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APPLICATION OF MODAL ANALYSIS TO MUSICAL BELL DESIGN

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1 INTRODUCTION

1.1 Pitch Perception

Pitch perception has been shown to depend on a variety of attributes of the sensation. Pitch height can be subjectively measured as the frequency of a pure tone with the same pitch height as the sound being measured[1]. Pitch strength can then be defined as the certainty of pitch height[2], reflected by the standard deviation of subjective pure tone matching results. These measures are influenced by frequency, sound pressure, masking, spectral complexity, and spectral harmonicity of the sensation. However, they are inappropriate where multiple strong pitches might be perceived in one sensation. Pitch strength might then be better defined by the salience and multiplicity of the simultaneously audible tones and pitch sensations that arise at various times in the sensation[3].

Terhardt's virtual pitch theory[4,5] attempts to describe the synthetic pitch percepts that arise where complex tones contain a number of partial frequencies at ratios close to subsets of the harmonic series. After masking effects have been determined for a sensation the audible partial tones are compared to a harmonic template to determine the relative strength of virtual pitch sensations at sub-harmonic frequencies. Pitch sensations arising in the 'strike tone' of carillon bells and tubular bells are examples of virtual pitch percepts.

Mathematical models of pitch multiplicity and virtual pitch have been developed and compared to human subjective responses to synthesized bell timbres, a range of musical chords and complex harmonic tones. A factor determined from results of human subjective tests can then be used to adjust for the listening attitude, where analytical listening favours hearing individual partials. The calculated and reported values of multiplicity for synthesized bell sounds were found to be higher than for complex harmonic tones with comparable fundamental tone frequencies[3]. The tone salience of individual pure tone or complex tone percepts is proportional to their calculated audibility[3]. The multiplicity and tone salience of European carillon bells calculated in this manner have been contrasted to a range of new harmonic bell designs with very low pitch multiplicity due to the tuning of the first seven partials to the harmonic series[6].

Multiplicity and tone salience have also been reported for bells designed to produce multiple pitch sensations[7]. Virtual pitch theory predicts that pitch percepts will be stronger when more partial frequencies of a sensation are in harmonic relationships. It has also been employed to

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design *polytonal* bells in which two or more pitch percepts of approximately equal strength arise from the sound of one bell, where the partial frequencies are tuned to overlapping subsets of two or more harmonic series.

1.2 FE Modelling and Shape Optimisation of Bells

Bells and other idiophones are unique amongst musical instruments in that they naturally produce inharmonic overtones. This is because, unlike air-columns and strings that vibrate predominantly in one dimension only, bells vibrate flexurally in three dimensions. Flexural vibrations are much more difficult to describe analytically than longitudinal vibrations and it is common to use numerical methods such as Finite Element Analysis (FEA) to predict the behaviour of bells and gongs[8,9].

A number of 'major third' carillons have been designed using Finite Element Analysis in the Netherlands[10]. In this work numerical optimisation was applied to the bell geometry starting from a typical European carillon bell geometry. The frequency ratios of the first five, radiating, partial tones in carillon bells is usually 1; 2; 2.4; 3 and 4. Thus, the European bell could be tuned to a series of partials that included the first four harmonics. However the presence of the partial at 2.4 times the fundamental (a minor third musical interval) is likely to create complex acoustical percepts of pitch and dissonance that change in time as partial frequencies decay at varying rates[6]. In the 'major third' bells, the partial normally found at 2.4 times the fundamental frequency is tuned to 2.5. This was believed to be an advantage in limiting the dissonance caused by this partial in most musical contexts where major third intervals predominate.

Given the ability to solve complex eigenvalue problems using computational methods, shape optimisation can be applied to bell models to adapt wall profiles and where possible, arrive at specific tuning ratios of partial frequencies. Pre-developed, commercial Finite Element (FE) software was employed to implement a classic linear finite element method in this work[11]. The 'solid property' elements used for the FE models consisted of *Tetra* and *Penta* elements in which the nodes' coordinates determine some subsequent shape parameters such as thickness along the bell length. The accuracy of FEA predicted frequencies for the lower frequencies of bells have been shown to be around 1% of measured results[9] when working with two-dimensional FE models. It is reasonable to expect greater accuracy from three-dimension solid property models.

Design optimisation applied to finite element models usually involves creating the model with a number of shape parameters that can be varied during the optimisation process to achieve a certain objective such as tuning to a target set of natural frequencies. Geometrical constraints such as maximum allowable mass, and behavioural constraints such as specific natural frequencies, often need to be applied to the design process to obtain useful solutions. For bells, the shape parameters used to define the bell profile could include angles, lengths, and offsets of lines, or the relative positions of points used to create spline curves. Optimisation could then involve a systematic evaluation of the design space (defined by the design constraints) through the generation of a set of models representing all possible combinations of shape parameters at certain levels of discretisation. The optimum design solution can then be found by polynomial curve fitting to the values of the objective across all design variables. This is likely to be the most time consuming approach! Other optimisation methods can be used to reduce the number of analyses undertaken to achieve a satisfactory solution. These include curve fitting to data generated by making random jumps through the design space, or gradient projection methods.

A gradient projection method[11] was utilized in this work in order to optimise the objective function by changing the coordinates of the FE nodes themselves, rather than global shape parameters. The software application is designed to compute the nodal vector sensitivities of the objective (such as modal natural frequencies) as a function of the FE nodes' coordinates. The user may select a zone of active nodes with coordinates that will vary during optimisation. The sensitivity is calculated from differences in the objective parameter after displacement of each active finite element node. The process of optimisation then iterates towards a target in accordance with geometric constraints that preserve shape parameters of the model such as symmetry about the vertical axis.

To achieve the final goal of optimisation, the process would ideally be performed separately on each vibrational mode. Behavioural constraints can be applied to limit the allowable changes of frequency of particular modes so that optimisation processes on other modes do not change them. These behavioural constraints are met by computing the nodal vector sensitivities to the constraint parameters as a function of the nodes' coordinates. The final displacement vector for each active FE node is calculated from sensitivities to both constraint and optimisation parameters during each iteration of the optimisation process. The optimisation process stops if progress toward the optimisation target cannot be achieved without altering the constrained parameters beyond a given tolerance.

The user sets the step size used in the first iteration of the optimisation process. It is also possible to define a reduction rate for the step size in the following iterations in order to prevent the optimisation from overshooting the target and then alternating on either side of it. The initial step size and its reduction rate were carefully selected based on experience with similar models.

2 BELL MODAL ANALYSIS

It was quickly observed that the behaviours of many bending modes are strongly correlated with respect to changes in the geometry of the model. Therefore it is often not possible to raise or lower any given partial frequency using gradient projection method optimisation applied to positions of FE nodes without affecting other partial frequencies. Constraining all the modes except the mode being optimised restricts the optimisation process, and in most cases the objective value cannot be achieved.

A number of strategies may be employed to overcome this problem. The maximum strain energy of the bending modes to be tuned may occur in different locations along the length of the bell, and therefore the magnitude of their sensitivity vectors may also vary. It may be possible for the constrained modes to remain relatively unaffected by the optimisation process if a careful selection of the active nodal zones is made on the basis of the relative positions of vibrational maxima.

A study of the behaviour of modes for changes in model geometry could enable the user to identify which mode types are correlated for certain geometrical changes. By careful selection of the sequence of modes to be optimised, and the constraint sets, groups of highly correlated modes could be moved simultaneously toward a set of target frequencies. Furthermore, the deviation tolerance of the constrained modes' frequencies may be increased to allow them to change more during the optimisation. This may be sufficient to achieve a set of optimisation targets within an acceptable error.

In order to better understand these correlated behaviours, a series of experiments were conducted in which only one geometric parameter of an FE model was varied, and the resultant changes to the predicted partial frequencies of various modes was tabulated and plotted. Results for purely circumferential modes (modes which produce only nodal lines in plane with the axis of symmetry of the bell), purely axial modes (transverse modes such as those found in tubular bells) and mixed modes (producing both nodal lines and rings) are reported below for changing lengths and cone angles of capped cylinders and cones. Wave number pairs (m, n) are used to describe bending modes such that the first number refers to the number of nodal lines in plane with the axis of symmetry (i.e. circumferential wave number), and the second number refers to the number of nodal rings perpendicular to the cylinder length (i.e. axial wave number).

Figure 1 shows the variation in frequency of a series of modes due to changes in length relative to a 600mm long, by 240 mm circumference capped cylinder, and changes in width relative to a 200mm long capped cylinder. The wall thickness was 10mm. For variation of length and width the behaviour of the 2,2 mode is very closely related to the behaviour of the sum of the 2,0 and 0,2 frequencies. This suggests that the behaviour of the mixed modes may be considered to be due to combinations of changes in circumferential and axial stiffness for varying geometry where mass distribution is constant over the range of these shape parameters.

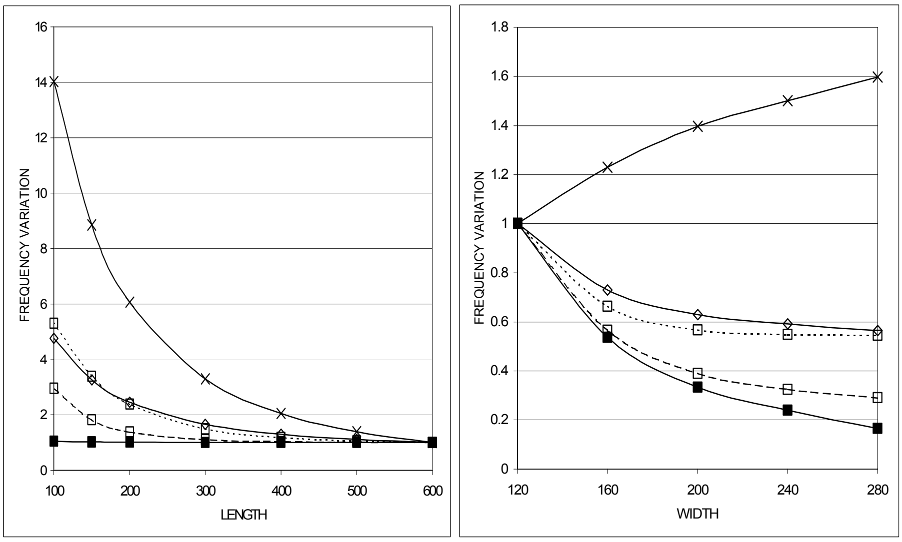

Figure 1. a) Calculated frequency variation of modes due to decreasing length of a capped cylinder, and b) due to increasing width.

A similar study was undertaken for a typical church bell when scaled in height and width. It should be noted that the normal method of supporting a church bell will substantially alter the 0,2 mode so that it is unlikely to be heard, and it is discussed here as a guide to the changing axial stiffness of the bell. In this case, the behaviour of the mixed modes do not correlate as well with the sum of the 2,0 and 0,2 modes because of the more complex geometry involved. The rim is much thicker than the rest of the bell, leading to greater circumferential stiffness for modes with maxima near the rim, and very uneven mass distributions along the height of the bell. The results are too complex to discuss in here detail, except to report that the behaviour of the modes was consistent with changes in axial and circumferential stiffness along with other effects. Previous studies on the modes of vibration of European bells have failed to identify the presence of purely axial modes even under unconstrained conditions[9].

The effects of geometric parameters such as wall curvature and taper, and cone angle for cylinders and cones with closed ends were investigated for a wider range of modes in models able to vibrate freely. The results for varying cone angles have been reported in a previous paper[6]. The data from these experiments could then be used to select a model geometry in which the partial frequencies were reasonably close to the required set and, given an understanding of the behaviour of the modes, likely to be able to be tuned correctly using gradient projection method shape optimisation. Silicon bronze (95% copper, 1% manganese, and 4% silicon) and its material properties were used for all the models and cast bells described in this paper. The mass density and Young's modulus of silicon bronze are 8.4×10^3 kg/m^3, and 9.4×10^{10} N/m^2 respectively.

Figure 2 shows plots of FE predicted frequencies for varying cone angle, wall taper, and wall curvature of a capped cone of 200mm length, 220mm top circumference and 10mm wall thickness. These parameters are defined in Figure 3.

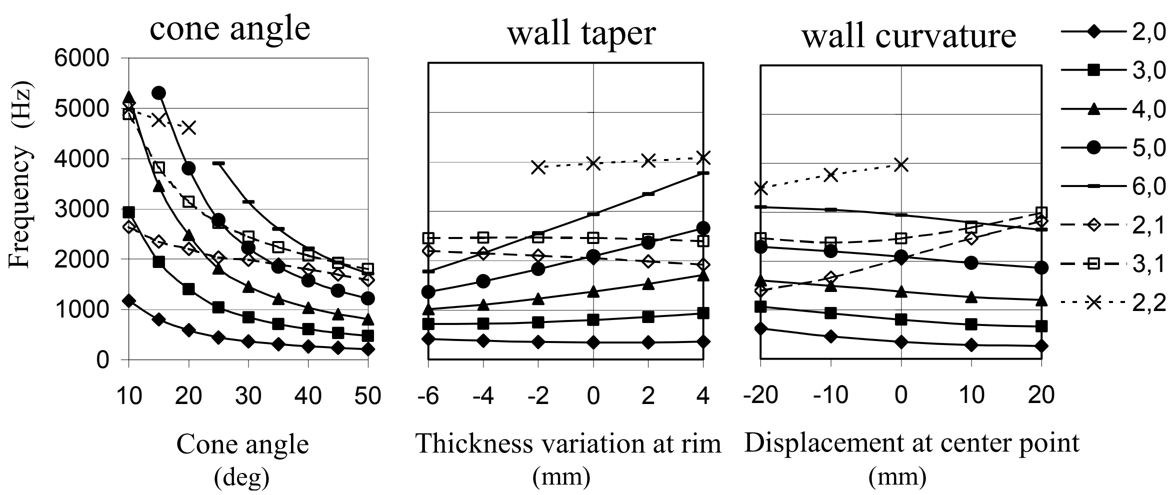

Figure 2. FEA predicted frequencies for varying the cone angle, wall taper and wall shape of freely vibrating capped cones.

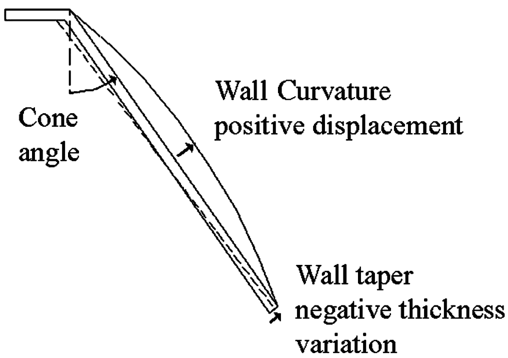

Figure 3. Shape parameters used to generate data in Figure 2.

From Figure 2 it can be seen that the frequencies of the m,0 modes remain approximately equally spaced as the range of their frequencies change with these geometric changes. Effects similar to this have been reported for European bells in that it was observed that the correct tuning of the hum (2,0 mode) and tierce (3,0 mode) to a ratio of 2.4 would ensure that the 4,0 and 5,0 were at least approximately in harmonic ratios of the fundamental[12]. Figure 2 also shows that the spacing of the m,0 modes do generally compress as the mode number increases as would be required to satisfy this statement. Mixed modes decrease in frequency more slowly than purely circumferential modes with increasing cone angles and increasing wall tapers toward thinner rims. Increasing curvature away from the axis of symmetry (concavity) increases the frequency of the mixed modes and decreases the frequency of purely circumferential modes.

3 POLYTONE BELL DESIGN

A number of tuning opportunities for polytonal bells arise for nearly cylindrical bell geometries, including tunings very close to the minor and major third European carillon bells. Inspection of Figure 2 reveals that at a cone angle of 10° the sound radiating modes are in the same order as those found in church bells although slightly compressed in frequency range. Adding a positive taper to the wall thickness will increase the frequencies of the purely circumferential modes relative to the fundamental (due to increased circumferential stiffness at the rim) without substantially affecting the frequency of the mixed modes. Shortening the length of the bell will increase the frequencies of the mixed modes (due to increasing axial stiffness) without substantially effecting purely circumferential modes.

The overtone tuning ratios considered ideal for polytonal bells are based on the hypothesis that tuning the bell overtones to two or more harmonic series can produce a bell sound with two or more pitch sensations of approximately equal strength. The fundamental of each harmonic series being the desired pitch sensations. However many other psycho-acoustic factors need to be considered in the tuning design. For example, it may not be desirable to have two low frequency partials tuned closely to each other, as they could produce rapid beating and perceptual roughness in low-pitched bells. Major or minor third bells are examples where it may be desirable to omit the fundamental of the higher pitch percept as they would be at frequencies close to the fundamental frequency. Otherwise the ideal tuning ratios are chosen to maintain an approximately even distribution of overtones, given that certain bell geometries lead to either generally compressed or widely dispersed overtone frequencies.

Another important factor is the possibility that the sub-harmonic of the two harmonic series will emerge as the strongest pitch percept. Terhardt's virtual pitch theory indicates that the higher the frequency of the missing fundamental tone in an incomplete harmonic series (within the normal audible range), the more likely a listener is to perceive a virtual fundamental tone. Furthermore, the simpler the just ratio between the intended pitches in a polytone bell, the higher the frequency of the sub-harmonic and therefore the stronger its pitch strength. Consequently, no attempt was made to design a polytone bell with a 3/2 ratio (chromatic interval of a 5^{th}), and the bells must have a relatively low fundamental frequency.

Table 1 shows the ideal tuning ratios and the ratios achieved for FE models of minor third and major third cylindrical polytonal bells given the above considerations. The tunings are similar to European carillon bells except that harmonic overtones of the ‘third’ intervals are included and more partials are tuned. It should be noted that the 2,2 modes do not belong to the ideal tuning schema. These modes proved very difficult to adjust without disrupting the remaining ‘in tune’ modes. Given their high mode number (7) they are expected to contribute very slightly to the overall pitch multiplicity. Table 2 also includes the tuning of cast minor and major third cylindrical bells. The modes are well tuned apart from the 3,0 and 4,0 modes of the major third bell and the 2,1 and 4,0 modes of the minor third bell. These modes are all about 2% flat due to the rim in the cast bells being slightly too thin. Given the expense of recasting the 300 kg bells no attempt has been made to cast better tuned bells to date.

Cyl. Min. 3	**Ideal**		**2**	**2.4**	**3**	**3.6**	**-**	**4**	**-**	**4.8**	**5**
Type	modelled	2,0	2,1	3,0	3,1	4,0	0,2	3,2	2,2	4,1	5,0
Ratio	modelled		2	2.42	3.05	3.6	3.8	4.09	4.36	4.85	4.94
Ratio	cast	335 Hz	1.95	2.38	3.02	3.53	-	4.07	-	4.79	4.92
Cyl. Maj. 3	**Ideal**		**2**	**2.5**	**3**	**3.75**	**-**	**4**	**-**	**5**	**5.25**
Type	modelled	2,0	2,1	3,0	3,1	4,0	0,2	3,2	2,2	4,1	5,0
Ratio	modelled		2.02	2.49	3.04	3.77	3.87	4.03	4.3	4.9	5.26
Ratio	cast	293 Hz	1.99	2.45	3.02	3.68	-	3.96	4.22	5.03	

Table 1. The ideal tuning ratios and the ratios achieved for both FE models and cast minor third and major third cylindrical polytonal bells.

Many refinements of the starting geometry were made so as to enable the automated shape optimisation by FE nodal position to tune the bell. A successful attempt that resulted in a cylindrical 'minor third' bell is shown in Table 2. The starting geometry was a cylinder that was 1000mm long by 317mm rim diameter, and had a 5.5^{o} cone angle and wall tapering to a thick rim. The optimisation parameters refer to the mode being shifted in frequency and the modes being constrained to arrive at the values in that row. For example D1; C5 means drop the frequency of mode 1 while constraining the frequency of mode 5. Figure 4 shows the resultant profiles of the cylindrical minor and major third bells. It is notable in this figure that these two profiles are very similar. This is due to the high sensitivity of the third partial frequency to the geometric variations.

Model	Optimisation parameters	Freq. 1 (2,0)	Rat. 2 (2,1)	Rat. 3 (3,0)	Rat. 4 (3,1)	Rat. 5 (4,0)	Rat. 6 (3,2)	Rat. 7 (2,2)	Rat. 8 (4,1)	Rat. 9 (5,0)
Cyl		365 Hz	1.77	2.27	3.01	3.53	4.01	3.92	4.89	4.98
	D1; C5	352 Hz	1.95	2.34	3.07	3.67	4.30	4.31	4.99	5.16
	D1; C2	346 Hz	1.98	2.37	3.09	3.64	4.19	4.31	4.90	5.08
	D6; C1	345 Hz	1.98	2.38	3.08	3.62	4.03	4.12	4.79	5.01
Min. 3	U7; C1, 2, 5, 6	346 Hz	2.00	2.42	3.05	3.60	4.09	4.36	4.85	4.94

Table 2. Steps in the optimisation of a minor 3rd cylindrical polytonal bell.

a) **b)**

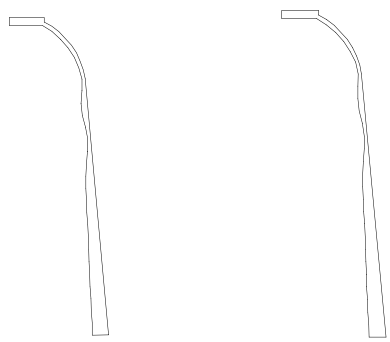

Figure 4. Profiles for a) major and b) minor third cylindrical bells.

4 PSYCHO-ACOUSTIC ANALYSIS

The acoustic spectra of the cast bells are shown in Figure 5. The actual frequencies shown in the spectra of the bells were used to derive the ratios shown in Table 1. The acoustic spectra were produced from recordings taken at 1 meter from the bell perpendicular to the axis of symmetry of the bells. The bells were struck near the rim by a bronze hammer of about 1.5 kg mass. The mallet was chosen as appropriate for producing a tone balance predominantly consisting of the full range of tuned partials. The mallet velocity was such as to produce an 'A weighted' sound pressure level of around 85 dB (fast response) in the early part of the sound. The FFT spectrum was produced at about 50 milliseconds after the sound onset by using a Hamming window of 4,096 samples for a sample rate of 44.1 kHz.

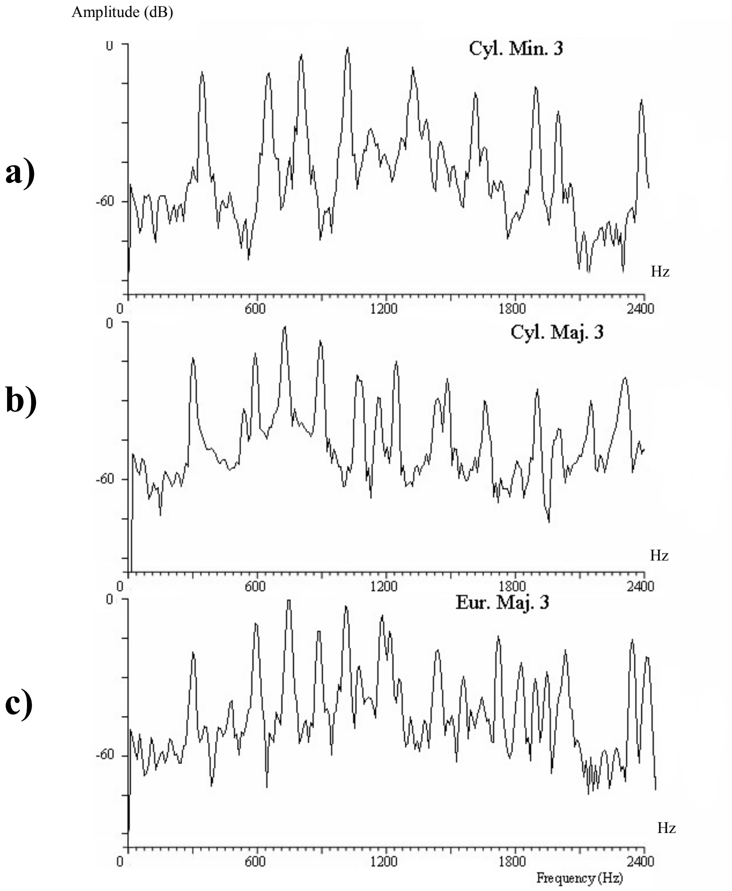

Figure 5. a) and b) Acoustic spectra for minor and major third cylindrical bells respectively, and c) for a European major third bell.

FE modal analysis revealed that the maxima of the 3,0 and 4,0 modes at 718 Hz and 1,078 Hz respectively for the cylindrical major third bell are situated near the centre of the bell due to the higher circumferential stiffness at the thickened rim. A spectrum recorded with a strike at the centre of the bell reveals much higher relative amplitudes for these modes. A spectrum for a European major third bell is included in Figure 5 for comparison. Notice the strong peak at 1003 Hz (a frequency ratio of 3.42) that is not usually reported for European style bells. This peak decays relatively quickly (an RT60 of about 0.7 seconds), and by comparison to ratios of modal frequencies calculated by FE modelling can be ascribed to the 0,2 mode. This mode was not observed in the spectra of the cylindrical bells due to the bells being firmly constrained at the head whilst the recordings were made.

Tone saliencies for both virtual and spectral pitch percepts were calculated using Terhart's algorithm[5] applied to FFT spectra of the bells calculated using a Hamming window of 2,048 samples at a sample rate of 12.8 kHz[7]. Spectral percepts are due to bell partials that are heard as individual tones rather than virtual pitches that are the root of a harmonic complex tone. Multiplicity is also of interest here because we are studying bells designed to yield more than one distinct pitch. For the present study, Parncutt's model[4] was applied directly to the output of Terhardt's model. Figure 6 shows the tone saliencies and multiplicity calculated over 80 millisecond intervals for recordings of the cylindrical minor and major third bells and compared to minor and major third European bells recorded in a similar manner.

a) Major 3rd Bells **b) Minor 3rd Bells**

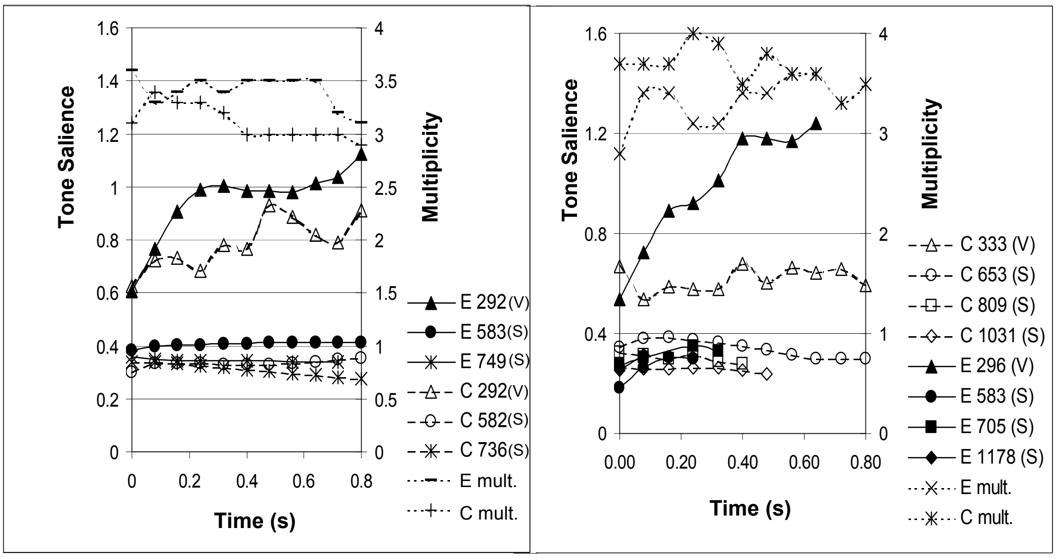

Figure 6. Calculated tone salience and multiplicity of Cylindrical and European a) major and b) minor third bells. Predicted virtual pitch sub-harmonics of the lowest frequency partial have not been shown for the sake of clarity.

In all these bells the ‘third’ partials produce spectral percepts despite the presence of higher frequency partials tuned to harmonic frequencies of the ‘thirds’ in the cylindrical bells. The psycho-acoustic model calculates that masking phenomena obscures these partials. A feature of the data shown in Figure 6a and b is that the audibility of the major third partial is greater and more sustained for both styles of bells than the minor third partial when struck in a similar manner. This may be due to the greater frequency difference between these partials and the partials tuned to an octave of the lowest frequency partial, thereby reducing masking of the ‘third’ partials. Figure 7 shows the calculated tone saliences and multiplicity for the cylindrical ‘major third’ when struck at the wall center. The increased amplitude of the 3,0 and 4,0 modes is shown to produce a major third virtual pitch sensation at 357 Hz.

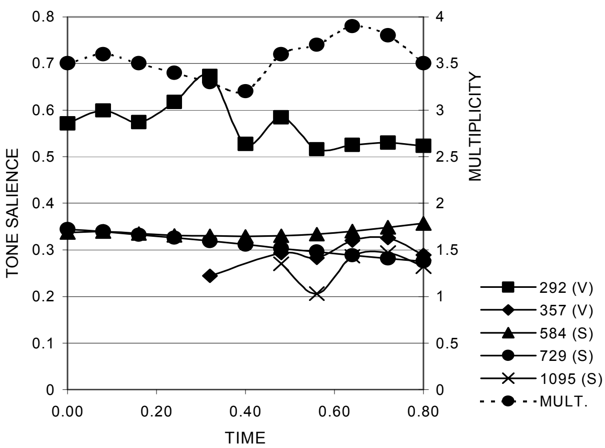

Figure 7. Calculated tone saliencies and multiplicity for the cylindrical ‘major third’ when struck at the wall centre. Predicted virtual pitch sub-harmonics of the lowest frequency partial have not been shown for the sake of clarity.

5 CONCLUSION

Application of the gradient optimisation technique in a classical FE method resulted in the design of two cylindrical polytone bells with frequency ratios similar to the European minor and major third bells. To select an initial geometry and to direct the optimisation process to achieve its objective functions, extensive computational modal analyses were required in order to understand the modes behavior with respect to the model's geometric parameter variations. The importance of a previously unreported mode type for the behaviour of European bells is described.

The designed bells were then cast and their acoustic spectra were established after recording[13]. The psycho-acoustic analysis of both major and minor third cylindrical polytone bells show that pitch multiplicity is higher than for a single harmonic complex tone and that pitch sensations at the minor and major third intervals do arise. The expected virtual pitch sensation for the major third interval was elicited by striking the bell closer to its center and thereby increasing the relative amplitude of modes tuned to produce this pitch sensation.

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13. To see and hear a wide range of harmonic and other bells see the Australian Bell website, www.ausbell.com.