

Simulation of a Ripple Tank

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Abstract: Visualisation of the reflection of waves off objects is useful in education, because we cannot see sound waves. Techniques for visualising sound fall into four groups: listening, mechanical analogues, measurement, and computer simulation. This paper describes efforts to develop the computer graphics equivalent of a ripple tank.

1. INTRODUCTION

The most common means of visualising sound propagation is a ripple tank, where an observer can watch the motion of waves in an artificial physical environment. The aim of our work is to produce the computer equivalent of a ripple tank by displaying the propagation of sound waves on a graphics display. While our motivation is to produce a visualisation system for research into ultrasonic sensing for mobile robots[1], such a system is very useful for teaching wave motion.

Educational software must run on a personal computer, placing considerable importance on the selection of algorithms. Also, educational software requires a good user interface to enable the student to interact with the system by changing parameters to see what happens.

The following models of wave propagation are examined: physical models, Transmission Line Matrix models and Lattice Gas models. Models for visualising wave propagation, the interference between multiple sources, reflection, and diffraction are discussed in Sections 3, 4 and 5.

2. VISUALISATION TECHNIQUES

Techniques for visualising sound fall into four groups: listening, mechanical analogues, measurement, and computer simulation.

2.1 Listening

People usually perceive sound through their ears. Humans have highly developed auditory perception for speech, music and other sounds. Wenzel et al. utilised this ability to develop an acoustic display for a virtual environment workstation[2]. This system generates localised acoustic cues in real time over headphones. Their aim is to make their virtual world sound real as well as look real.

2.2 Mechanical Analogues

Physics teachers use mechanical analogues to visualise sounds during laboratory classes. They demonstrate one dimensional wave propagation with springs and wave machines, and two dimensional wave motion with ripple tanks. Some ripple tanks have transparent bases to enable an image of the waves to be projected onto a screen using an overhead projector.

However, repeatable demonstrations are difficult to achieve due to the cumbersome nature of ripple tanks. For this reason, teachers refer their students to photographs of wave motion printed in most physics text books. A ripple tank can be used to visualise wave propagation, reflection, interference and diffraction.

2.3 Measurement

While a researcher can observe a ripple tank and record it photographically, accurate measurement of wave propagation is required in order to do calculations. All measurement techniques use microphones to measure the instantaneous sound pressure. These measurements can be recorded and displayed for visualisation purposes. To build up a picture of a sound field, a microphone is scanned or an array of microphones is used.

Prior to the availability of low cost computers, Winston Kock developed a method for recording an acoustic field on a photographic plate[3]. He attached a microphone to the end of a scanning device: a long rod oscillated by a motor to move the microphone in an arc transverse to the axis of the field to be measured. A second motor moved the scanning device linearly along the axis of the field to be measured. Fixed to the microphone was a lamp whose intensity was modulated by the measured sound field. A time lapse camera recorded the intensity of the lamp as it moved in the two dimensional scanning plane.

To show the intensity of the sound waves at points in the scanning plane, the measured and reference signals were summed to produce a standing wave pattern. With this apparatus he was able to photograph sound including diffraction at the edges of shadowing objects.

2.4 Simulation

The problem with measurement is that many points have to be measured in the acoustic field to gain an accurate representation of wave motion. This process takes a long time and requires expensive equipment. To gain an understanding of wave propagation and scattering, we are developing simulations.

To develop a useful simulation we have to choose a suitable model. This choice is constrained by the desired accuracy of the simulation, the required update rate of the

graphical display and the method of presenting the information on the display. Complex models and complex rendering often result in simulations that are too slow for dynamic visualisation.

It is difficult to achieve dynamic displays on a personal computer with all but the simplest models. An example of a simple model used to produce a dynamic display for robotics research is the arc model[4]. In this technique, a sound chirp is modelled as an arc with arc angle equal to the beam angle of the transducer. This model has helped us to understand the problems that occur when using ultrasonic sensing to guide a mobile robot (Figure 1).

Sound waves can be modelled using geometric, physical and discrete models. Physical models, based on the wave equation, are the most accurate but take the longest to compute. Geometrical models, based on ray tracing, are useful for tracing the path of a wave envelope but give no detail of wave structure. In order to reduce the execution time of simulation, discrete models are solved with numeric techniques, with resultant loss of information. In the following sections, we examine the Transmission Line Matrix model (TLM)[5], the wave model, and the Lattice Gas model (LG)[6].

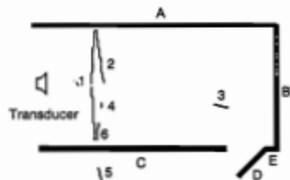


Figure 1. Simulator using arc model to show motion of chirp and reflection off specular surfaces. Reflection off 1. surface B, 2. surface A then B, then A, 3. surface D, then A, 4. corner between surface B and E, 5. surface B travels through opening between C and E, and 6. surface C during return of chirp.

3. DISCRETE MODEL - TLM

TLM modelling is a numerical method for solving scattering problems. This method produces a computer simulation of electric fields in both space and time. The two dimensional TLM model has a network analogue in the form of a mesh of orthogonal transmission lines (Figure 2). There is a direct equivalence between the voltages and currents on the lines in the mesh and the pressure and intensity of sound. With this mesh we can model two dimensional wave problems.

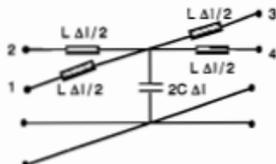
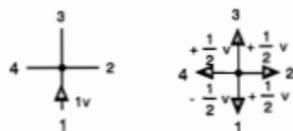
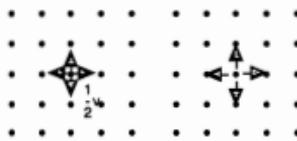


Figure 2. The building blocks of the two-dimensional TLM network. Equivalent lumped element model.

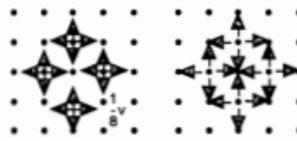
The model operates by propagating sine waves along the lines and, due to the simulated discontinuity at the nodes, results in transmitted and reflected waves being scattered back into the lines. These scattered waves then become incident on adjoining nodes at the next time instant (Figure 3).



a) Incident wave on line 1. b) Reflected waves.



c) Scatter (t_0). d) Connect (t_0).



e) Scatter (t_1). f) Connect (t_1).

Figure 3. Transmission Line Matrix model.

The analogous relationships among the electrical and acoustic wave parameters are as follows:

Voltage V	Pressure P
Current i	Particle speed u
Inductance L	Density ρ
Capacitance C	Compressibility B
Impedance $\sqrt{\frac{L}{C}}$	Impedance $\sqrt{B\rho}$

Each iterative step includes two processes, scattering and connection (Figure 3, c, f). The scattering process is that waves scatter from a node after impulses incident on the node; the connection process is that waves propagate toward its neighbours after waves scatter from a node. The computation for the scattering process at each node, within each iteration, is the weighted sum of impulses incident on the node.

$${}_{k+1}V_n^i = \frac{1}{2} \left(\sum_{m=1}^4 {}_kV_m^i \right) - {}_kV_n^i, \quad (n = 1, 2, 3, 4) \quad (1)$$

where

$V_n^{r,i}$ is the incident impulse at time step on k line n , and
the r superscript of V_n^r denotes the scattering impulse.

The formulae for the connection process are:

$$\begin{aligned} k_{+1}V_1^{r,i}(z,x) &= k_{+1}V_3^{r,i}(z,x-1) \\ k_{+1}V_3^{r,i}(z,x) &= k_{+1}V_1^{r,i}(z,x+1) \\ k_{+1}V_2^{r,i}(z,x) &= k_{+1}V_4^{r,i}(z-1,x) \\ k_{+1}V_4^{r,i}(z,x) &= k_{+1}V_2^{r,i}(z+1,x) \end{aligned} \quad (2)$$

The mathematics of the TLM model is simple and hence rapid to compute. At each iteration step, values are calculated at nodes in the mesh. To render a continuous or a smooth image, we must have sufficient nodes per wavelength to enable accurate interpolation of the values between nodes. If we use linear interpolation, we can get the result quickly, but the image looks rough. If we use nonlinear interpolation, we get a better result at the cost of increased calculation time.

To smooth the image and to avoid the long time required for nonlinear interpolation, we use grey scale to render the sound pressure at all points in 2D space. A human's eyes are much more sensitive to the straightness of a line than variation in grey scale. The images in Figure 4 were calculated at time steps proportional to wave travel of one wavelength, with 20 nodes per wavelength. Then they were stored and displayed at the end of the simulation. Even with grey scale rendering, noise produced by this method can be seen in the images.

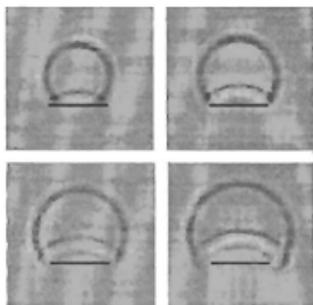


Figure 4. Play back of a time sequence of grey scale images of wave propagation and reflection off a flat surface calculated with the TLM model for one cycle.

4. CONTINUUM WAVE MODEL

To obtain a more accurate visualisation we can use continuous models based on the physical properties of sound waves. However, each property must be modelled separately and then combined to produce the final visualisation. Thus, we require models for propagation, interference, reflection, and

diffraction. As the properties of sound waves are similar to the properties of light waves, models for light can be applied to acoustics after modification for medium properties and wave energy characteristics.

4.1 Propagation

Sound propagation is described by the wave equation:

$$\nabla^2 P = \frac{1}{c^2} \frac{\partial^2 P}{\partial t^2}, \quad (3)$$

where

P is the instantaneous pressure of the wave,

∇^2 is the Laplacian operator (the divergence of the gradient),

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2},$$

c is the propagation velocity of the wave, and

t is the time.

This expression applies to waves moving through non-dispersive mediums. The solution of the wave equation for a cylindrical source [8, p. 357] is:

$$P = A[J_0(kr) + iN_0(kr)]e^{-i\omega t} \quad (4)$$

$$\xrightarrow{r \rightarrow \infty} A\sqrt{\frac{2}{\pi kr}} e^{i(kr - \omega t) - i(\pi/4)} \quad (5)$$

$$\xrightarrow{r \rightarrow 0} i\frac{2A}{\pi} \ln(r)e^{-i\omega t} \quad (6)$$

where

A is constant,

ω is the angular frequency,

k is the wavenumber,

r is the distance from the source,

J_0 denotes the Bessel function of first kind of zero order, and

N_0 denotes the Bessel function of second kind of zero order.

To model wave propagation in two dimensions, as required to produce the graphical equivalent of a ripple tank, we use Equation (5) and place objects at several wavelengths from the source. Although not correct near the source it is a sufficiently good approximation to Equation (6) to produce an acceptable visualisation (Figure 5). Using Equation (5) results in faster calculation and in smooth rendering.

4.2 Interference

Another important area of ripple tank simulation is the interference of waves, as this allows us to study beam forming by arrays of sources. In a linear medium, one can apply the principle of superposition to obtain the resultant disturbance.

The superposition principle states that the actual displacement of any part of the disturbed medium equals the algebraic sum of the displacements caused by the individual waves.

Suppose we have waves P_1 and P_2 emitted from two sources, $P_1 = P(r_1, t)$ and $P_2 = P(r_2, t)$, where r_1 and r_2 are the distances of the wavefronts from the two sources at time t . The interference wave $P_{int,j}$ is

$$P_{int,j} = P_1 + P_2 = P(r_1, t) + P(r_2, t) \quad (7)$$

Figure 5 shows the interference between waves from two point sources.

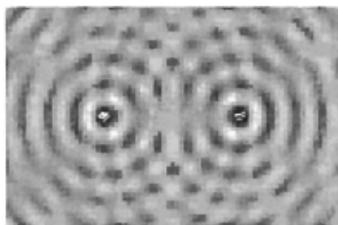


Figure 5. Wave interference from two point sources, calculated with the Equations (5) and (7).

4.3 Reflection

Waves are reflected when they reach a barrier. The law of reflection for specular surfaces states: the angle of reflection θ_r equals the angle of incidence θ_i .

To calculate the wavefront reflected off a barrier, we use the mirror equation [7] from optics which approximates the law of reflection when the incident waves strike the obstacle at points in the paraxial area. This approximation speeds up the calculation. The mirror equation describes the relationship between the wavefront curvature of incident waves and reflected waves. That is:

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \quad (8)$$

where

s is the distance of the wave source from the obstacle,

s' is the distance of the focus point from the obstacle, and

R is the radius of the obstacle.

In the case of a concave barrier, the reflected rays near the axis pass through a focal point in front of the object (S' in Figure 6). In the case of a flat barrier, the reflected spherical waves appear to come from a point behind the barrier. This point, called the imaginary point, is the same distance from the flat reflector as the source S is.

Spherical waves reflected from a convex curved barrier appear to originate from a focal point behind the barrier but closer to the barrier than the source (S' in Figure 7).

Each geometric shape has to be modelled separately and the results combined using superposition (Figure 8).

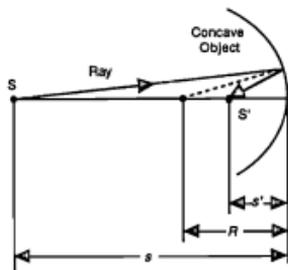


Figure 6. Graphical representation of mirror equation for a point source S and a concave object. S' is the focal point for the reflected paraxial rays.

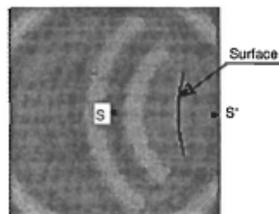


Figure 7. Spherical waves reflected from a convex surface appears to originate at the virtual source S' .

We use the mirror equation to simulate wave reflection off obstacles with simple shapes, - planes and arcs. For more complex curved shapes, we are experimenting with algorithms to calculate the amplitude of a reflected wave at a point in space from the curvature of the object.

4.4 Diffraction

When waves encounter an obstacle, they tend to bend around the obstacle. Diffraction is apparent by the waves in the shadow region and the interference between the diffracted waves and the incident and reflected waves near the edges of the object.

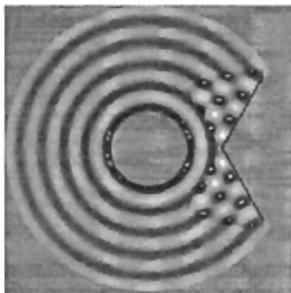


Figure 8. Reflection of spherical waves from a straight edge obstacle.

One way to model diffraction is to place secondary point sources at the edges causing diffraction. A simple source produces a wave front with equal intensity in all directions. As this is an inaccurate model of diffraction, we use the following equation to display the diffraction from a knife edge [9]:

$$P(r, \phi) = A e^{-i k r \cos \phi} E \left[\sqrt{2 k r} \cos \frac{1}{2} \phi \right] + A e^{i k r \cos(\phi + 2\psi)} E \left[\sqrt{2 k r} \cos \left(\frac{3}{2} \pi - \Psi - \frac{1}{2} \phi \right) \right] \quad (9)$$

where

$P(r, \phi)$: wave pressure at a point (r, ϕ) relative to the edge (Figure 9),

r : the distance from the knife edge to observation point,

ϕ : the angle of the direction of incident waves to the direction of the observation point, (Figure 9),

Ψ : the angle of the obstacle plane with the orthogonal direction of incident waves, and

$E(z)$ is defined as:

$$E(z) = \frac{1}{\sqrt{i\pi}} \int_{-\infty}^z e^{it^2} dt$$

$$= \begin{cases} \left[1 - \frac{1}{2} [1 - C(z^2) - S(z^2)] - \frac{1}{2} i [C(z^2) - S(z^2)] \right] & z \geq 0 \\ \left[\frac{1}{2} [1 - C(z^2) - S(z^2)] + \frac{1}{2} i [C(z^2) - S(z^2)] \right] & z \leq 0 \end{cases} \quad (10)$$

Fresnel integrals $C(z^2)$, $S(z^2)$ are defined as:

$$C(w) = \int_0^w \cos\left(\frac{\pi u^2}{2}\right) du$$

$$S(w) = \int_0^w \sin\left(\frac{\pi u^2}{2}\right) du$$

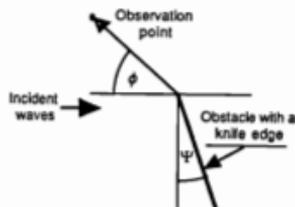


Figure 9. Parameters used in the model of diffraction (Equation 9).

Equation (9) is used to model the diffraction which occurs on both the transmitted waves and reflected waves as shown for diffraction from a knife edge in Figure 10. The diffraction

of the transmitted waves can be seen in the region (B). The diffraction of the reflected waves is visible in the region (D).

5. LATTICE GAS MODEL

The complexity of continuous models results in long computation times. To reduce this time we use discrete techniques to solve the physical model. One approach is the LG model which is based on cellular automata theory. A cellular automata model is a grid of cells each with a finite number of states. At each time step a set of rules defines the evolution of these states. The rules for a cell involve a finite number of neighbouring cells and can be either deterministic or nondeterministic.

The LG model emerged as a means of modelling fluid dynamics by modelling the molecular dynamics of the fluid in order to calculate transport coefficients. The first application of this technique to sound waves was reported by Kadanoff and Swift [10], who used a continuous time model. Krutar et al. [6] were the first to apply a discrete time model to sound waves. The discreteness of the Lattice Gas approximation introduces some noise into the visualisation.

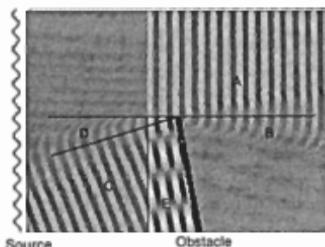


Figure 10. Wave reflection and diffraction, showing A the transmitted waves, B diffracted waves from a knife-edge, C reflected waves, D diffracted waves from reflected waves, E interference of incident waves with reflected waves.

5.1 Lattice Gas Calculation

The LG method models macroscopic wave motion by modelling microscopic particle motion. The cells in the lattice are connected in a square configuration [10] and, thus, each cell is connected to four others. The particles can be modelled as single particles, counts of particles or pressures [6]. These particles (or pressures) move along the links between cells. At each time step, the particles collide at the cells and then propagate out from the cells. Krutar developed a finite difference solution (Equation (11)) for the acoustic wave equation to calculate the derivative of pressure at a cell at each time step to generate a set of rules for the cellular automata.

$$dP(x, t + dt/2) = P(x, t + dt) - P(x, t)$$

$$ddP(x, t) = dP(x, t + dt/2) - dP(x, t - dt/2)$$

$$ddP(x, t) = \sum m_a P(x + dx_a, t) \quad (11)$$

where

$P(x, t)$ is the pressure at time t and location x , equivalent to an integer number of particles,

dt is the time step,

the subscript a represents one of the five directions, N, S, E, W, and at the cell, and

m_a is weighted coefficient of the pressure P at the direction a .

The first two equations are discrete derivatives. The third equation is the finite difference equation used to represent the wave equation (Equation (3)). It calculates the double derivative of pressure as a weighted neighbourhood average of pressure. By changing the speed of sound at a cell we can simulate a different medium (Figure 11).

From Krutur's model, we derived the following equations to be applied at each cell at each time step,

$$P(x, y, t + 1) = 2P(x, y, t) - P(x, y, t - 1) + c_s^2(x, y)(P(x + 1, y, t) + P(x - 1, y, t) + P(x, y + 1, t) + P(x, y - 1, t) - 4P(x, y, t)) \quad (12)$$

where

$c_s(x, y)$ is the simulation speed (cells propagated / simulation step), and

$P(x, y, t)$ is the wave pressure at location (x, y) and time t .

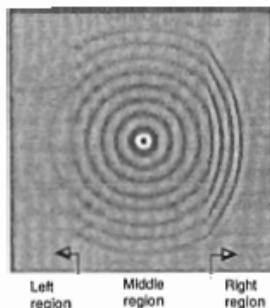


Figure 11. Wave propagation through different media. Left region : Speed = $2c$. Middle region: Speed = c . Right region: Speed = $0.5c$.

In a square grid the maximum speed of the simulated wave is 0.707 cells per time step[6]. Thus, the maximum speed of sound in any of the mediums, (there could be more than one medium), in the simulation is equivalent to this rate. In practice, the user wants to specify the frequency (f), spatial resolution (model scale) and the mediums used in the simulation. From this, the simulator calculates the simulation rate, based on the maximum speed of sound (C_{max}) in any of the mediums, where

$$\text{simulate rate } R = \frac{C_{max} * S}{0.707} \text{ steps / second,}$$

$$\text{model scale } S = \frac{m * f}{C_{max}} \text{ cells / meter, and}$$

m is the number of cells / wavelength.

Figure 12 shows that the LG model can simulate all wave phenomena: propagation, interference, reflection and diffraction. In this figure specular reflection is visualised by setting all cells on a straight line to have zero speed to represent an obstacle. As there are 20 cells per wavelength, diffusion may be modelled by modelling the texture of the surface.

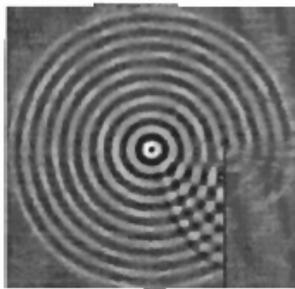


Figure 12. Wave propagation, reflection and diffraction visualised with the LG model.

6. VISUALISATION

Physical parameters of waves include: space, time, velocity, pressure, and intensity. In the figures in this paper space is represented by the 2D graphics screen, time by animation and wave pressure by grey scale or wire frame. Grey scale gives an image that looks like a photo of waves moving on a ripple tank.

Wave motion can be visualised with a variety of graphical representations. In Figure 13, wave generation by a point source is visualised with wire frame rendering.



Figure 13. Wire frame rendering of a point source, modelled with the wave equation.

Most of the images in this paper are grey scale images at an instant in time, that is a snap shot. More information can be visualised by a time sequence of images to show the motion of waves (Figure 4). Animation is achieved by displaying the simulation results at the end of each time step. However, on a personal computer the update rate is very slow, so better animation is obtained by recording the images from each time step and replaying them.

7. CONCLUSION

A simulation of a ripple tank is a useful tool for teaching wave motion concepts. Further, it is useful in the study of sound propagation in natural systems, such as bat echolocation, and artificial systems, such as ultrasonic sensing for mobile robots. Visualisation helps us to understand a physical phenomenon that we cannot see. However, the results of visualisation are only as meaningful as the accuracy of the underlying models.

We have discussed the theoretical basis for wave simulation, and demonstrated current results. We have simulated sound with three models: two discrete and one continuous. The discrete models are simple to iterate and automatically show wave properties, such as propagation, reflection and diffraction. The TLM model is based on electrical field theory, and the LG model on acoustic theory.

Both models use a square grid and hence are suitable for real-time execution on a parallel computer. Also, they include all wave motion properties of interest in their equations. Due to the discretisation of space, the execution of the simulation is independent of the complexity of the environment. However, a number of problems arise as a result of the discretisation. First, special processing has to be done at the boundaries. Second, quantisation errors in the discrete calculations appear as small noise waves (Figure 4). Third, wire frame rendering shows this noise as a zigzag on all the lines. The result is rough looking images, unless nonlinear interpolation is used. These variations are not so visible if grey scale rendering is used.

The continuous model requires separate models for each wave motion property, which must be combined using superposition to produce the final result. Also, the complexity of the calculation increases rapidly with the number of obstacles in the environment. As pressure is calculated at every point in space without reference to neighbouring points, continuous models do not have the window edge problem.

When calculating the pressure at a point at an instant in time the calculation is fast. But a simulation using a wave model may only be fast enough to display a snapshot of the output at regular time intervals. As the output is based on a continuous model, it is suitable for both grey scale and wire frame rendering.

Finally, a very powerful computer is required for animation. For this reason, discrete models will be used in educational tools which must execute on personal computers. Researchers who are interested in the greater accuracy of continuous models will either require access to a powerful computer or be content with snapshots. With all models, it is difficult to achieve real-time animation, so the computer equivalent of a ripple tank will calculate and store the images and then play them back in real time.

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