LOW-FREQUENCY ABSOLUTE CALIBRATION OF ACCELEROMETERS

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Abstract: At the National Measurement Laboratory, Australia, apparatus and techniques have been developed for calibrating vibration-measuring transducers in the low-frequency range, 2Hz - 250Hz. This paper describes the apparatus and the methods used to calibrate the built-in reference accelerometers absolutely in terms of the primary standards of length, time and voltage.

1. INTRODUCTION

Traceability, and estimation of uncertainty, are often of interest to engineers who measure low-frequency vibration, particularly if there is a possibility of legal challenge [1], in which case the component of uncertainty due to the calibration of the measuring transducers may need to be taken into account.

The National Measurement Laboratory (NML) maintains a horizontal air-bearing electrodynamic vibration exciter, which is big enough to accommodate most triaxial geophones and similar large transducers. Such transducers are routinely calibrated by comparison with the built-in low-frequency references, a set of servo accelerometers.

This paper is concerned with the apparatus, methods and techniques which are used at NML to calibrate the low-frequency reference accelerometers. To "calibrate" is defined as to determine the magnitude of the transfer function, generally referred to as the sensitivity of the accelerometer, in terms of the physical standards of length, time and voltage. In calculating the uncertainty of the sensitivity values, the methods used are taken from the ISO "Guide" [2].

2. STATIC CALIBRATION AT ZERO HERTZ

This method is a simple adaptation of the familiar "2g turnover" which is frequently used for field calibration of accelerometers which have a response down to zero hertz.

The accelerometer is first aligned with its sensitive axis vertical and the "positive" direction pointing up (Fig. 1). In the case of the NML reference accelerometers, this is achieved by standing on end the armature from the air-bearing shaker. The signed dc voltage output *V1* is then measured using a calibrated voltmeter.

Next, the vertical orientation is reversed, ie the accelerometer is turned over so that the "positive" direction is pointing down, and the signed dc output voltage *V2* is measured.

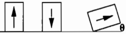

Figure 1. Static calibration of accelerometer using Earth's gravity field

The sensitivity S_0, in Volts per m s^{-2} , is then obtained as

$$S_0 = (V1 - V2) / (2 * g_l) \qquad (1)$$

where g_l = local value of the acceleration due to gravity, in m s^{-2} .

To convert this to Volts per g_n , S_0 is multiplied by g_n/g_l where g_n = ISO standard gravity, the value of which is defined to be 9.80665 m s^{-2}.

At the NML location of the calibration apparatus, the value of g_l is 9.79638 m s^{-2}, with an uncertainty of about 5 parts in 10^6. However, typically the true sensitive axis of an accelerometer may differ from the geometric axis by about 1.5 degrees, and the uncertainty in setting the geometric axis to vertical is about 1.5 degrees also, when the armature is simply stood on end on the bench. Hence for this calibration, the total uncertainty in the applied acceleration is approximately ±0.1% .

The inherent dc offset, V_0, in the accelerometer output is also obtainable from

$$V_0 = (V1 + V2) / 2 \qquad (2)$$

2.1 LINEARITY (i)

The static method gives an accurate value for the sensitivity at zero hertz, amplitude $1.0g_l$, but it is not reasonable to extrapolate to lower accelerations without knowledge of the linearity of the accelerometer. For an initial linearity check, the accelerometer was mounted on a precision rotary table

(Optical Measuring Tools Ltd), which was then rotated incrementally such that the sensitive axis was rotated in a vertical plane, and at each increment a reading was taken of the dc voltage output $V(\theta)$, where θ is the angle between the sensitive axis and horizontal. Linearity was then assessed from a plot of $V(\theta)$ versus $g_l \sin\theta$.

This method was found to be not entirely satisfactory. At very small angles, there is significant error due to finite transverse sensitivity of the transducers, and non-coincidence of the sensitive axis with the geometric axis. The two factors are not independent, but the nett error can be reduced by aligning the direction of minimum transverse sensitivity to lie in the plane of rotation (resulting in the smallest minimum output when rotated through "zero" angle), by which the effective transverse sensitivity was reduced by a factor of ten. At an angle of $\theta = 6$ degrees, corresponding to approximately $0.1g_l$, transverse sensitivity of 1% can produce an error of 9.5%, but the alignment strategy reduced this to a just acceptable 0.95%. The practical lower limit to this method is thus considered to be about $0.1g_l$ (1 m s^{-2}), corresponding to $\theta = 6$ degrees approximately.

3. ABSOLUTE CALIBRATION USING INTERFEROMETRY

In principle, the accelerometer is subjected to rectilinear simple harmonic motion (SHM) along the direction of its sensitive axis, and the displacement and frequency are measured, thereby defining the applied acceleration. The displacement measurement utilises the so-called "frequency ratio" method (FR) of counting optical interference fringes [3,4,5,6]. The voltage output is measured at the same time, hence the sensitivity is obtained.

3.1 THE APPARATUS

At NML, SHM is produced by an electrodynamic vibration generator ("shaker"), the drive coil of which is attached to an aluminium armature ("shaker table") which is constrained to horizontal rectilinear motion by air bearings (Fig. 2). The drive coil has no separate supports, and the only other constraints are four light rubber bands which centre the oscillation. Peak-to-peak displacements of 25mm are attainable, but displacement is restricted to about 10mm peak-to-peak for the lowest distortion.

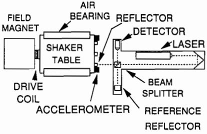

Figure 2. Air-bearing shaker and interferometer for low-frequency calibration.

Two servo accelerometers are permanently mounted in one end of the armature, and mounted adjacent to each of them is an optically flat mirror which serves as one of the reflectors in a simple Michelson interferometer. The entire shaker assembly, of mass approximately 100 kg, rests on a concrete and brick block approximately 2m long by 0.7m wide by 0.9m high, which in turn is cemented to the concrete ground floor of the NML building. There is a layer of about 3mm of bituminous felt between the shaker assembly and the concrete block.

Also resting on the top of the concrete block, at the opposite end to the shaker, is a steel cruciform structure which houses the rest of the Michelson interferometer: a beam splitting cube, a plane reference reflector, a 3mW HeNe laser, and a photodetector. The assembly can be moved laterally, to point at any of the plane reflectors on the shaker table. The light from the laser reaches the beam splitter after reflection by two mirrors, the purpose of which is to increase the distance between the laser source and the reflectors of the interferometer. Thus, by a deliberate tiny misalignment (approximately 2 mrad) of the interferometer, reflected light is prevented from re-entering the laser, and the resulting error in displacement measurement is insignificant.

In addition to the built-in accelerometers, there is provision for calibrating a small piezoelectric accelerometer, which can be attached directly to the back of one of the reflectors on the shaker table. This is routinely used to extend downwards the frequency range of absolute calibration of piezoelectric reference accelerometers.

The apparatus is used to calibrate the reference accelerometers over the frequency range 2Hz - 250Hz, with acceleration of up to 9.8 m s^{-2} available at frequencies not less than 10Hz. At 10Hz, the corresponding peak velocity is about 0.16m/s, equivalent to 500,000 fringes per second. This is easily handled by the photodetector, which is a PIN diode with a close-coupled wide-band amplifier giving an overall frequency range of 3 MHz.

3.2 ACCELERATION MEASUREMENT

The shaker is driven from a programmable oscillator via a power amplifier (Fig. 3), and the displacement, D, is measured by counting the passage of interference fringes past the detector during several complete cycles of excitation (see appendix A). The counter is "gated" with a signal from the shaker drive oscillator. The drive frequency f is measured with a second counter, the time base for which is derived from the NML Caesium Beam Frequency Reference. As the wavelength of the laser is known in terms of the legal standard of length, to an uncertainty of a few parts in 106 in the controlled NML environment, the applied acceleration ($= (2\pi f)^2 D$) can be known very precisely. Simultaneous measurement of the accelerometer output voltage completes the calibration.

In applying the FR method of displacement measurement, a correction is made for ambient displacement of the reference reflector. At the NML location this includes random noise with mean amplitude less than 0.5μm, plus components

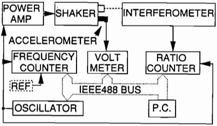

Figure 3. Block diagram of low-frequency accelerometer calibrator

at about 4.5Hz, 20Hz and 25Hz (attributed to structural resonances), with amplitudes which vary randomly between 0.5μm and 1μm. Denoting the mean ambient displacement by *Da*, measured by the FR method over an integral number of cycles of the drive frequency *f* with the drive coil disconnected, subsequent displacement measurements D_{meas} at frequency *f* are corrected as

$$D \text{ (corrected)} = \sqrt{(D_{meas}{}^2 - D_a{}^2)} \qquad (3)$$

which is justifiable as D_{meas} and D_a are uncorrelated.

The ambient vibration is also of a suitable amplitude and velocity to provide "jitter" of the photoelectric signal at the turning points of the motion. By averaging counts made as the reference reflector is slowly moved, it is possible to extend the resolution below the expected ±1 fringe count (see appendix A, and Fig. 4). Hohmann & Martin [5] noted a tenfold improvement in resolution from moving the reference reflector with mean velocity much less than the velocity of the vibration being measured. In comparing the FR method with other methods at NML [7], it was found that, by moving the reference reflector, the displacement was measurable by the FR method with a resolution of 1 nm and an uncertainty of approximately ±2 nm. Von Martens [9] achieved even greater accuracy for SHM at frequencies > 500Hz.

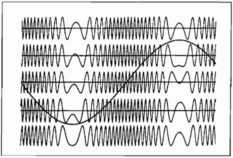

Figure 4. Successive observations of the photoelectric signal over one cycle of constant amplitude SHM, as the reference reflector is slowly moved.

3.3 LINEARITY (ii)

At 2Hz, 1 m s^{-2}, the displacement amplitude is 6.33257 mm, thus the peak-to-peak displacement corresponds to the passage of 20 014.46 interference fringes. Acceleration of 0.001 m s^{-2} at the same frequency corresponds to about 20 fringes peak-to-peak, which can be measured with an uncertainty of about ±2 nm, ie ± 0.03%. Thus linearity measurement can cover a 60 dB range at a single frequency. Similar measurements at 10 Hz, from 1 m s^{-2} to 10 m s^{-2}, effectively extend the linearity measurement to an 80 dB range.

3.4 VOLTAGE MEASUREMENTS

Several different instruments have been used for voltage measurement, and in each case the instrument calibration is traceable to the Josephson Volt Standard, via thermal transfer standards.

The first instrument used was a Fluke 931B True RMS Differential Voltmeter. This is a 5$^1/_2$ digit instrument which is calibrated "in house" by the electrical standards section of NML, over its full range of use. At frequencies not less than 30 Hz, the uncertainty in those calibrations is ±0.05%, increasing to ±0.3% at 2 Hz. The disadvantage in using this instrument is that each reading must be made manually, thus measurement is slow, tedious and cannot be easily automated.

A Hewlett-Packard hp3458A digital multimeter overcomes the above difficulties. This 8$^1/_2$ digit instrument measures by fast digital sampling and computation of true rms, with or without dc as required, and its response is flat within a few parts in 10^6 over the voltage and frequency range required. Uncertainty in its calibration is similarly small, of the order of ±30 parts in 10^6, and the instrument can be programmed and read via an IEEE488 general purpose interface bus (GPIB).

As the acoustics and vibration standards program has only one hp3458A, it has been decided to retain it as a reference voltmeter against which to check our other voltmeters. These include several hp34401A meters which are used at frequencies down to 20Hz, and an Analogic DP6100 analyser which is calibrated as a voltmeter for the frequency range 2Hz-500Hz. All of these have IEEE488 interfaces.

The hp34401A multimeter, on medium filter setting, can take 1 reading/sec with a claimed accuracy of ±0.3% for full scale at 20Hz, and about ±0.15% at higher frequencies. It was calibrated by comparison with the hp3458A, the uncertainty of this calibration was estimated as ±0.1% for full scale at 20Hz.

The DP6100 analyser is a computing instrument with a 16-bit 8MHz mainframe, a 14-bit A/D input which can sample at up to 100k samples/sec, and a built-in voltage reference. It can be programmed to compute true rms in the same way as the hp3458A, and when calibrated against the hp3458A the correction is less than 0.1% for the voltage range of interest. The additional uncertainty is about ±0.05%, with an extremely flat frequency response. To attain this performance, the program ensures that sampling is over an integral number of cycles.

The DP6100 can also compute Fast Fourier Transforms (FFTs), and has been used in this mode as a narrow-band voltmeter. After applying to the transformed data a correction for the so-called "picket fence effect" [7,8], it was found that these measurements could be made with an uncertainty of approximately ±0.1% .

4. CONCLUSIONS

At the National Measurement Laboratory, apparatus and techniques have been developed to calibrate reference accelerometers "absolutely" at frequencies down to 2Hz. By exercising great care in making measurements, correcting for errors, and by maintaining strict traceability to the primary standards of length, time and voltage, an uncertainty of approximately ±0.5% is achieved for these calibrations. The interferometric method is complemented by a static calibration using the local g field, which effectively extends the calibration down to zero hertz. When accelerometers and other transducers are calibrated by comparison with these references, the sensitivity value can have a least uncertainty, at the time of the calibration, of about ±0.6% in the 2Hz-20Hz frequency range, though this may be much greater for large transducers. Uncertainties quoted here are at the 95% level of confidence [2].

5. REFERENCES

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APPENDIX A`

MEASURING VIBRATION DISPLACEMENT WITH AN INTERFEROMETER

Assume that the reference reflector and the reflector on the armature are initially stationary, and that the distance from the beam splitter to each of the reflectors is exactly the same, or differs by an exact number of half-wavelengths of the laser light. Light returning from the two reflectors is thus in-phase at the beam splitter, and there is constructive interference of the re-combined light emerging from the beam splitter. If the armature is now displaced by a distance of one quarter wavelength, light reflected from it travels an extra half-wavelength, and the resulting re-combined light exhibits destructive interference. A further quarter-wavelength displacement again produces constructive interference, and at the photodetector the re-combining light has now gone through one complete cycle of intensity variation. By converting such cyclic variations (or "interference fringes") of intensity to electric signals, the photodetector makes it possible to count them, and thereby measure the distance through which the armature has moved, in terms of the wavelength λ of the laser light. If the armature is subjected to rectilinear SHM with a displacement amplitude equal to one half-wavelength, then one "fringe" will be counted 4 times in each cycle of the SHM. Any given displacement amplitude D is thus measureable by counting the number of fringes per cycle of SHM, ie by measuring a mean frequency ratio R, thus

$$D = R\lambda / 8$$

This is essentially the principle of the "frequency ratio" method of measuring displacement interferometrically. If R is not an integer, then the extra fraction may or may not be counted, depending on the position of the fringe pattern with respect to the photodetector at the turning point of the SHM. Figure 4 shows successive observations of the fringe pattern, over one cycle of constant amplitude SHM, as the reference reflector is slowly moved. Averaging a large number of such observations gives the value of R complete with the fractional part.

APPENDIX B

ESTIMATION OF UNCERTAINTIES

Uncertainty values are calculated using the methodology in the ISO "Guide" [2]. The following describes how this is done for absolute calibrations of low-frequency reference accelerometers.

Type A Uncertainties

From n repeat measurements of the sensitivity of an accelerometer, the MEAN and STANDARD DEVIATION (S.D.) are calculated.

The S.D. gives an estimate of the uncertainty of individual measurements, but to estimate the TYPE A STANDARD UNCERTAINTY OF THE MEAN, (corresponding to the older concept of the Standard Error of the Mean, or S.E.M), S.D. is divided by $\sqrt{n}$.

Before factoring in type B uncertainties, the type A uncertainty is converted to per unit form, thus U_A (per unit) = S.D./($\sqrt{n}$ * MEAN)

Type B Uncertainties

These are all evaluated in per unit form:

Voltmeter resolution	U_1	(resolution/(√3 x reading))
Voltmeter calibration uncertainty	U_2	(0.001-0.003, see section 3.4)
Voltmeter sensitivity drift	U_3	(0.001)
Voltage uncertainty due to total noise & distortion (TND)	U_4	(after correction, 0.0002)
Frequency resolution	U_5	(resolution/(√3 x reading))
Frequency reference uncertainty	U_6	(1 x 10^{-6})
Uncertainty in displacement due to total noise and distortion(TND)	U_7	(after correction, 0.0002)
Transverse & rocking motion	U_8	(0.0005)
Temperature coeff(accelerometer)	U_9	(0.00018 for 1° shift)
HeNe Laser Wavelength	U_{10}	(2 x 10^{-6})
f ratio resolution(fringe counting)	U_{11}	(0.001/(√3 x ratio count))

The above type B uncertainties are considered to be uncorrelated, and can be combined by Root Sum of Squares to obtain U_B (per unit).

If the type A uncertainty is "very large" by comparison with all type B uncertainties, then an expanded uncertainty, at 95% level of confidence, can be estimated by multiplying U_A by $t_{0.025}$ for (n-1) degrees of freedom, where t is from "Student's t" distribution, and corresponds to *k*, the "coverage factor" of the ISO Guide. For these purposes, "very large" means that no significant figures of the expanded uncertainty would be changed by the addition of type B uncertainties. In general, however, an expanded combined uncertainty, at 95% level of confidence, is obtained as

$$U_{exp} = k * \sqrt{(U_A^2 + U_B^2)}$$

where k = 2.0 if ν >30, otherwise k = $t_{0.025}$ for ν degrees of freedom, where ν is calculated from the Welch-Satterthwaite formula (appendix G of [2]).

Typically for a calibration at 5Hz, U_{exp} is about 0.005, ie ±0.5% uncertainty.