj} \end{bmatrix}^T
\end{aligned} \tag{4}$$

A volume constraint is employed to limit the amount of material distributed in the design domain in order to save weight and cost. Here, β is the volume fraction of allowable material and takes a value between 0 and 1.

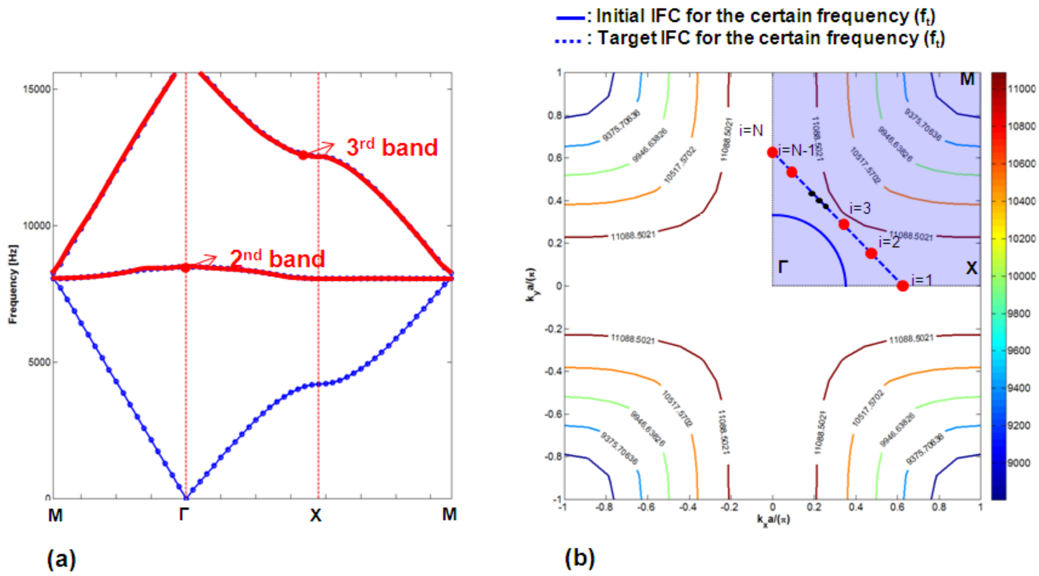

Figure 2 - Acoustic dispersion relations of the acoustic metamaterial: (a) band structure (b) iso-frequency contour (IFC)

3.2 Design sensitivity analysis of the acoustic dispersion

In this study, we focus on the design of acoustic metamaterial using IFC. In order to perform a design optimization of the acoustic metamaterial using IFC, the design sensitivity analysis of it is indispensable. As mentioned prior, the IFC can be obtained by computing the propagation frequency at the certain wavevector. Hence, we can ultimately accomplish a design sensitivity analysis of the IFC through computing design sensitivity of the propagation frequency. Note that we consider only the case of that the propagation frequencies are independent of each other (i.e., distinct eigenvalue case). We will extend the proposed design sensitivity analysis to the case of repeated propagation frequency in the near future. The design sensitivity formulation for the propagation frequency ($\omega_i\left(\mathbf{k}^*\right)$) at the certain wavevector ($\mathbf{k}^*$) is represented by Eq. (5).

$$\frac{d\omega_i\left(\mathbf{k}^*\right)^2}{d\gamma_e} = \hat{\boldsymbol{\Phi}}_i^T \frac{\partial\hat{\mathbf{K}}\left(\mathbf{k}^*\right)}{\partial\gamma_e}\hat{\boldsymbol{\Phi}}_i - \omega_i\left(\mathbf{k}^*\right)^2 \hat{\boldsymbol{\Phi}}_i^T \frac{\partial\hat{\mathbf{M}}\left(\mathbf{k}^*\right)}{\partial\gamma_e}\hat{\boldsymbol{\Phi}}_i \qquad (5)$$

Where γ_e is the *e*-th design variable, and the subscript *i* means index of the *i*-th band. Mass density and bulk modulus of the medium can be considered as the design variable.

4. NUMERICAL EXAMPLES

A topology optimization of the acoustic metamaterial satisfying the specified IFC is carried out, in order to verify the proposed design sensitivity analysis of the IFC. In the previous section it was explained that the design of various acoustic metamaterials (e.g., negative mass density, negative bulk modulus, negative refractive index, anisotropic acoustic metamaterial, acoustic zero-index metamaterial, etc.) can be performed by tailoring the specified IFC and group velocity. The design domain of acoustic metamaterial is like Fig. 3(a).

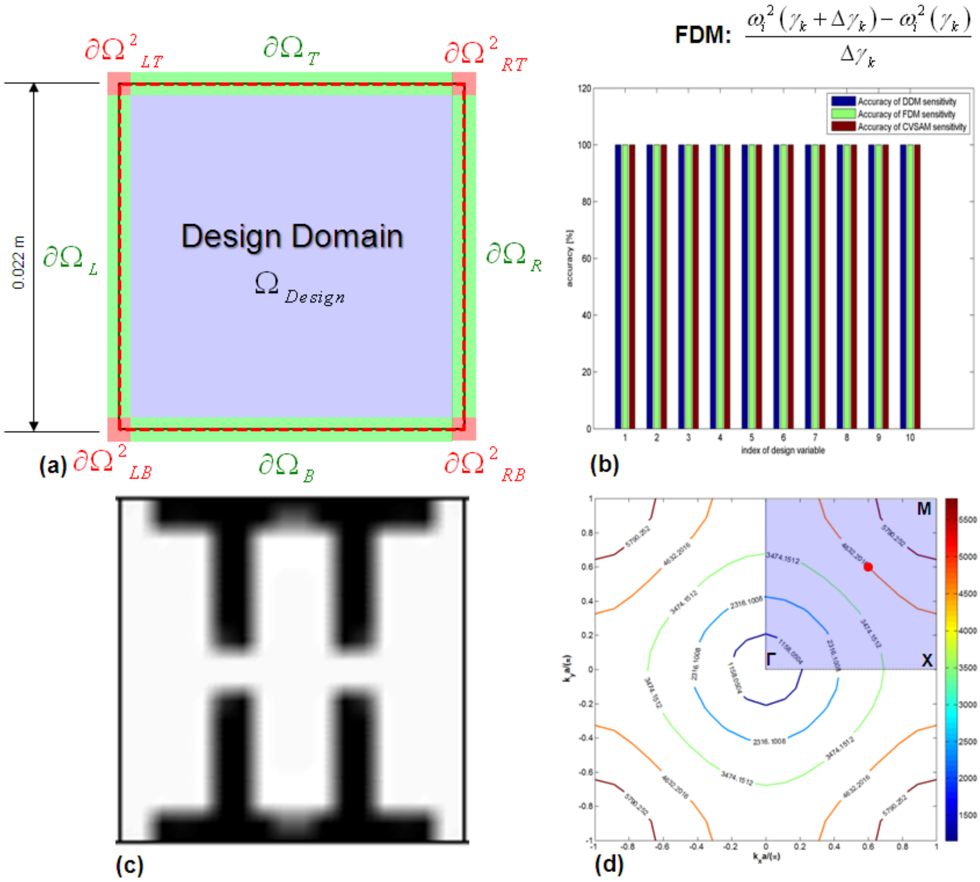

Figure 3 - Design optimization result for the acoustic metamaterial tracking the specific IFC: (a) unit microstructure and design domain, (b) comparison of the accuracy between the analytical-based design sensitivity and the numerical-based design sensitivity, (c) optimized acoustic metamaterial shape, and (d) optimized IFC

We perform the topology optimization for the certain wavevector (i.e., kx=0.6 and ky=0.6), and the results are shown well in Fig. 3. First of all, let us see the design sensitivity analysis result of the propagation frequency. An analytical method using the direct differential method (DDM) is in good agreement with a numerical method using the finite difference method (FDM), as shown in Fig. 3(b). We can know from this that the proposed design sensitivity analysis has been performed correctly. Moreover, the optimized acoustic metamaterial and the corresponding IFC are shown in Fig. 3(c) and Fig. 3(d), respectively.

5. CONCLUSIONS

In this paper, it was shown that sensitivity analysis of the acoustic dispersion relation could be employed to design acoustic metamaterial for satisfying a specified IFC. Finally, we expect that the proposed design sensitivity analysis of the acoustic dispersion relation described in this study would be a good way to design the various acoustic metamaterials (e.g., anisotropic acoustic zero index metamaterial, A-AZIM, etc.).

ACKNOWLEDGEMENTS

This work was supported by Mid-career Researcher Program though the National Research Foundation of Korea grand funded by the Korea government (MSIP) (NRF-2014R1A2A1A01005264).

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