# DESIGN OF HELMHOLTZ RESONATORS IN ONE AND TWO DEGREES OF FREEDOM FOR NOISE ATTENUATION IN PIPELINES

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A thorough design methodology of one and two degrees of freedom Helmholtz resonators leading to optimised transmission loss is described and validated in this paper. Numerical simulations of acoustic wave propagation in pipelines fitted with designed resonators have shown great agreement with analytical modelling and experimental tests. The Helmholtz resonator concept has been analysed in various configurations to evaluate the effect of the size and arrays on the overall noise attenuation performance. Using this method to directly dimension geometry aspects of the resonators followed by numerical computation of the sound pressure levels has shown that considerable sound attenuation could be achieved.

## INTRODUCTION

Excessive noise from compressors is a major concern in industries and refineries. The biggest impact of this noise is the discomfort to the personnel working at the facility as observed in practice. The primary concern is that the noise may drown/ hide the sound of the emergency alarms of the facility. The noise levels in compressors vary over a wide range from 70-120 dB(A) [1-3]. As the compressor operates over its lifetime, the noise and vibration levels expectedly increase, since centrifugal compressors are continuous flow machines and are extensively used in Saudi Arabia at crude oil processing facilities such as Saudi Aramco. Maintenance is periodic and stopping the operation every time noise levels exceed the desired threshold can be very expensive. Currently Dresser Rand uses Duct Resonator arrays (DR arrays) as an add-on solution [1, 2]. This solution was applied successfully to a 2528 PSIG (172 BARG) multistage centrifugal compressor on a platform in the North Sea and was shown to successfully give a reduction of up to 12 dB(A). Although the DR arrays give appreciable noise reduction since they are machined directly on the diffuser, the challenge lies in the manufacturing and application cost of this solution. The aim of this work is to formulate a design procedure for Helmholtz resonators that will be considered as an add-on device to existing pipelines to further reduce noise levels in a compressor line (Fig. 1).

Two levels of noise reduction are usually requested in a compressor line i.e. compressor and pipeline. The first one is at the compressor level as introduced previously and the other one is within the pipelines. In this paper, the focus will be on the design of a fit resonator for pipelines bearing in mind that characteristics of the compressor are known. The objective is then to identify a design method to fit the right resonator in terms of shape and size on the current pipeline connected to a known compressor.



Figure 1. Source of noise reduction in a compressor line

# SOURCES OF NOISE IN CENTRIFUGAL COMPRESSORS

Noise originates from various sources within compressors in downstream oil sector. The most critical source of noise in centrifugal compressors is considered to be the blade passing noise which mostly characterizes the gas flowing to the pipelines. This noise arises from the interaction between the impeller blade and the stationary diffuser vanes [1-3]. It is widely known that blade passing frequency (BPF) noise components come from the circumferential flow distortions upstream and downstream of the impeller [4]. The interaction between the impeller blades as it passes by the stationary diffuser vane causes a pressure pulsation which leads to the development of positive and negative vortices.

The interaction of these vortices as they move along the flow path creates the discrete frequency noises of the blade passing frequency. Conventionally the BPF falls between 1000 Hz to 4500 Hz, usually depending on the speed of the compressor and the number of impeller blades [1]. This range falls within human hearing sensitivity which adds to the irritating nature of this noise. Although the BPF may be considered to be the most annoying aspect of compressor noise, at supersonic flow conditions another source of noise arises in the form of buzz saw noise. The BPF noise and the buzz saw noise coupled together can lead to structural failure due to fatigue especially at pipe nipples, stubs, and instrumentation connections. In any centrifugal compressor as the fluid flow exits the impeller, the

flow distribution is distorted. Specifically, such distorted flow is characterised by a low angle (relative to a tangent to the impeller circumference) fluid flow exiting most prominently adjacent to the shroud side of the diffuser. In the past, this distorted flow has been shown to cause severe compressor performance problems [5]. Due to the design of the compressor, the inlet and discharge pipes are relatively more susceptible to noise transmission that the compressor casing itself. Noise propagates through the medium of least resistance and since the piping at the inlet has thinner walls when compared to the compressor casing, this provides a path of lower resistance for noise propagation. Between the inlet and the discharge, investigations have found that higher vibration and noise levels emanate from the discharge. At the inlet, the primary source of noise is the rotor-alone noise, while at the diffuser the BPF noise is dominant [2].

## HELMHOLTZ PRINCIPLE

However in recent years, an add-on solution using the Helmholtz concept has been developed in the form of Helmholtz resonators. A Helmholtz resonator operates on the phenomenon of air resonance in a cavity, the pressure inside a cavity increases when air is forced into it. When the air source is removed, the air pressure inside the cavity flows outwards. This outward air pressure tends to overcompensate due to the inertia of the air in the neck this causes the pressure inside the cavity lower than the outside letting the air to come back into the cavity. This continues with a decrement in the pressure magnitude every time. There exist many variations of the Helmholtz resonators in the form of a quarter-wavelength resonator [6], branched type resonator [5] and duct resonators [1, 2]. Some of them are already studied and published with some of their characteristics [7]. The authors have focused on a lumped element model for this study because of the expected practical application in industrial plants.

# LUMPED ELEMENT MODEL OF THE HELMHOLTZ RESONATOR

The Helmholtz resonator acts as an acoustic filter element. The dynamic behavior of the Helmholtz resonator can be modelled as a lumped system if the dimensions of the Helmholtz resonator are smaller than the acoustic wavelength. The air in the neck is considered as an oscillating mass and the large volume of air is taken as a spring element [4]. Damping appears in the form of radiation losses at the neck ends and viscous losses due to friction of the oscillating air in the neck. Figure 2 shows this analogy between the Helmholtz resonator and a vibration absorber with defined parameters [8]. In Fig. 2,  $M_a$  is the acoustic mass of the resonator and  $M_m$  is the mass of the mass-spring-damper system. F is the force applied at the resonator neck entrance and P is the pressure at the neck entrance. V and  $R_a$  are respectively the cavity volume and acoustic damping capacity of the Helmholtz resonator. K and  $R_m$  are respectively the stiffness and damping capacity of the mass-spring-damper system.  $\omega$  is the excitation frequency.



Figure 2. Helmholtz resonator and vibration absorber

## DESIGN PROCEDURE FOR ONE AND TWO DOF RESONATORS

It is aimed in this part of the paper to establish a parametric design procedure which will be used to dimension resonators of one and two degrees of freedom (DOF) capable of reducing noise and securing a great transmission loss of noise starting from a known compressor line where the blade passing frequency and the pipeline geometry are already known. The procedure proposed by the authors considers the resonating frequency and transmission loss equations from [9] and [10]. The procedures are explained hereafter. The one DOF resonator has only one resonant frequency hence one peak for transmission loss while two DOF resonators exhibit two resonant frequencies, thereby giving two peaks for transmission loss [11].

### Design of a one degree of freedom resonator

The resonating frequency and transmission loss for a one DOF Helmholtz resonator are represented by Eqs. (1) and (2), respectively [8, 10]. There are four design parameters corresponding to  $L_c$ ,  $L_n$ ,  $a_c$  and  $a_n$  that represent the cavity and corrected neck lengths, and the cross sections, respectively, as shown in Fig. 3. Damping appears in the form of radiation losses at the neck ends, and viscous losses due to friction of the oscillating air in the neck.



Figure 3. Single degree-of-freedom Helmholtz resonator

Relationships need to be defined to proceed with a suitable design. The design procedure to estimate the optimal size for the resonator needs to satisfy a couple of conditions derived from Eqs. (1) and (2).

$$f = \frac{c}{2\pi} \sqrt{-\frac{3L_n + L_c A}{2L_n^3} + \sqrt{\left(\frac{3L_n + L_c A}{2L_n^3}\right)^2 + \frac{3A}{L_n^3 L_c}}}$$
(1)

where A is the area ratio defined in Eq. (4) and c is the speed of sound in the medium. The only restriction in the theory is the cavity diameter that must be less than a wavelength at the resonance frequency. On applying the transfer matrix method [7], the transmission loss is obtained as

$$TL = 10\log_{10} \left[ 1 + \left( \frac{a_n}{2a_d} \frac{(1/A)\tan(kL_c) + \tan(kL_n)}{(1/A)\tan(kL_n) + \tan(kL_c) - 1} \right)^2 \right]$$
(2)

where *k* is the wave number.

Expressing the areas  $a_n$  and  $a_c$  given by Eqs. (A2) and (A3) respectively, and using the condition of solution existence (see appendix A), this leads to a condition on the frequency given by

$$f < 0.2756 \frac{c}{L_c} \tag{3}$$

This condition will be considered as an initial necessary condition for the design of the resonator when the BPF is known.

The second equation given by Eq. (4) is defined as the ratio between the cross sections. It is derived from the optimum transmission loss of Eq. (2) in which the denominator provides a relationship for the resonance frequency, as shown in Eq. (2). The frequency is a function of the cavity dimensions as expressed in Eq. (1) [12]. This second condition will be considered as the sufficient condition to complete the design of a single degree of freedom resonator.

$$A = \frac{a_c}{a_n} = \tan(kL_n). \, \tan(kL_c) \tag{4}$$

Since the procedure is based on one-dimensional wave propagation, for a successful noise attenuator, the length to diameter ratio of the neck connected should not be less than 1 as the discrepancy between the analytical results and the experiment is largest for *length/diameter* of this order [7, 13].

The design procedure is organised hereafter:

- i. Specify the frequency to be attenuated.
- ii. Assume values for A (sections ratio) within the range 0.1 to 1.
- iii. Determine the maximum value of  $L_c$  from Eq. (3).
- iv. Calculate the values of  $L_n$  from Eq. (4) using next step.
- v. Define a value of  $\Delta$  to determine a set of lower values for  $L_c$  using  $L_c = L_c \Delta$ , and hence the corresponding  $L_n$  set of values.  $\Delta$  is chosen to be small in the same unit as  $L_c$  and

 $L_n$ , if mm then  $\Delta$  will be 1, 2 or 3 mm for example (this will show optimum values of both  $L_c$  and  $L_n$  for maximum TL).

- vi. The optimum dimensions i.e.  $L_c$  and  $L_n$  are selected when satisfying Eq. (2) and to provide the maximum transmission loss.
- vii. In order to combine the effects of end correction factors, Eq. (5) can be used [7, 14]

$$l_n = L_n - \delta_1 - \delta_2 \tag{5}$$

The end correction factor  $\delta_2$  is given by [10]

$$\delta_2 = 0.48\sqrt{a_n}(1 - 1.25\sqrt{A}) \tag{6}$$

The end correction  $\delta_1$  between the circular neck and main duct is approximated by

$$\delta_1 = 0.46 \ \frac{\sqrt{a_n}}{2} \tag{7}$$

viii. After calculating the optimised dimensions, the resonator can be designed for  $l_c = L_c$  and  $l_n = L_n - \delta_1 - \delta_2$  for the *A* which gives maximum transmission loss. Figure 3 shows one of the examples based on this design methodology.

#### Design of a two degree of freedom resonator

Let the dual frequencies  $f_1$  and  $f_2$  to be attenuated,

a. Following the procedure for two DOF resonators calculate the values of the radius, neck length and the volume of the first resonator i.e.  $R_{n1}$ ,  $l_{n1}$ ,  $V_1$ , respectively. Calculate the ratio  $(\gamma_a / \gamma_l)$  which satisfies the necessary condition derived from Eq. (B1) (procedure explained in Appendix B).

b.

$$\frac{f_1^2}{f_2^2} + \frac{f_2^2}{f_1^2} \ge 2\left(1 + \frac{2\gamma_a}{\gamma_l}\right)$$
(8)

where  $2\gamma_a = \frac{a_{n2}}{a_{n1}}$ ,  $\gamma_l = \frac{l_{n2}}{l_{n1}}$ , and  $a_{n1}$ ,  $a_{n2}$  are the area of cross sections of the first and the second neck and  $l_{n1}$  and  $l_{n2}$  their respective lengths.

c. Let 
$$\alpha = \frac{a_{nl}}{l_{nl}}$$
 and  $\beta = \frac{a_{n2}}{l_{n2}}$ 

 $\frac{\gamma_a}{\gamma_l}$  can be determined from Eq. (8) as well as  $\frac{\alpha}{\beta}$  since it is equal to the same ratio.

d. By using the following frequency equation,  $V_1$  and  $V_2$  can be calculated in terms of  $\alpha$  and  $\beta$ .

$$f_{1,2} = \frac{c}{2\sqrt{2\pi}} \sqrt{\left(\frac{\alpha}{V_1} + \frac{\beta}{V_1} + \frac{\beta}{V_2}\right)} \pm \sqrt{\left(\frac{\alpha}{V_1} + \frac{\beta}{V_1} + \frac{\beta}{V_2}\right)^2 - 4\frac{\alpha}{V_1}\frac{\beta}{V_2}}$$
(9)

Equation (9) is the dual frequency expression rewritten in this form from Xu et al. [10].

e. Now the transmission loss can be calculated using Eq.(10) but rewritten and plotted with respect to  $\alpha$  and  $\beta$  for getting the optimum  $\frac{\alpha}{\beta}$  or maximum transmission loss. The corrected lengths can be converted to original lengths used for design by following step (vi) of the one degree-of-freedom design procedure.

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$$TL = 20\log_{10} \left| 1 + \frac{\alpha}{\left[ 2a_d \left( ik + \frac{\alpha}{ikV_1} \left( 1 - \frac{V_2}{V_2 + V_1 - \frac{V_2V_1k^2}{\beta}} \right) \right) \right]} \right|$$
(10)

where  $a_d$  is the cross section area and k is the wave number.



Figure 4. Dual Helmholtz resonator [10]

This method has been applied to design a two DOF resonator for a pipeline. Referring to Fig. 4, the values can be correlated. The corresponding volume of the first and second cavities simulated have  $V_1$ =3706 cm<sup>3</sup> and  $V_2$ =1853 cm<sup>3</sup>, respectively. Figure 5 shows the transmission loss distribution depending on the dimensional parameters ratios  $\alpha$  and  $\beta$ . An indepth analysis of the level of transmission loss that could be achieved shows that optimisation can be done based on the acceptable manufactured necks in terms of diameters and lengths. Hence as shown in Fig. 5(a) the maximum transmission loss obtained from simulation could be achieved with  $\alpha$  towards 1 while  $\beta$  within 0.2. Therefore, the first neck cross section  $a_{n1}$  is proportional to its length  $L_{n1}$  e.g. the neck length will be two orders higher than the neck's diameter  $(d_{n1}^2=4L_{n1}/\pi)$ , while  $\beta$ shows that the second neck cross section is less than 20% of the length  $L_{n2}$ . As  $\alpha$  moves towards zero and  $\beta$  towards 0.2, the first neck cross section becomes very small with respect to its

length  $(d_{n1}^2 = \alpha 4L_{n1}/\pi)$ , while the second cross section remains similar as in previous case. This trend is fairly conserved for the second frequency in Fig. 5(b).

It is worth noticing from the numerical simulations using COMSOL Multiphysics that these frequencies are higher enough so that higher modes, in addition to planar wave, can propagate depending on the size of the main duct.



Figure 5. Transmission loss distribution versus ratios  $\alpha$  and  $\beta$  for two degrees of freedom resonator applied to a pipeline for two different frequencies at 5000 Hz (a) and 3000 Hz (b)

# NUMERICAL SIMULATIONS FOR ONE AND TWO DOF RESONATORS

### **3-Dimensional numerical simulation for one degree of** freedom resonator

The following example is applied to an industrial plant. A cylindrical Helmholtz resonator was designed using the design procedure for one degree-of-freedom discussed earlier. Sound pressure levels showing transmission loss have been numerically computed using the COMSOL Multiphysics Acoustics module. The blade passing frequency that was attenuated was measured at the suction pipe of a compressor located in one of the plants of Saudi Aramco. Figure 6 shows

the corresponding sound pressure level versus frequencies. The simulation results are shown for a pipe with one resonator in Fig. 7, and with an array of resonators in Fig. 8. The maximum achieved noise attenuation was around 30 dB. The dimensions that were taken to model the resonators were found for A=0.1corresponding to  $r_{neck}$ = 1 mm,  $l_{neck}$ =3.74 mm,  $r_{cavity}$ =3.16 mm,  $l_{cavity}$ =6.21 mm. A sudden decrease in the sound pressure levels can be visualized in Fig. 9 when comparing a pipe fitted with a resonator whether it is single or in an array with a non-fitted pipe. When an array of identical resonators is added on the same location around the perimeter of the pipe (Fig. 8), it was observed from the numerical simulation that the sound pressure attenuation has improved by about 15% as shown in Fig. 9(b). This shows the advantage of using arrays rather than single resonator although the attenuation offered by the array has a very limited incremental range (Fig. 9). Also, the reduction of sound pressure level at the resonance frequencies is very narrow since damping and viscosity effect were not considered in the simulation. The little increase in noise level above the resonating frequency may indicate that the reference pressure used for the computations is higher than the measured pressure level. As known from Helmholtz resonators, the damping appears in the form of radiation losses at the neck ends, and viscous losses due to friction of the oscillating air in the neck. Neglecting the damping at this stage of numerical simulations by COMSOL will only have little effect on the results since the structure is stiff while the system with resonators is by definition damping-controlled.



Figure 6. Suction pipe narrow band sound pressure level of a measured compressor (from Dresser Rand)

# **3-Dimensional numerical simulation for two degrees of freedom resonator**

The dual resonators used by Xu et al. [10] were used as a benchmark test to validate the analytical design method described previously. The same dual resonator was used in the COMSOL simulation to validate the level of transmission loss obtained analytically.

### Validation of results

The dual resonator described in Fig. 4 was used in the numerical simulation. The main duct was considered with a square cross-section of 4.3 cm x 4.3 cm. The square main

duct is then connected to a circular impedance tube with smooth transitions that retain a constant cross-sectional area development. The following figures show the results of simulation as generated by COMSOL Multiphysics in the Acoustics module, and showing relative noise attenuation in Figs. 7 and 8 and in Figs. 12 and 13. The input frequency was varied and the acoustic response of the system was recorded. As the figures show, there is a noticeable reduction in noise levels. There is nearly a 20 dB decrease in noise levels at the 73 Hz resonant frequency and a 25 dB decrease in noise levels at the 166 Hz resonant frequency. The noise response of the dual Helmholtz resonator versus frequency is shown in Fig. 11. Using the stated equations for two DOF resonators, the results match the analytically calculated values with an error of 1%.



Figure 7. Sound pressure levels distribution at 3556 Hz on the surface of the pipe without and with one DOF single designed resonator with a closer view of the tuned resonator.



Figure 8. Sound pressure levels distribution at 3556 Hz on the surface of the pipe without and with one DOF of four designed resonators with a closer view of the tuned resonator.

### **Experimental tests**

The experiment included a main square section duct (43x43mm) and one two degree of freedom resonator formed of two cylinders which dimensions are shown in Fig 4 [10]. The main duct has a square cross section and was connected to a circular impedance tube. The apparatus used in the experiments to measure the transmission loss is based on two-microphone technique applied on the impedance tube set-up [15]. Along with the random sound input, B&K Multichannel Analysis System Type 3550 has been used. Throughout the frequency range of interest, the reflection coefficient measured on the downstream side of the resonator was ensured, by an appropriate termination, to remain below 0.1 which translates to accurately measured transmission loss.



Figure 9. Sound pressure levels comparison without and with resonators, (a) pipe with a single designed one DOF resonator, (b) pipe with four designed single DOF resonators. Sound pressure measured at the end of the main duct which length is 1.2m and hosting the resonators. The source is a random noise signal



Figure 10. Sound pressure levels at the resonating frequency (a) 73 Hz, (b) 166 Hz



Figure 11. A comparison of transmission loss with published experimental results [10] for a single 2 DOF resonator



Figure 12. Sound pressure levels distribution at 3556 Hz on the surface of the pipe without and with two DOF designed resonators. A closer view of sound pressure levels distribution at 3556 Hz on the surface of the pipe with and without two DOF designed resonators



Figure 13. Sound pressure levels distribution at 2712 Hz on the surface of the pipe without and with two DOF designed resonators. A closer view of sound pressure levels distribution

#### Simulation of array resonators

A 2 DOF cylindrical Helmholtz resonator was designed using the design procedure. This time two highest peaks from Fig. 6 were taken to be attenuated by designing resonators. The simulation shown in Figs. 12 and 13 gives an idea of the degree of attenuation received where the pipes with the array of resonators is able to attenuate the noise by around 30-40 dB for both frequencies. The values of  $\alpha$  and  $\beta$  used are 1.2 and 0.07142 so as to satisfy Eq. (9), i.e. the ratio,  $\alpha/\beta \le 0.07522$  for frequencies 3556 and 2712 Hz.

### CONCLUSIONS

A new design procedure has been proposed and validated in this paper for noise attenuation using Helmholtz resonators in pipelines. Applied to one and two of Helmholtz resonators, the designed models of resonators have been verified numerically using COMSOL. All analytical and numerical results were validated using experimental results from published data. Attenuation of around 40 dB has been achieved which proves not only the efficiency of the proposed design procedure but also the straightforward method to dimension the resonators.

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### Appendix A - One degree of freedom resonator

The equation for angular resonating frequency of one degree of freedom resonator is given by [7]

$$\omega = c\sqrt{-\frac{3L_n + L_c A}{2L_n^3} + \sqrt{\left(\frac{3L_n + L_c A}{2L_n^3}\right)^2 + \frac{3A}{L_n^3 L_c}}$$
(A1)

where

 $A = \frac{a_n}{a_c}$ 

 $a_n$  is the cross section area of the neck,  $a_c$  is the cross section area of the cavity,  $L_n$  is the corrected neck length,  $L_c$  is the length of the volume, and c is the speed of sound.

Solving Eq. (A1) for the two areas of cross sections results in

$$a_n = \frac{a_c \left(-3c^2 \,\omega^2 \,L_n L_c - \omega^4 \,L_n^3 \,L_c\right)}{c^2 \left(-3c^2 + \omega^2 \,L_c^2\right)} \tag{A2}$$

$$a_{c} = \frac{a_{n}c^{2} \left(-3c^{2} \omega^{2} L_{c}^{2}\right)}{\left(-3c^{2} \omega^{2} L_{n} L_{c} - \omega^{4} L_{n}^{3} L_{c}\right)}$$
(A3)

From these two equations it is obvious for the two cross section areas to be positive, the denominator should be negative for Eq. (A2) while the numerator should be negative for Eq. (A3), that is

$$-3c^2 + \omega^2 L_c^2 < 0 \tag{A4}$$

where

$$\omega < \sqrt{\frac{3c^2}{L_c^2}}$$

The resonating frequency then becomes

Therefore, Eq. (B1) can be expressed as

 $f < \left(\frac{1}{2\pi} \sqrt{\frac{3c^2}{L_c^2}} = 0.2756 \, \frac{c}{L_c}\right)$ 

## Appendix B - Two degree of freedom resonator

The equation for angular resonating frequency of two degree of freedom resonator is derived from Eq. (1) and is given by

$$f_{1,2} = \frac{c}{2\sqrt{2\pi}} \sqrt{\left(\frac{\alpha}{V_1} + \frac{\beta}{V_1} + \frac{\beta}{V_2}\right) \pm \sqrt{\left(\frac{\alpha}{V_1} + \frac{\beta}{V_1} + \frac{\beta}{V_2}\right)^2 - 4\frac{\alpha}{V_1}\frac{\beta}{V_2}}}$$
(B1)

where

$$\alpha = \frac{a_{n1}}{l_{n1}}$$
 and  $\beta = \frac{a_{n2}}{l_{n2}}$ 

 $f_{1,2} = \frac{c}{2\sqrt{2\pi}} \sqrt{\left(\frac{a_{n1}}{l_{n1}V_1} + \frac{a_{n2}}{l_{n2}V_1} + \frac{a_{n2}}{l_{n2}V_2}\right) \pm \sqrt{\left(\frac{a_{n1}}{l_{n1}V_1} + \frac{a_{n2}}{l_{n2}V_1} + \frac{a_{n2}}{l_{n2}V_1}\right)^2 - 4\frac{a_{n1}}{l_{n1}V_1}\frac{a_{n2}}{l_{n2}V_2}}$ (B2)

Equation (B2) can be solved for the volumes of the first and second cavities, which are given by

$$V_{1} = \frac{(8 a_{n1}^{2} a_{n2} c^{4})}{l_{n1} \left( 2a_{n1}a_{n2}c^{2}f_{1}^{2} + 2a_{n1}a_{n2}c^{2}f_{2}^{2} - \sqrt{\left( 4a_{n1}^{2}a_{n2}^{2}c^{4}f_{1}^{4} - 8a_{n1}^{2}a_{n2}^{2}c^{4}f_{1}^{2}f_{2}^{2} + 4a_{n1}^{2}a_{n2}^{2}c^{4}f_{2}^{4} - \frac{16a_{n1}a_{n2}^{3}c^{4}f_{1}^{2}f_{2}^{2}l_{n1}}{l_{n2}} \right) \right)} + \frac{(8 a_{n1}a_{n2}^{2}c^{4})}{l_{n2} \left( 2a_{n1}a_{n2}c^{2}f_{1}^{2} + 2a_{n1}a_{n2}c^{2}f_{2}^{2} - \sqrt{\left( 4a_{n1}^{2}a_{n2}^{2}c^{4}f_{1}^{4} - 8a_{n1}^{2}a_{n2}^{2}c^{4}f_{1}^{2}f_{2}^{2} + 4a_{n1}^{2}a_{n2}^{2}c^{4}f_{2}^{4} - \frac{16a_{n1}a_{n2}^{3}c^{4}f_{1}^{2}f_{2}^{2}l_{n1}}{l_{n2}} \right) \right)}$$
(B3)

$$V_{2} = \left(\frac{0.5\left((a_{n1}a_{n2}c^{2})(2f_{1}^{2} + 2f_{2}^{2}) - \sqrt{\left(a_{n1}^{2}a_{n2}^{2}c^{4}(2f_{1}^{2} + 2f_{2}^{2})^{2} - \frac{16a_{n1}a_{n2}^{2}c^{4}f_{1}^{2}f_{2}^{2}(a_{n2}l_{n1} + a_{n1}l_{n2})}{l_{n2}}\right)}{f_{1}^{2}f_{2}^{2}(a_{n2}l_{n1} + a_{n1}l_{n2})}\right)$$
(B4)

The two volumes  $V_1$  and  $V_2$  are real if the expressions in square root are positive or zero. Hence, a new condition emerges and is given by

$$\left(\frac{f_1^2}{f_2^2} + \frac{f_2^2}{f_1^2}\right) \ge 2\left(1 + \frac{2a_{n2}l_{n1}}{a_{n1}l_{n2}}\right)$$
(B5)

which can also be written as

$$\left(\frac{f_1^2}{f_2^2} + \frac{f_2^2}{f_1^2}\right) \ge 2\left(1 + \frac{2\gamma_a}{\gamma_l}\right)$$

$$\gamma_a = \frac{a_{n2}}{a_{n1}} \quad , \quad \gamma_l = \frac{l_{n2}}{l_{n1}}$$

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# **Distance Learning for Acoustics**

The Professional Education in Acoustics program was established some years ago on the request from the industry due to a lack of regularly available appropriate courses in the formal University programs. It is aimed at providing appropriate modules to meet the needs of those embarking on a career in Acoustics and has the support of the Association of Australian Acoustical Consultants (AAAC). It is also be of value for those working in government agencies and allied organisations needing a fundamental understanding of acoustics. The program is based on a similar program that has been offered via universities and the UK Institute of Acoustics (IOA).

The program is fully flexible and all undertaken in distance learning mode. This means the modules can be commenced at any time and there is no requirement to complete at a specific date. This is an advantage to those who are unsure of future work demands – but of course a disadvantage as the lack of a deadline means that completion depends on the commitment of the registrant.

Each module of the program will be offered separately in distance learning mode so that it can be undertaken throughout Australia or elsewhere in the world and can be commenced at any time. Each module comprises course notes, assignments and two modules include practical exercises and a test. Registrants work through this material at their own pace and in their own location submitting the work electronically. The practical work and the test are undertaken at the registrant's location under supervision of their employer. It is expected that those registrants working for acoustical consultancies will receive support and supervision by their supervisors. For registrants who are working on the program without support from their employer will be given assistance by phone or email from the course coordinator. This assistance can be supplemented with assistance from a company that is a member of the AAAC.

For more information on the program see http://www.aaac.org.au/au/aaac/education.aspx

