USE OF PiP TO INVESTIGATE THE EFFECT OF A FREE SURFACE ON GROUND VIBRATION DUE TO UNDERGROUND RAILWAYS

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The Pipe-in-Pipe model (PiP) is a quick and accurate tool for calculating vibration from underground railways and for assessing the performance of vibration countermeasures. The original model formulation simulates a tunnel buried in a fullspace but has recently been extended to account for a free-surface (i.e. halfspace). Results from the two versions are compared to quantify the effect of the free-surface on soil power spectral density (PSD) values. The study suggests that it is reasonable to assume the PSD surface results predicted from the free-surface model will be approximately 6dB more than those predicted by the fullspace model when the tunnel is at a depth of two tunnel-diameters or more. For tunnel depths less than two tunnel-diameters it seems beneficial to account for the free surface in the simulation as there is significant variation in the results invalidating the 6dB assumption.

1. INTRODUCTION

Underground railways are an effective means of transport in urban areas: locating the railway infrastructure below ground aids in reducing surface vehicle congestion and per-capita pollution emissions are lower than from the equivalent number of personal vehicles. A main concern with underground railways is vibration which propagates to nearby buildings causing annoyance to people [1]. The vibration may be perceived either directly through motion of floors and walls or indirectly as re-radiated noise. The vibration frequencies of interest typically range between 15-150 Hz [2]; higher frequencies are generally attenuated rapidly with distance along the transmission path through the soil [3]. The concern over vibration from underground railways has spurred the development of BS ISO 14837-2 to quantify acceptable vibration levels from underground railways.

Designers of underground railways and surrounding buildings rely on vibration predictions from numerical simulations to ensure they do not exceed these specified vibration levels; failure to do so can result in costly retrofitting of vibration countermeasures. Numerical methods commonly used for simulating ground vibration due to underground railways include two-dimensional [4, 5] and three-dimensional [6, 7] finite-element (FE) and boundary-element (BE) models; however, each method suffers from its own set of difficulties. Andersen and Jones [8] show that while 2D models require less computation effort they prove to be only qualitatively useful to indicate whether reductions in vibration can be achieved through structural changes. Three-dimensional models provide more quantitative results but require significantly more computational effort.

Semi-analytical modeling is also an accurate and efficient method for simulating ground vibration. Forrest and Hunt [9, 10] present a computationally efficient, three-dimensional semi-analytical model for calculating soil vibration in a fullspace from underground railways, known as the Pipe-in-Pipe model (PiP). As the name implies, the PiP model represents the tunnel and soil as concentric, coupled "pipes" as shown in Figure 1. The tunnel pipe is modeled using thin-shell theory while the soil pipe is modeled using elastic continuum theory. The outer radius of the tunnel pipe is equal to the inner radius of the soil pipe, and the outer radius of the soil pipe is infinite to simulate a fullspace.

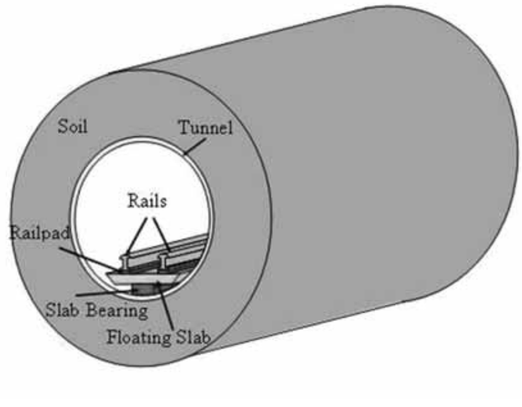


Figure 1. A floating-slab track coupled to the tunnel-soil model

Forrest [9] describes the method of transforming the governing equations of motion for the tunnel and the surrounding soil into the frequency, wavenumber, and circumferential ring-mode domains using Discrete Fourier Transforms (DFT); this allows the transfer functions for the tunnel and soil to be written in the simplified manner as follows

$$\tilde{\mathbf{U}} = [\tilde{\mathbf{A}}]_{tunnel}\left(\tilde{\mathbf{F}} - \tilde{\mathbf{P}}\right) \qquad \tilde{\mathbf{U}} = [\tilde{\mathbf{A}}]_{soil}\tilde{\mathbf{P}} \qquad (1)$$

where $\tilde{\mathbf{U}}$ represents the cylindrical displacements at the tunnel-soil interface, $\tilde{\mathbf{F}}$ describes the traction applied to the inner surface of the tunnel, and $\tilde{\mathbf{P}}$ is the reactionary traction developed at the tunnel/soil interface. The transfer function matrices $[\tilde{\mathbf{A}}]_{tunnel}$ and $[\tilde{\mathbf{A}}]_{soil}$ are both 3×3 matrices where the components are functions of material properties, wavenumber, frequency and ring-mode only. The tunnel and soil are coupled at the interface by enforcing continuity of displacements and equilibrium of reaction forces which results in the coupled equation of motion

$$\tilde{\mathbf{U}} = \left([\mathbf{I}] + [\tilde{\mathbf{A}}]_{tunnel}[\tilde{\mathbf{A}}]_{soil}^{-1}\right)^{-1} [\tilde{\mathbf{A}}]_{tunnel}\tilde{\mathbf{F}} \tag{2}$$

The actual displacements are then calculated by transforming the solution back into the spatial domain using inverse DFT. For full details on this method, please refer to references [9, 10].

A slab and rails are mounted to the bottom of the tunnel using Euler-Bernoulli beam theory; rail pads and slab bearings are represented by two separate, continuous layers of hysteretically damped springs shown schematically in Figure 1. Hussien and Hunt [11] show that the transfer function in the frequency and wavenumber domain for the coupled rail-slab section subjected to harmonic moving loads can also be expressed in a simplifed matrix form as

$$\tilde{\mathbf{U}}_{rail} = [\tilde{\mathbf{A}}]_{track}\tilde{\mathbf{F}}_{rail} \tag{3}$$

where $\tilde{\mathbf{U}}_{rail}$ describes the vertical motion of the rail and the slab, $\tilde{\mathbf{F}}_{rail}$ describes the moving harmonic force, and $[\tilde{\mathbf{A}}]_{track}$ is the 2×2 transfer matrix built from the coupled Euler-Bernoulli equations. The slab displacements are used to calculate the forces at the wheel/rail interface which are used as inputs into the fully coupled PiP model.

The theory of random vibration [12] is used to calculate vibration levels in the soil due to an infinite train passing along a rough track. The model used for this purpose is shown in Figure 2 where an infinite number of axles (only 3 axles, i.e. 6 wheels are shown in the figure) are used to determine the resultant moving loads due to an uncorrelated rail roughness spectrum; it is assumed that the roughness on the two rails is identical. It is reasonable to ignore the sprung masses of the train (i.e. main body of the carriages) due to the low stiffness of the primary suspension isolating the carriage from the axle assembly (i.e. the unsprung mass). Standard random vibration theory states that the power spectral density (PSD) for displacement response at any point in the pipe-in-pipe soil model can be calculated using

$$PSD(\omega) = \sum_{j=1}^{N} \left|H_j(\omega)\right|^2 S_\delta(\omega) \tag{4}$$

where $H_j(\omega)$ is the transfer function describing the displacement at the observation point in the soil due to a unit harmonic roughness applied under the jth axle of the train, and $S_\delta(\omega)$ is the single-sided rail-roughness spectrum experienced by an axle at an angular frequency ω. A realistic value of the rail roughness spectrum can be calculated from the following empirical formula [13]

$$S_\delta(\omega) = \frac{a}{v\left(b + \frac{\omega}{2\pi v}\right)^3} \tag{5}$$

where v is the load velocity (m/s), ω is the forcing frequency (rad/s), and a and b are constants describing the rail unevenness (eg. average rail roughness values are given as 1.31×10^{-2} mm^2/m^2 and 2.94×10^{-2} /m, respectively). This allows the calculation of power spectral density and insertion gain (IG) of the vertical displacement for any point in the soil; IG is defined as the ratio between the PSD displacement before and after changing parameters of the track, tunnel or soil.

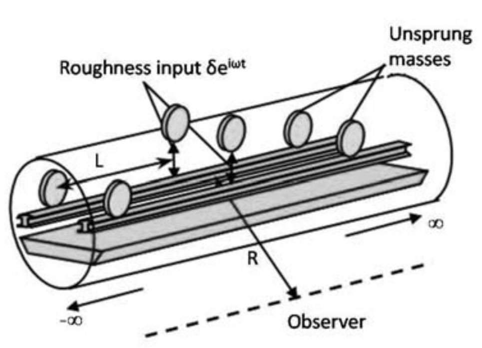


Figure 2. Unsprung masses moving over rough rails causing random force input

The PiP model has been validated against a coupled BE-FE model and shown to have good agreement over the frequency range of interest [14] but with a computational cost which is orders of magnitude less than the BE-FE model. The combination of model accuracy and computational efficiency makes PiP a powerful computational tool for calculating vibration from underground railways and for assessing the performance of vibration countermeasures. The reader is invited to evaluate a free version of the software at www.pipmodel.com.

Hussein and Hunt [15] have recently extended the PiP model to account for a tunnel embedded in a halfspace rather than a fullspace. The standard pipe-in-pipe arrangement is not suitable for including a free-surface since the soil is modeled as a cylinder with infinite radius. To account for the free surface an extension to the model is incorporated which calculates the Green's functions for a homogeneous halfspace [16]; the standard PiP method predicts the loading at the tunnel-soil interface and is used as input into the halfspace model to calculate surface response.

The purpose of the current work is to investigate the effect of this free-surface on ground vibration levels. Equivalent models are run using the fullspace model (PiP version 3) and halfspace model (PiP version 4) and compared to determine how the inclusion of the free-surface affects the vibration levels along the surface. The paper is broken into three sections: a description of the parameters used for the comparison between fullspace and halfspace models, a discussion of the results, and concluding remarks.

2. MODEL DESCRIPTION

Figure 3 shows the software user-interface for PiP v4 (note the halfspace schematic of the buried tunnel at the top-center of the screen). Default parameters for the soil, tunnel, floating-slab track and train can be altered by the user as necessary for specific simulations. The observation point can be set to any location in the fullspace (v3) or halfspace (v4) at which the PSD of the vertical displacements will be calculated. As a demonstration of the computational efficiency of the software the default parameters are used as inputs for a simulation with frequency range of 2-150 Hz with 2Hz intervals. The running time to produce the response curve shown in the plot window of Figure 3 is 42 seconds using PiP v4; to run the equivalent simulation in PiP v3 takes only 6 seconds. The difference in computational time is due to the homogeneous halfspace Green's function formulation required for simulating the free-surface in v4. As PiP v3 is less computationally expensive some users may prefer to trade the added accuracy of accounting for the free-surface for maintaining quick run-times. It would be useful to understand the effect that the free-surface has on PSD values so the uncertainty associated with making this compromise can be quantified.

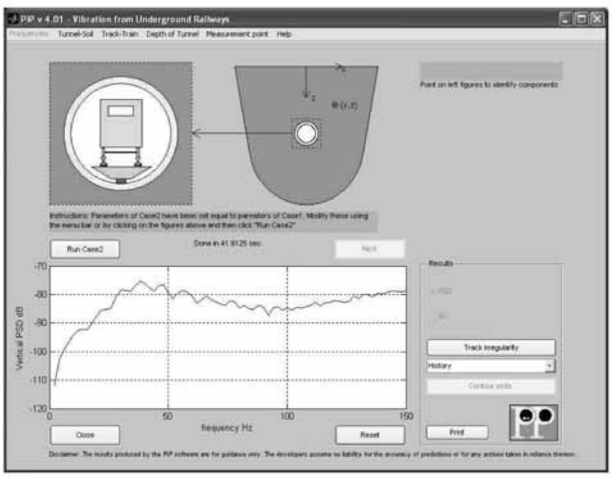


Figure 3. The graphical user interface of the PiP software

Table 1. Model Properties

Soil	
Elastic modulus	0.55 GPa
Density	2000 kg/m^3
Poisson's ratio	0.44
Shear loss factor	0.06
Dilation loss factor	0
Tunnel	
Elastic modulus	50 GPa
Density	2500 kg/m^3
Poisson's ratio	0.3
Shear loss factor	0
Dilation loss factor	0
Outer radius	3 m
Wall thickness	0.25 m
Train	
Unsprung axle mass	500 kg
Spacing between axles	20 m
Rails	
Mass of one rail	50 kg/m
Bending stiffness of one rail	5 MNm^2
Bending stiffness loss factor	0.02
Railpads	
Railpad stiffness per rail	200 MN/m/m
Railpad loss factor	0.3
Slab	
Bending stiffness	1430 MNm^2
Bending stiffness loss factor	0.05
Slab mass	3500 kg/m
Slab bearing	
Bearing stiffness	221 MN/m/m
Bearing loss factor	0.5

To investigate the effect of a free-surface on vertical soil PSD values, a set of representative properties were chosen to simulate an underground railway in the London, UK area. The parameters tabulated in Table 1 are used in both PiP v3 and v4 over a frequency range of 2-150Hz using 2Hz spacing. Figure 4 details the location of five observation points (x =0, 2.5, 5, 10, 20m) used to compare the results from the two models. The tunnel depth is defined as the distance between the free surface and the center of the tunnel; as there is no free surface in PiP v3 equivalent observation points are placed in the fullspace. Five tunnel depths are investigated: 3.5m, 5m, 10m, 20m, 40m. The results at the observation points for the two models are compared in the following section to determine the effect of the free surface.

3. RESULTS AND DISCUSSION

The vertical power spectral density at each observation point was calculated using PiP v3 and v4 for the parameters specified above. The results comparing the two models at x=10m for a tunnel depth of 5m are presented in Fig 5. This typical response shows the model containing the free-surface (PiP v4) predicts PSD values which are offset by a relatively constant margin from those of the fullspace model (PiP v3).

The differences in PSD response for the two models at all observation points are presented in Fig 6. The PSD difference is calculated in dB (ref 1 mm^2/Hz) by subtracting the free-surface response by the fullspace response: a positive value indicates PiP v4 predicts higher PSD values than does PiP v3 at equivalent points.

The results suggest the offset between the two models is relatively insensitive to the observation point, increasingly so at greater tunnel depths. At the deepest tunnel depth used in this study (Fig 6(e) at TD=40m) the results for the five observation points are closely banded around the 6dB level. As tunnel depth decreases the spread between the observation points' responses increases but stays relatively centered around the 6dB offset level.

This 6dB offset is not entirely unexpected. Consider axial vibrations in the infinite and semi-infinite bars shown in Fig 7. If the bars are nominally identical and subjected to the same incoming pressure wave-field, it is known that the displacement at the free end of the semi-infinite bar (u_2) will be twice that of the infinite bar at the same location (u_1) [17]; in the decibel scale this doubling of the response is equivalent

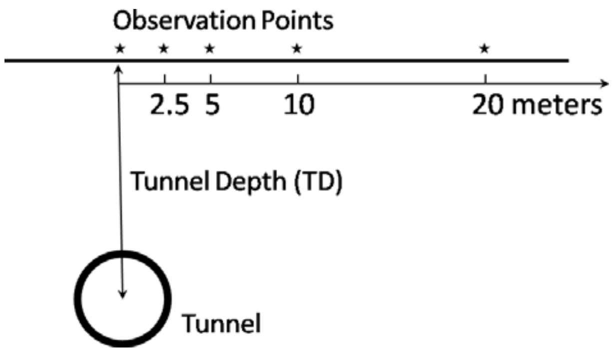


Figure 4. Schematic showing location of observation points

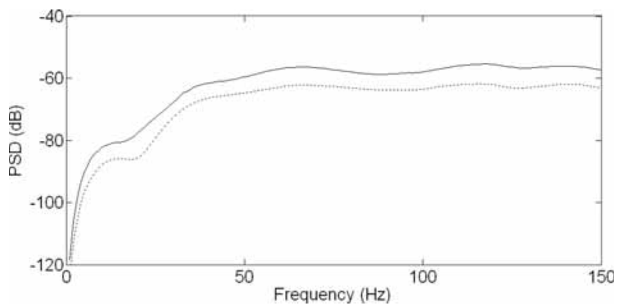
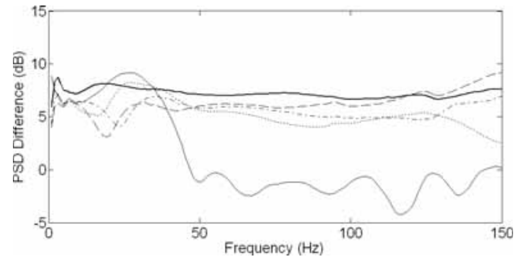


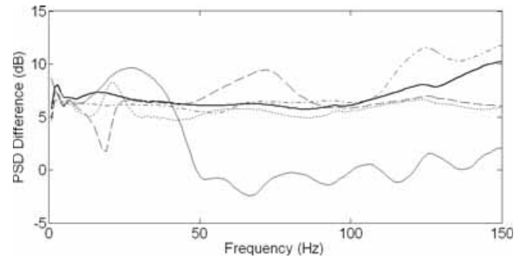
Figure 5. Response at x=10m for a tunnel depth of 5m: (solid-line)-PiP v4 with free-surface; (dotted-line)-PiP v3 using fullspace solution; results presented in dB (ref 1 mm^2/Hz)

to an increase of 6dB. The "doubling effect" is attributed to the superposition of the incoming and reflected waves. This effect can be extrapolated to the fullspace vs. halfspace argument if the incoming wave field is traveling perpendicular to the surface (i.e. wavefronts parallel to the surface). In the case of a tunnel buried in a halfspace, waves are emitted from the tunnel exterior with cylindrical wavefronts; if the tunnel is at great depth the radius of curvature of the wavefronts will be quite large thus the waves will be virtually parallel to the surface when they are reflected.

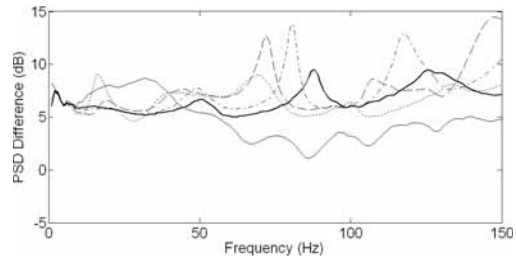
The increase in spread around this 6dB offset for shallow tunnel depths is attributed to the complexities of wave-surface interaction above the tunnel. When the tunnel is near the surface the wavefronts still have relatively small radii of curvature when they are reflected by the free-boundary. The amount of energy that is reflected and the superposition of that energy with the incoming field depends on the incident wave angle. Therfore changing the location of the observation point on the surface (i.e. the angle between the source and the surface point) significantly affects the amount of energy transformed into vertical particle motion on the surface. This general trend in angle dependance can best be seen in Fig 6(a) where the PSD difference drops as the observation point moves away on the surface. At $x = 20$m the incident angle is steep compared to the other observation points thus little superposition of reflected energy occurs, resulting in a PSD difference which approaches 0dB.



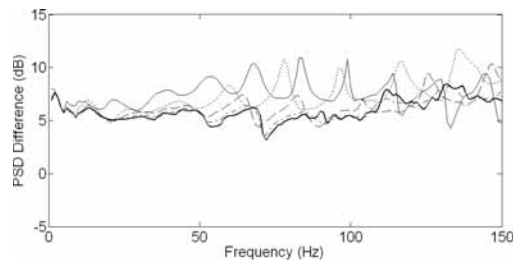
(a) Observation points for 3.5m tunnel depth



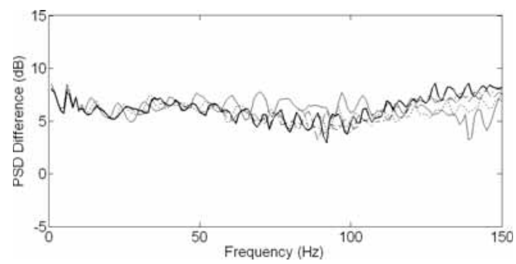
(b) Observation points for 5m tunnel depth



(c) Observation points for 10m tunnel depth



(d) Observation points for 20m tunnel depth



(e) Observation points for 40m tunnel depth

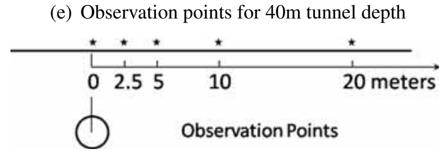


Figure 6. PSD difference [dB (ref 1 mm^2/Hz)] between free-surface model (PiP v4) and fullspace model (PiP v3) at various observation heights; positive value corresponds to an increase in PSD when free-surface is included compared to fullspace model. Legend: (thick-solid) x=0m; (dash-dot) x=2.5m; (dashed) x=5m; (dotted) x=10m; (thin-solid) x=20m

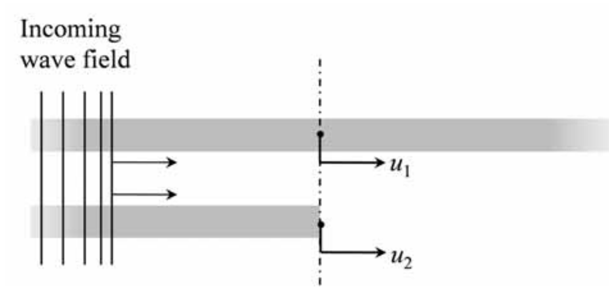


Figure 7. Schematic showing an infinite bar (upper) and semi-infinite bar (lower) subjected to an equivalent pressure wave field and the respective displacements at the boundary of the semi-infinite bar

4. CONCLUSIONS

The Pipe-in-Pipe model (PiP) is a powerful computational tool for calculating vibration from underground railways and for assessing the performance of vibration countermeasures. PiP v3 simulates a tunnel buried in a fullspace using analytical models for the tunnel and soil which results in rapid computational times (6 seconds for 150 Hz frequency sweep). PiP v4 accounts for a free surface by extending the model to include homogeneous halfspace Green's functions; however this added calculation complexity increases the computational time (42 seconds for a 150 Hz frequency sweep). Results from the two models are compared to quantify the effect of the free-surface on soil power spectral density (PSD) values. The study suggests that it is reasonable to assume the PSD surface results predicted from the free-surface model will be approximately 6dB more than those predicted at an equivalent "surface" location by the fullspace model when the tunnel is at a depth of two tunnel-diameters or more, regardless of the surface location of the observation point. This is a useful finding as the computationally efficient fullspace model can be confidently used early in the design process when numerous sensitivity studies are necessary without concern for how the free-surface alters the results. Once the design parameters are narrowed down to specific cases a free surface can be incorporated into the model to determine the soil response more accurately. If the tunnel depth is less than two tunnel-diameters there is significant variation in the results between the fullspace and halfspace models; this is attributed to the increased incident angle of the incoming vibration wavefield. For these instances the results suggest it would be beneficial to include a free surface in the simulation to obtain more accurate predictions of the surface PSD levels.

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