Modelling of Fluid-Structure Interactions in the Hydraulic Circuit of Passive Interconnected Suspensions

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ABSTRACT

The pressure changes in the liquid-filled fluid circuits of Hydraulically Interconnected Suspensions (HIS) often lead to vibration of the whole pipeline and associated structures and hence become a source of structural noise, which degrades ride comfort.

This paper presents the numerical and experimental investigation into the vibration of this kind of hydraulic circuit. The one-dimensional wave theory is employed to formulate the equations of motions that govern the dynamics of the fluid-structural system. Axial and lateral vibrations as well as the effects of shear deformation on the lateral vibration of the pipe are considered. The dynamic interaction between fluid and pipe is modelled by considering Poisson and junction coupling.

The Transfer Matrix Method (TMM) is applied to determine the steady state response of the fluid-structural system, which consists of various pipe sections, hose sections, damper valves, accumulators, supports and joints.

Laboratory experiments are performed on a liquid-filled hydraulic circuit with pulse trigger. The measured steady-state responses of the fluid circuit are compared with those obtained from the numerical simulations of the developed hydraulic circuit model. It is found that the developed model of the hydraulic systems including the coupling with boundaries has a reasonable accuracy in the frequency range of interest.

Keywords: Piping, Noise, FSI   I-INCE Classification of Subjects Numbers: 21.6.6; 25.2

1. INTRODUCTION

The suspension system of a car basically consists of springs, shock absorber, and linkages that connect the vehicle body and wheels. This system mainly has two functions: one is for safely driving and handling pleasure, and the other is for people comfort by reasonably isolating road bumps and vibrations from passengers.

There are three different types of vehicle suspension systems, namely, passive suspensions, semi-active suspensions and active suspensions. For the conventional passive suspension system, the driving safety and handling generally resists ride comfort because the roll stiffness and bounce stiffness are always changed together. The increased bounce stiffness results in the reduced capacity of separating bumps and vibrations between road and passengers. Semi-active or active suspensions can overcome this compromise by applying control assembly, but the advantage is constrained by the cost, packaging, weight, reliability, and/or the other challenges.

Interconnected suspension, on the other hand, can offer some advantages over solving this problem by decoupling different vehicle vibration modes in a passive manner, which does not increase any extra part and could be realised by mechanical, hydraulic or pneumatic means. The HIS system has been successfully applied to race cars and passenger cars for improving drive performance and preventing rollover accidents. The interconnection between wheel stations can not only overcome the ride/handling compromise, but can also provide more control on suspension performance.

Figure 1 illustrates the basic structure of the HIS on the base of front half car. This simple diagram represents a sealed liquid-filled pipe guided hydraulic circuit system, which consists of double-acting cylinders, diaphragm-type hydraulic accumulators, damper valves, steel pipes and rubber hoses.

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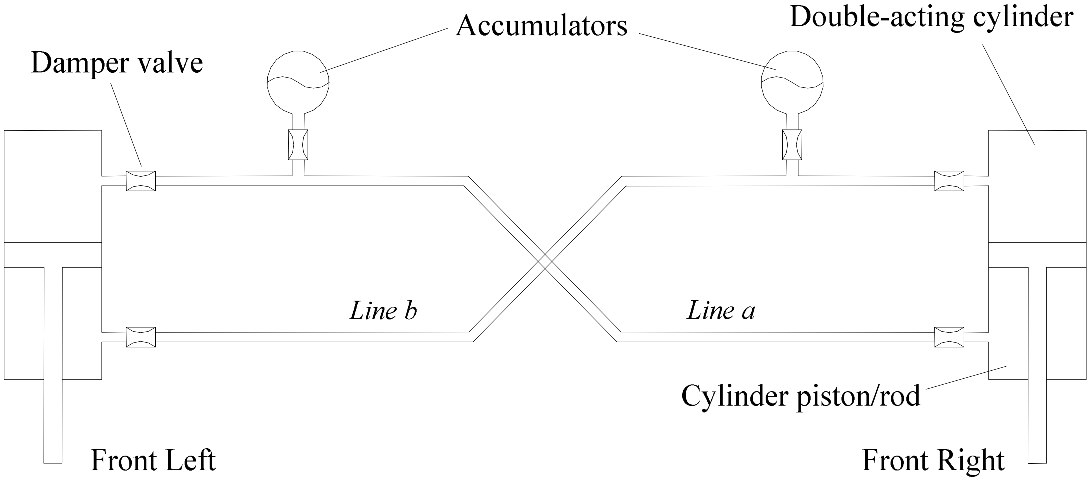

Figure 1 – Schematic diagram of half-car HIS system (1)

Being installed on a vehicle, the pistons of the cylinders are fixed on wheel stations while the cylinders and the hydraulic circuits mount on the chassis of vehicle. Therefore, when the vehicle runs, the relative movement between the vehicle body and its wheels result in the relative movement between the cylinders and pistons. The pressure ripple induced by the movement is propagated within the hydraulic circuit and generates objectionable vibrations. The vibrations can be transferred to the vehicle structure and become an excitation force to the vehicle. The low- and mid-frequency vibrations can influence vehicle handling and ride whilst the high-frequency vibrations are associated to the vehicle noise.

It has been shown that the HIS system can significantly increase rollover stability and improve vehicle handling effectiveness compared with conventional suspension systems without considerably sacrificing the vehicle capability of reducing shocks (2). However, since the system is mounted on the chassis of a vehicle, the vibration induced by the fluid flow dynamically interacts with some of the vehicle assemblies and/or parts. Thus, the resonance of high-frequency vibrations between the HIS and vehicle structures cannot be avoid and the consequent noise cannot be ignored.

Due to the competition of commercial environment of the modern automotive market, the level of Noise and Vibration Harshness (NVH) is one of the important quality indicators of a passenger car and has to be eliminated or minimized cost-effectively. It has a significant impact on not only the direct commercial profit but also the future success of automotive companies. In order to effectively reduce the vehicle noise, an experimentally validated mathematical model of the HIS system is required to understand the system dynamics and provide theoretical basis to optimise the structure design and assembly manner. This paper will delve such a mathematical model of the HIS system.

2. MATHEMATICAL MODELLING

2.1 Method

The hydraulic circuit of the HIS can be regarded as a series of discontinuous points connected by pipes and/or hoses. The natural frequencies and mode shapes of a system are normally considered as the key characteristics to analyse the system dynamics at frequency domain, which is the proper way to understand the noise problem of this system. According to Chaudhry (3), the impedance method and the TMM should be used to analyse the system in frequency domain. Therefore, the TMM is employed in this study because it is simpler and more systematic than the impedance method (4).

Adopting the TMM to model the FSI system requires the integration of field transfer matrices and point transfer matrices. The field matrices of pipes and hoses express the forces and displacements at one section of the structure in terms of the corresponding forces and displacements at an adjacent section (5, 6). The point matrices, which express the relation of the two sides of the discrete locations, include structural discontinuities such as supports, mass or bend and fluidic discontinuities like accumulator and orifice.

The entire transfer matrix of the whole system can be developed by combining the field transfer matrices and point transfer matrices. Equation 1 shows the relationship of these matrices, in which **P** represents the point matrix, **F** indicates the field matrix, and **S** stands for the state vector. After applying boundary conditions, Equation 2 can be derived to determine the natural frequencies of the system, in which the order of matrix **TT** is smaller than the order of the entire transfer matrix **T** and the dimension of vector $\mathbf{SS}_1^L$ is smaller than the dimension of vector $\mathbf{S}_1^L$. The mode shape of one frequency is obtained by substituting the frequency into the **T**.

$$\mathbf{S}_N^R = \mathbf{P}_N\left(\prod_{i=N-1}^{1}\mathbf{F}_i\ \mathbf{P}_i\right)\mathbf{S}_1^L \quad \Rightarrow \quad \mathbf{T} = \mathbf{P}_N\left(\prod_{i=N-1}^{1}\mathbf{F}_i\ \mathbf{P}_i\right) \tag{1}$$

$$\mathbf{0}_N^R = \mathbf{TT}\,\mathbf{SS}_1^L \tag{2}$$

where

$\mathbf{S}_1^L$ the state vector of the left end of the system

$\mathbf{S}_N^R$ the state vector of the right end of the system

$\mathbf{0}_N^R$ zero vector

2.2 Modelling

The modelling of pipe elements will be based on the following considerations. The ratios of fluid velocity to wave speed, pipe wall thickness to inner radius of pipe section, inner diameter of pipe section to wavelength, and liquid pressure to fluid bulk modulus are small with respect to unity, as discussed in (7). The pipe element is liquid-filled, slender, straight, prismatic, and with circular cross-section; and the liquid and the pipe-wall are linearly elastic and friction free, as given in (7-9). Axial, lateral and torsional vibrations are assumed not to influence each other along the straight pipe section (5, 6). The phenomenon of cavitations is not considered in this sealed high pressure hydraulic circuit.

The pipe element transmits torsional waves, transverse shear and bending waves in the pipe wall, and axial compression waves in both the pipe wall and the liquid. Such motion can be described by 14 equations (10, 5, 6). The axial and one lateral direction (y-z) of the piping is studied in this paper and the hose element is considered as thin-wall and "soft" pipe element. The minor errors of the given equations are corrected by the author. The friction between hose and oil is ignored due to this assumption.

The bends of pipes, pipe supports and concentrated masses are considered as discontinuous point, at which the structural dynamics changes. They are modelled as point transfer matrices that were derived by Lesmez (5). The valves and accumulators, on the other hand, are treated as fluidic discontinuities. A damper valve is simulated as a sharp-edged constant-area orifice. A diaphragm-type hydraulic accumulator consists of two chambers: a pre-charged gas chamber and a fluid chamber connected to a hydraulic system, which are separated by a kind of elastic diaphragm (11). Their point matrices are shown as Eq. 3 and Eq. 4 respectively.

$$\begin{Bmatrix} U_z \\ P \\ V \\ F_z \\ U_y \\ \Psi_x \\ M_x \\ F_y \end{Bmatrix}_i^R = \begin{bmatrix} \begin{matrix} 1 & 0 & 0 & 0 \\ j\omega\dfrac{A_f}{k_{orf}} & 1 & -j\omega\dfrac{A_f}{k_{orf}} & 0 \\ 0 & 0 & 1 & 0 \\ -j\omega\dfrac{A_f^2}{k_{orf}} & 0 & j\omega\dfrac{A_f^2}{k_{orf}} & 1 \end{matrix} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{Bmatrix} U_z \\ P \\ V \\ F_z \\ U_y \\ \Psi_x \\ M_x \\ F_y \end{Bmatrix}_i^L \quad k_{orf} = \frac{0.7^2}{3}\frac{\pi r_{orf}^3}{\mu} \tag{1}$$

$$\begin{Bmatrix} U_z\\ P\\ V\\ F_z\\ U_y\\ \Psi_x\\ M_x\\ F_y\end{Bmatrix}_i^R = \begin{bmatrix} 1 & 0 & 0 & 0 & & & & \\ 0 & 1 & 0 & 0 & & & & \\ 0 & -\frac{1}{A_f Z_a} & 1 & 0 & & \mathbf{0} & & \\ -\omega^2 m_a & 0 & 0 & 1 & & & & \\ & & & & 1 & 0 & 0 & 0\\ & & & & 0 & 1 & 0 & 0\\ & \mathbf{0} & & & 0 & 0 & 1 & 0\\ & & & & -\omega^2 m_a & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} U_z\\ P\\ V\\ F_z\\ U_y\\ \Psi_x\\ M_x\\ F_y\end{Bmatrix}_i^L \qquad Z_a = \frac{\gamma p_0^2}{p_{pre} V_{pre}} - \omega^2 L_a \qquad (2)$$

2.3 Numerical Simulations

2.3.1 Orifice's Influence in Axial Direction

AB = BC = CD = DE = 0.5m, EF = 0.1m, L = AF = 2.1m

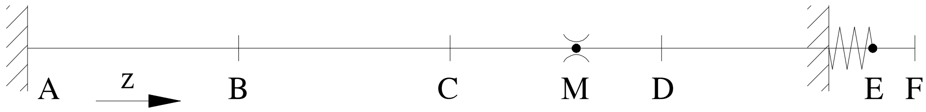

Figure 2 – Orifice investigation system

This section is focused on the impact of orifice and accumulator, thus the hose is not included in the assumed systems and the axial dynamics is analysed. Figure 2 shows a pipe guided fluid circuit system, consisting of pipe sections, a support, and an orifice. The orifice location is changed to show its influence. Figure 3 shows the first and second vibration modes of fluid displacement for the system without the orifice, which represents the fluid velocity distribution along the pipe.

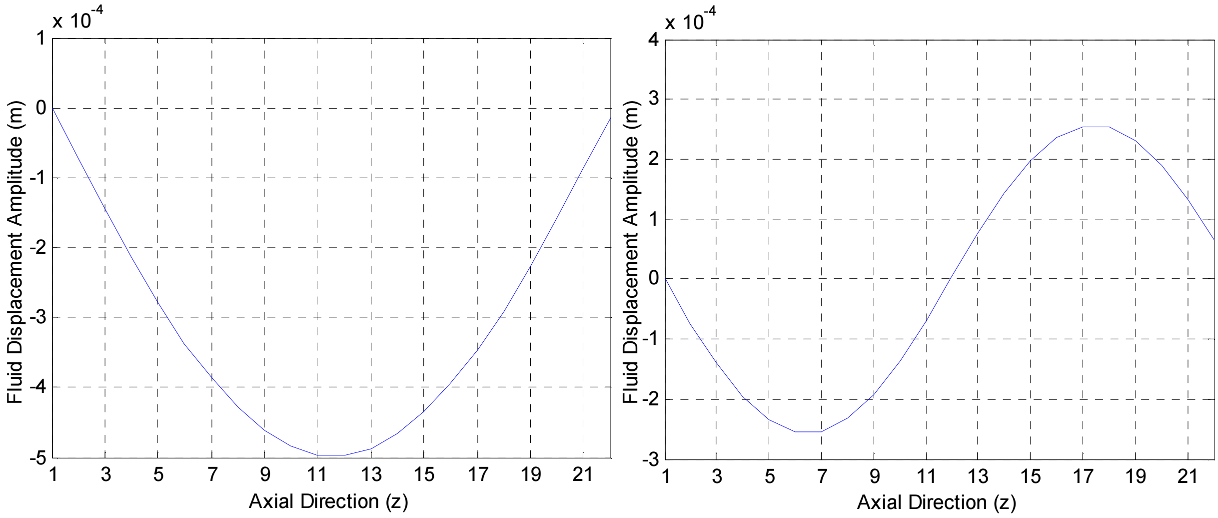

Figure 2 – 1st and 2nd vibration modes of fluid velocity

The orifice shows its impact on the system dynamics only when the radius of the cross hole is small enough. This effect given in Table 1 demonstrates that the orifice has no impact if it is fitted at Location A. For the first system natural frequency, it has quite little effect when it is very near Location E. When the orifice is fitted between Location C and D, the first natural frequency is much larger than that of other locations, but close to the second natural frequency of other locations. The two values are almost same if the orifice is located at M and the distance of ME is equal to 0.79m. It can be concluded that the orifice has significant influence on the first vibration mode but not on the second vibration mode at the same location.

Table 1 – Influence of the orifice's location on system natural *frequencies*

Length (m)	support on E	f1 (Hz)	f2 (Hz)
AE = 2.0, EF = 0.1	no orifice	293.6	524.3
	orifice on A	293.6	524.3
	orifice on B	383.1	540.5
	orifice on C	518.0	608.8
	orifice on M	524.7	700.6
	orifice on D	428.6	529.2
	orifice very near E	305.4	542.9

2.3.2 Accumulator's Influence in Axial Direction

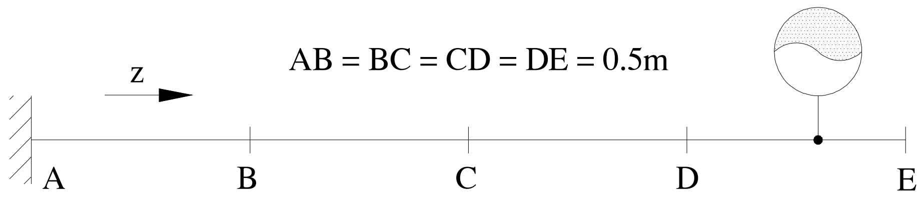

Figure 4 – Accumulator investigation system

The system shown in Figure 4 is to analyse the accumulator influence on the system. To reduce the impact from other factors, only an accumulator and pipe sections are included. The axial direction of this system is analysed and the accumulator position is changed to obtain results of different situations. Figure 5 shows the first and second vibration modes of fluid pressure for the system without the accumulator, which represents the pressure distribution along the pipe. The x axis is the series number of pipe sections and the y axis is the inside pressure.

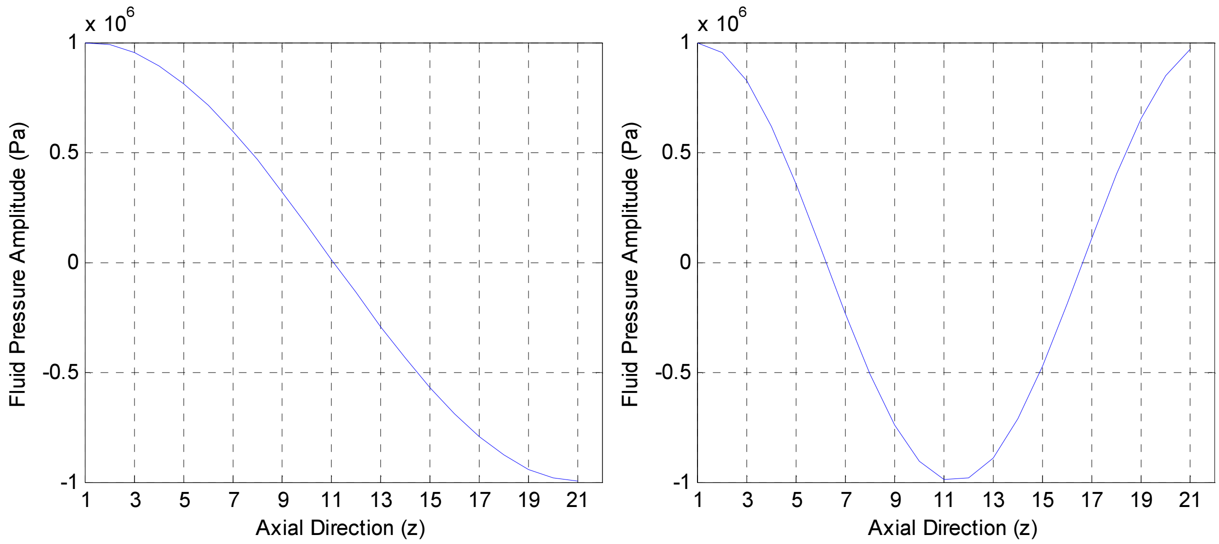

Figure 5 – 1st and 2nd vibration modes of fluid pressure

Table 2 shows the influence of the accumulator's location on the natural frequencies. The accumulator has no impact on the first system natural frequency when it is fitted on the middle of the piping (Location C) because the pressure amplitude almost equals zero at this point. The function of the accumulator is to reduce water hammer influence, i.e. the fluid pressure peak value, in piping systems. Therefore, the accumulator cannot have its influence if it is located at the point that the pressure is always equal to zero. The same phenomenon is shown on the second system natural frequency when the accumulator is equipped on Location B or D.

Table 2 – Influence of the accumulator's location on system natural frequencies

Length (m)	Accumulator position	f_1 (Hz)	f_2 (Hz)
AE = 2.0	no accumulator	309.1	598.7
	accumulator on C	309.1	643.1
	accumulator on B	219.6	598.6
	accumulator on D	220.7	600.0

According to conservation of energy, the total amount of the potential energy and the kinetic energy of the system is a constant at any point. Large pressure implies large potential energy and large velocity means large kinetic energy. At one location on which the potential energy is dominant, the kinetic energy must be small, and vice versa. This can also be concluded by comparing Figure 3 and 5.

3. EXPERIMENT

3.1 Test Process

3.1.1 Test Rig Configuration

Figure 6 shows the liquid-filled pipe guided hydraulic circuit system investigated in this paper, which includes steel pipes, steel-reinforced rubber hoses, supports, three pressure transducers, and an accumulator. The system equipped on a test rig is initially pressurised and sealed. The experiments basically have two stages: measuring system natural frequencies and identifying partial mode shapes under defined frequencies. During the tests, only the pressure responses are measured and analysed to obtain the relevant data. Therefore, the system natural frequencies and mode shapes are related to fluid dynamics.

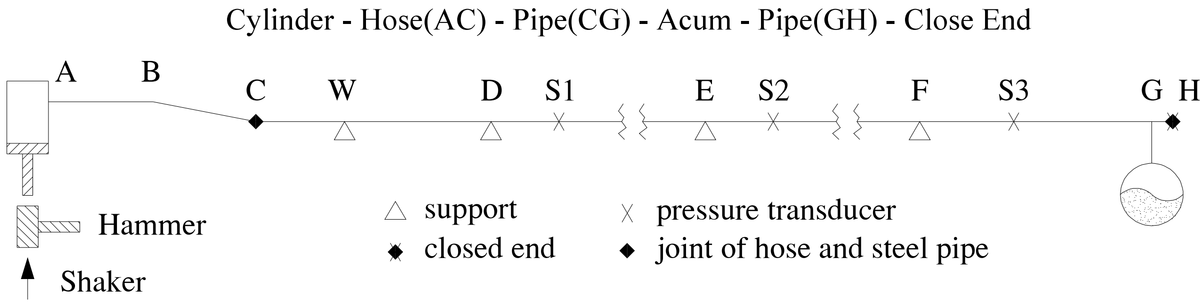

Figure 6 – Pipe guided hydraulic circuit of test rig

3.1.2 Method to Measure the Natural Frequency and Mode Shape

The Dynamic Signal Analyzer (DSA) is the core instrument to measure the system natural frequencies. The hammer shown in Figure 6 is connected to the channel 1 of the DSA. By hitting the piston, the pulse signal is input into the DSA as trigger signal. The 1st pressure transducer (S1) is connected to the channel 2 of the DSA and the pressure change is input into the DSA as response signal. Since the pulse trigger can produce broad range of frequencies as input, the response frequencies are the system natural frequencies. The input frequency influence is filtered from the response frequency spectrum through dividing the response spectrum by the input spectrum.

As shown in Figure 6, a shaker is connected with the piston and provides harmonic vibration under certain frequency to the system. By measuring the pressure response along the pipeline, the pressure mode shape can be derived from the data. In this experiment, the pressure response is measured only on the three pressure transducers, thus the pressure mode shape of the whole system cannot be demonstrated. However, the relationships of amplitudes and phases between the three positions can be derived from the measuring data and they can display partial mode shapes of the hydraulic circuit.

3.2 Comparison of Results

Comparison of the measured system natural frequencies and the simulation results calculated from the mathematical model is given in Table 3. It can be seen that the deviation between test results and simulation results is considerably small for all of the frequencies.

Table 3 – Frequency comparison between test and simulation

Frequency (Hz)	f_1	f_2	f_3	f_4
Test Data	128	369	700	992
Model	128.1	364	703.6	992.2

When the piston is activated by harmonic vibration under certain natural frequency, the response pressure at the locations of the three pressure transducers can be measured. Figure 7 compares the pressure relationship of the three locations between experimental results and simulation results under different system natural frequencies. It can be seen that the phase relationship shows good consistency between tests and simulation, but all simulation amplitudes are higher than test results. This derivation is due to the neglect of the oil viscosity, i.e. oil damping, which decreases the magnitude of the response pressure in the pipeline.

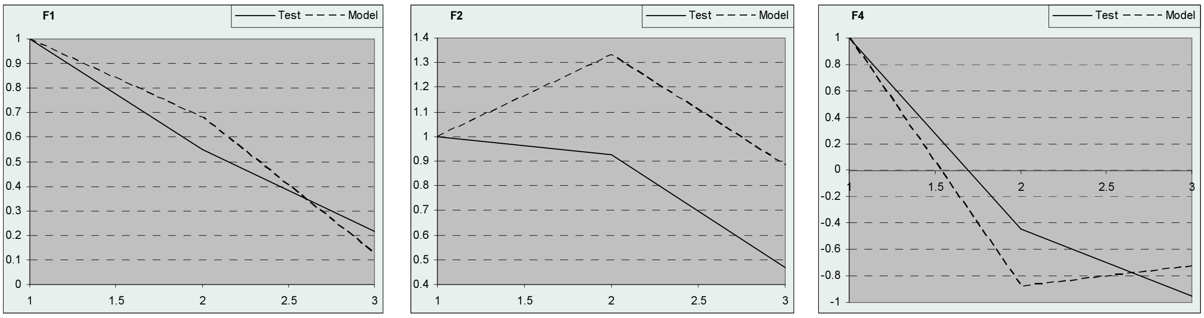

Figure 7 – Mode shape comparison

3.3 Hose Influence

3.3.1 Figures and Photographs

By comparing the preliminary model (4) and the further model in this study, the hose shows significant impact on system natural frequencies. Table 4, 5, and 6 show that the influence of hose thickness, hose length, and elastic modulus of hose on system natural frequencies respectively. The tables display that these characteristics of the hose show obvious impact on both low (f_1) and high (f_4) system natural frequencies, but slight influence on the middle range of the frequencies (f_2, f_3). Therefore, the mid-frequencies cannot be shifted by justifying hose parameters.

Table 4 – Impact of hose thickness

Frequency (Hz)	f_1	f_2	f_3	f_4
Test Data	128	369	700	992
$e_h = 0.003$	119.5	362.4	710.8	971.3
$e_h = 0.004$	128.1	364	703.6	992.2
$e_h = 0.005$	131.9	367.6	706.3	1040

Table 5 – Impact of hose length

Frequency (Hz)	f_1	f_2	f_3	f_4
Test Data	128	369	700	992
l_h = 0.1	147.2	358.9	703.8	1031
l_h = 0.2	128.1	364	703.6	992.2
l_h = 0.4	134	371.8	702.8	990.6

Table 6 – Impact of hose elastic modulus

Frequency (Hz)	f_1	f_2	f_3	f_4
Test Data	128	369	700	992
E_h=0.01, G_h=0.003	137.5	363.2	707.4	991.4
E_h=0.1, G_h=0.03	128.1	364	703.6	992.2
E_h=1, G_h=0.3	145.3	360.2	703.8	969.3

However, it is not easy to change the parameters of a hose except the hose length because the hose is designed according to defined specification. For each type of hose, the material, inner radius, and thickness are fixed and they cannot be freely modified. Furthermore, for an actual HIS system, when a hose is selected, other factors such as system pressure and assembly convenience have priority to be considered than the vibration resonance.

3.4 Model Application

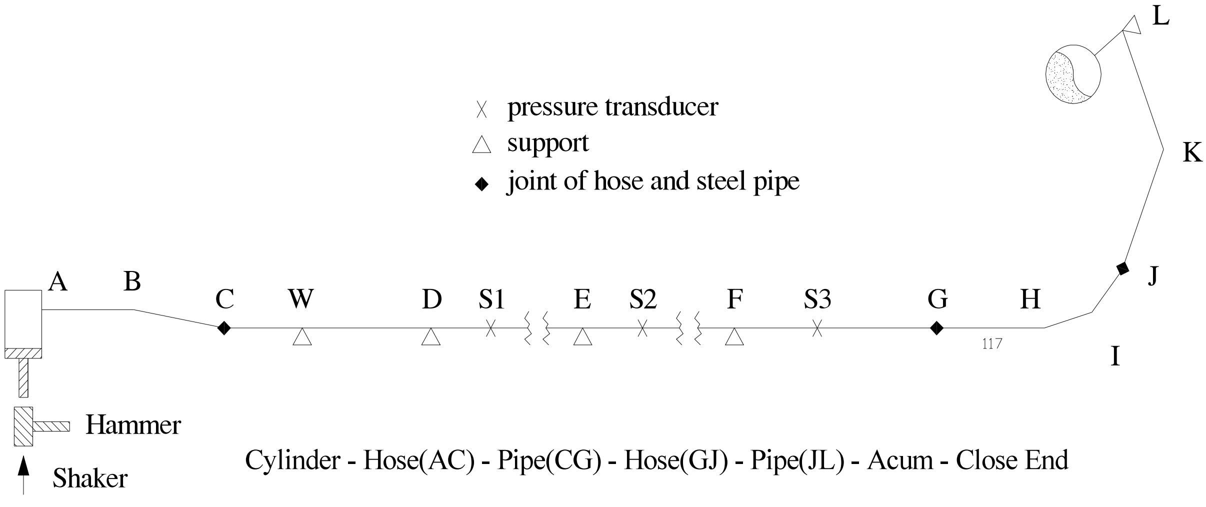

Figure 8 –Configuration of a complex piping system

The simulation model is applied on a more complex system (Figure 8) that includes more pipe bends and hose bends. Further experiments provided the system natural frequencies of the complex system. The comparison of the simulation results and the experimental results is shown in Table 7. The large deviation may result from the increase of hose bends.

Table 7 – Impact *Frequency comparison of complex system*

Frequency (Hz)	f_1	f_2	f_3	f_4	f_5	f_6
Test Data	88	264	276	496	512	624
Model	66	222.8	276.9	467	518.5	616.4

In this study, a hose bend is simply simulated like a pipe bend. For the simple system in Figure 6, the hose bends very gently and it is almost a straight hose, thus this assumption has little impact on the system dynamics. However, the complex system includes a big hose bend and the model of hose bend shows its influence. This means that the “soft-pipe” assumption can be applied to model straight hose, but for curved hose, more studies are needed to describe the actual situation.

4. CONCLUSIONS

This paper presented an extended transfer matrix method for free vibration analysis of liquid-filled pipe guided fluid-structure systems and the experimental validation of the mathematical model. The system model icludes field matrices for pipe sections and hose sections and point matrices for fluidic discontinuities such as orifices and the accumulator and structural considering like elbows and supports. The system dynamic characteristics have been investigated by considering various combinations of system configurations. The obtained results show that the influence of orifice and accumulator can be explained by their physical implication.

The comparison between experimental and simulation results shows that the theoretical model can properly describe this FSI system and the deviation can be explained by using different assumptions of model derivation. The sensitivity analysis of hose’s parameters shows that the system steady state characteristics can be modified by changing properties of components. Both the material and dimension of hoses shows significant impact on system dynamics, especially on low and high frequencies. For middle range frequencies, the hose shows limited influence.

The mathematical model can be applicable to various combinations of system configurations. The comparison of experimental and simulation results for another system shows that at certain extent, the developed model can properly describe the liquid-filled pipe guided fluid-structure systems. In this paper, the measured frequencies come from pressure responses. More experiments that include structual factors can provide investigations on structural dynamics of the system.

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