Investigation into the feasibility of using a parametric array control source in an active noise control system

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ABSTRACT

Conventional Active Noise Control (ANC) systems that minimise pressure at a point in space have typically used loudspeakers as control sources, which are virtually omnidirectional within the low frequency range of interest. One obvious disadvantage of this is that locations remote from the desired control point may actually experience an increase in sound pressure level. The parametric array is capable of producing a highly directional beam of low frequency sound via the nonlinear interaction of emitted ultrasonic waves with air. Although significant research and development of the parametric array has been undertaken, the feasibility of using a parametric array as the control source in an ANC system has not yet been fully investigated. Within this paper, the theory governing the operation of the parametric array and the resulting restrictions upon the production of low frequency sound are discussed. Experimental testing of a commercially available parametric array indicates that although highly directive low frequency noise of sufficient amplitude for some ANC applications can be produced, there are a number of practical concerns. The noise floor of the parametric array is high at low frequencies, resulting in poor signal to noise ratio. The nonlinear nature of the sound production process means that the precise control in amplitude and phase required for ANC cannot, as yet, be achieved. In addition, there are concerns regarding the safety of the high amplitudes of ultrasound emitted by the parametric array. These limitations would all need to be overcome before the parametric array could be successfully implemented as a control source in an ANC system.

It was known as far back as the 19th Century that as a sound wave propagates it also distorts (Ingard and Pridmore-Brown 1956). Non-linear interaction between two sound waves, of greater than 130 dB sound pressure level (SPL), of angular frequencies causes demodulation to occur, producing sound waves at the sum and difference frequencies.

From the 1930s until the early 1960s, both theoretical and experimental analyses of the production of extraneous frequencies as a result of distortion of propagating sound waves were undertaken (Ingard and Pridmore-Brown 1956, Westervelt 1957, Bellin and Beyer 1960, and Dean 1962). Westervelt (1962) presented a theoretical model of what has come to be termed the parametric array. The parametric array emits high amplitude ultrasonic waves which demodulate into directional audible sound due to non-linear interaction with the medium through which they propagate. A highly directional beam is therefore produced. A parametric array is able to produce a secondary sound beam with similar directivity to that of the primary carrier beam. For example, if a 48 kHz ultrasonic beam is modulated with a 500 Hz beam, the demodulated 500 Hz beam will have directivity comparable to the 48 kHz carrier. This difference in directivity between a low frequency audible beam at 500 Hz and a high frequency ultrasonic beam at 48 kHz is depicted graphically in Figure 1. The 500 Hz beam is virtually omnidirectional, while the 48 kHz beam is highly directional.

Berktay (1965) presented a quasilinear analysis of the parametric array. From his analysis he was able to predict the parametric array working in air.
Versions of the parametric array have been developed and commercialised by both American Technology Corporation (ATC 2001) and Holosonic Research Labs (HRL 2003). Intended applications of these commercial devices include point to point communication over ranges of up to approximately two hundred metres, non-intrusive individual communication to patrons/customers in museums and shopping centres, as well as directional entertainment systems.

The use of parametric arrays for applications such as landmine detection and analysis of underground structures (van Wijk et al. 2005), medical focussing (Cahill and Baker 1998) and the determination of sea bed properties (Hines 1999) have also been considered. The feasibility of using a parametric array as the control source in an Active Noise Control System is yet to be fully considered.

This paper outlines the basic underlying theory of the parametric array. A theoretical analysis of the commercially available ATC parametric array is presented. Results from experimental testing are also presented. The directivity and an on-axis response of the source are discussed, followed by a discussion on three main areas which have the potential to limit the performance of the parametric array in an ANC system: temporal behaviour of the demodulated signal, harmonic distortion levels and ultrasonic levels.

**BACKGROUND THEORY**

The primary carrier frequency pressure of a modulated high frequency sound source is expressed as (Yoneyama et al. 1983):

$$p_1 = P_0 \left( 1 + mg(t) \right) e^{-\alpha r} \sin \left( \omega \left( t - \frac{r}{c_0} \right) \right)$$  \hspace{1cm} (1)

where,

$$g(t) = \sin \left( \omega \left( t - \frac{r}{c_0} \right) \right)$$  \hspace{1cm} (2)

$p_0$ is the amplitude of the primary carrier frequency at the source, $m$ is the modulation index, $c_0$ is the speed of sound within the propagation medium, $r$ is the axial distance from the source, $\alpha$ is the attenuation coefficient at the source frequency in Np/m, $\omega$ is the modulation frequency and $t$ represents time.

The demodulated secondary frequency pressure is approximated using Berktay's theory (Berklay 1965):

$$p_2 = \frac{\beta \rho c^2 a^2 m^2 d^2}{16 \rho_0 c_0^2 \alpha} d t^2 E^2(t)$$  \hspace{1cm} (3)

where $\beta$ is the coefficient of nonlinearity, $a$ is the effective source radius, $\rho_0$ is the density of the propagation medium and $E$ is an envelope function. Berktay's approximation was developed using quasilinear analysis and is only applicable on-axis and in the far field (outside the nonlinear interaction region).

An envelope function of the form:

$$E(t) = 1 + mg(t) = 1 + m \sin \left( \omega \left( t - \frac{r}{c_0} \right) \right)$$  \hspace{1cm} (4)

is assumed. Differentiating the square of Eq. (4) twice with respect to time, to enable substitution into Eq. (3), yields:

$$\frac{d^2}{dt^2} E^2(t) = -2m^2 \omega^2 \sin \left( \omega \left( t - \frac{r}{c_0} \right) \right)$$

$$+ 2m^2 \omega^2 \cos \left( 2 \omega \left( t - \frac{r}{c_0} \right) \right)$$  \hspace{1cm} (5)

From Eq. (5) it can be observed that components at both the desired fundamental frequency and its first harmonic are produced. Substituting the fundamental frequency component of Eq. (5) back into Eq. (3) yields the demodulated fundamental frequency pressure:

$$p_2 = \frac{\beta \rho c^2 a^2 m^2 \omega^2}{8 \rho_0 c_0^2 \alpha} \sin \left( \omega \left( t - \frac{r}{c_0} \right) \right)$$  \hspace{1cm} (6)

From Eq. (6) it can be observed that the pressure amplitude of the secondary (audible) frequency, $p_2$, is proportional to the square of the modulating frequency, $\omega$. This results in a 12 dB per octave roll-off. Hence, at low frequencies, much higher primary pressure amplitudes are needed to produce secondary wave amplitudes comparable to what could be produced at higher secondary frequencies.

Substituting the first harmonic frequency component of Eq. (5) back into Eq. (3) yields the first harmonic of the demodulated secondary frequency pressure:

$$p_d = \frac{\beta \rho c^2 a^2 m^2 \omega^2}{8 \rho_0 c_0^2 \alpha} \cos \left( 2 \omega \left( t - \frac{r}{c_0} \right) \right)$$  \hspace{1cm} (7)
From Eq. (6) and Eq. (7) the second harmonic distortion ratio can be defined as:

\[
\% HD = \left| \frac{p_{2}}{p_1} \right| \times 100 = 100m
\]

If the modulation index is set to unity, 100% harmonic distortion occurs. Reducing the modulation index reduces the percentage harmonic distortion; however the amplitude of the desired secondary frequency is also reduced. The harmonic distortion component can also be reduced by integrating the envelope function twice and taking the square root (Kite et al. 1998). This is the preferred method as it does not adversely affect the amplitude of the desired secondary pressure.

**THEORETICAL ANALYSIS**

**Preliminary tests**

Parametric arrays were obtained from two commercial vendors: Holosonic Research Labs (HRL 2003) and American Technology Corporation (ATC 2001). Preliminary testing of parametric arrays from both of the commercial vendors was undertaken. The parametric array from HRL was shown to produce higher levels of harmonic distortion and therefore the ATC device was selected for further theoretical and experimental analysis. The noise floor of both parametric arrays was also observed to be high at low frequencies, resulting in a poor signal to noise ratio.

**Application of Berktay’s theory**

Demodulation, which occurs over the absorption length, \( l_a \), can be visualised conceptually as low frequency sound production from a virtual array. Non-linear effects cease at a distance termed the Rayleigh distance. A schematic of the production of sound from a parametric array is included as Figure 2.

Berktay’s theory assumes that the absorption length is less than the Rayleigh distance at the primary source frequency, ensuring that nonlinear absorption is confined to the near field (Weiguo 2003). The absorption length, \( l_a \), is simply the inverse of the attenuation coefficient:

\[
l_a = \frac{1}{\alpha}
\]

The Rayleigh distance is defined as:

\[
Ra = \frac{\pi l_a \omega^2}{c}
\]

where \( \omega \) is the source frequency. Assuming the air to be at atmospheric pressure, 20°C and 20% relative humidity, the thermoviscous attenuation coefficient is 0.2303 Np/m (Piercy et al. 1977) and the speed of sound is 343 m/s. The ATC parametric array has an effective radius of approximately 0.15 m and operates at a primary frequency of 48 kHz. The corresponding absorption length and Rayleigh distance are 4.3 m and 9.9 m respectively, satisfying the assumption that nonlinear absorption is confined to the near field. Assuming a 130 dB primary ultrasonic source modulated with a 500 Hz tone, the on-axis sound pressure level (SPL) of the ultrasonic carrier and the demodulated fundamental frequency were calculated and are plotted as a function of range in Figure 3. The Rayleigh distance and absorption length are also indicated. A coefficient of nonlinearity of 1.2, density of 1.21 kg/m³, and unity modulation are assumed. Since non-linear interaction terminates at the Rayleigh distance, Berktay’s approximation is only valid at distances from the source greater than this. An ANC system incorporating the parametric array would most likely have distances of much less than 10 m between the source and control locations. However, the performance of the array cannot be accurately modelled within this region using Berktay’s theory. Outside the region of non-linear interaction, the maximum sound pressure level of the demodulated fundamental frequency is only 45 dB, well below the control source levels desired in an ANC system.

![Figure 2. Schematic of the parametric array.](image)

![Figure 3. On-axis sound pressure level of the ultrasonic carrier and demodulated fundamental frequency for a 130 dB ultrasonic source modulated with a 500 Hz tone.](image)

The quasilinear approximation upon which Berktay’s theory is based also assumes that the nonlinear effects are relatively weak, that is, the Goldberg number, \( \Gamma \), is less than unity (Kim and Sparrow 2002). The Goldberg number is defined as:

\[
\Gamma = \frac{l_a}{r'}
\]

where \( r' \) is the plane wave shock formation distance defined as (Pierce 1989):

\[
r' = \frac{\alpha_0 \omega_0^2}{k \beta_0}
\]

where \( k = \omega_0/c \) and \( \alpha_0 \) is the primary source frequency. Substituting Eq. (12) into Eq. (11) yields:

\[
\Gamma = \frac{\alpha_0 \omega_0^2 \beta_0}{\alpha_0 \omega_0^2 \beta_0}
\]
Goldberg number to below 1, the primary ultrasonic level would need to be decreased to below 124 dB.

EXPERIMENTAL ANALYSIS

All testing was undertaken in the anechoic chamber at the School of Mechanical Engineering, University of Adelaide, Australia. In each experiment, an HP35665a spectrum analyser was used to deliver a 100 mVpk sinusoidal input to the parametric array, well below the maximum specified nominal input level of 1.0 V p-p per channel. Briel & Kjær (B&K) type 4133 (half inch) and type 4136 (quarter inch) microphones were used to record signals within the audible frequency range and ultrasonic frequency range respectively. Calibration of the microphones was done using a B&K type 4220 piston phone.

Directivity

A microphone was set-up to rotate at a radius of 1.5 m from the source on a B&K type 3921 turntable, which was operated by a B&K type 2305 level recorder. A stationary control microphone was set up at a distance of 1.7 m from the source, along the source axis. A 500 Hz signal was input to the source and the rotating microphone was swept around an arc from -90° to 90° relative to the source. The signal measured by the microphone, which was passed through a low pass filter to eliminate the ultrasonic carrier, was recorded at 1° increments using a PICO logger. A directivity plot of the recorded SPL, normalised to the maximum level, is shown in Figure 4.

From the figure it can be seen that the sound pressure level decreases by 20 dB within 10° either side of the speaker axis and by 30 dB within 20°, indicating that the parametric array is highly directional at this frequency. Levels of more than 30 dB less than the maximum were not recorded due to the noise floor of the measurement technique. The signals received by the PICO logging equipment were recorded as voltages within a specific range. The voltage range could have been expanded, however, for each case, the signal was seen to be highly directional.

Axial SPL

The on-axis response of the parametric array was measured experimentally for a range of 0.1 m to 3.6 m in 0.1 m increments at frequencies of 500 Hz, 1 kHz and 3 kHz. The sound pressure level of the ultrasonic carrier, the fundamental audible demodulated signal and the first harmonic of the demodulated signal, which were measured twice and then averaged, are depicted in Figures 5-7 as a function of distance. The theoretical ultrasonic carrier and fundamental demodulated sound pressure levels, as calculated using Berkty’s theory, are also shown. It has already been mentioned that Berkty’s theory is not necessarily applicable within the experimental range or for the experimental set-up in general. However, it was noted that the experimental results do follow the general trend of Berkty’s predictions.

The SPL curves are not as smooth as would be desired. One reason for this is that the demodulation process is non-linear with frequency. Another reason is that the source level output by the device was seen to vary temporally, even when the input frequency and amplitude were held constant. The SPL of the second harmonic is approximately 10 dB less than that of the fundamental signal for a range of greater than 1.5 m for both the 500 Hz and 1 kHz cases, indicating significant levels of harmonic distortion. The 3 kHz case shows lower levels of harmonic distortion, however at some ranges the SPL difference between the fundamental and its first harmonic is less than 20 dB. The ultrasonic levels produced by the source were in excess of 130 dB at some ranges for all three cases.
The temporal variation of the signal produced by the source, the high levels of harmonic distortion and the high ultrasonic levels were all highlighted as areas requiring further investigation.

**Temporal behaviour of demodulated signal**

The temporal behaviour of both the amplitude and phase of the demodulated signal were investigated. The signal received by a microphone at a distance of 1 m from the source along the source axis was passed through a low pass filter and the resulting time trace was observed on an oscilloscope. A snapshot of the time trace of both a 200 Hz modulation signal input into the source and the demodulated signal output from the source is shown in Figure 8.

No difference between the phase behaviour of the demodulated signal and that of the input signal was observed but the amplitude was seen to vary significantly. A variation of up to 60% was observed from the 1.25 s snapshot; however, the amplitude was seen to vary by even greater amounts over longer periods of time.

In order to ensure that the amplitude variation was due to the parametric array rather than the associated signal producing and measuring equipment, the parametric array was temporarily replaced with an ordinary loudspeaker. A signal of 250 Hz was input into the speaker. Over a period of 30 minutes, the amplitude of the signal produced by the speaker, which was chosen to be approximately 75 dB for comparability, was observed to vary by no more than 0.12 dB, indicating that the variation was not due to the measuring equipment.

The SPL of the fundamental demodulated signal was measured every 5 seconds until 35 samples were obtained, for modulating signals of 250 Hz, 1 kHz and 5 kHz, to determine limits on the amount by which the amplitude of the demodulated signal varies. The SPL of each sample at each frequency is depicted in Figure 9.

The mean SPL, as well as the range and standard deviation are included for each frequency in Table 1. The amplitude range and standard deviation are significant at all three frequencies, although it can be seen that the variabilities are not as great in the higher frequency case as they are at the lower frequencies.

A control source experiencing amplitude variations of this level would not perform well in an ANC system. The control levels would fluctuate within this range of variability. An adaptive controller could be used to change the filter weights as the amplitude varies; however, the effects of the filter weights would also be non-linear and subject to amplitude variation. It would therefore still be difficult to eliminate range fluctuations altogether.

**Table 1.** Mean SPL, range and standard deviation of 35 samples measured at 5 second increments, at frequencies of 250 Hz, 1000 Hz and 5000 Hz.

<table>
<thead>
<tr>
<th>frequency (Hz)</th>
<th>mean (dB)</th>
<th>range (dB)</th>
<th>standard deviation (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>67.9</td>
<td>6.5</td>
<td>1.8</td>
</tr>
<tr>
<td>1000</td>
<td>77.3</td>
<td>6.6</td>
<td>1.7</td>
</tr>
<tr>
<td>5000</td>
<td>63.5</td>
<td>4.2</td>
<td>1.0</td>
</tr>
</tbody>
</table>

It is likely that the most effective means of controlling the source amplitude lies within the design of the parametric array itself. It is possible that through better understanding of the non-linear phenomenon, parametric array design may be improved, ameliorating amplitude fluctuations.

**Harmonic distortion**

The harmonic distortion produced by the parametric array was measured along the axis of the source at a distance of 1 m from the source for multiple frequencies within the range.
of 100 Hz to 10 kHz. The measured SPLs of the fundamental demodulated signal and the first harmonic are depicted in Figure 10. The first harmonic level is not plotted at frequencies above 6 kHz as it was not observable above the noise floor.

Figure 10. SPLs of the fundamental demodulated signal and its first harmonic as a function of frequency.

At 100 Hz the harmonic distortion is greater than 100%. At frequencies higher than this and up to 1.3 kHz, the first harmonic is, on average, slightly more than 20 dB less than the fundamental signal. The higher levels of harmonic distortion at low frequencies are due to the 12 dB roll-off per octave inherent in the process. To maintain low frequency SPLs comparable to those in the mid to high audible frequency range, the modulation index can be increased (since the demodulated fundamental frequency pressure is proportional to the modulation index). However this compromises the harmonic distortion. It is possible that these levels of harmonic distortion may have little effect upon the perceived sound when the parametric array is used to produce music or speech; however the effects upon an ANC system, which is generally used to control noise within this low frequency range, would be more significant. The production of additional energy at frequencies other than those at which noise control is being implemented simply add to the total energy within the region. Even if noise at the target frequencies is minimised there is the potential that the increase in noise level at other frequencies would be problematic.

Ultrasonic levels

The frequency response of the source was recorded at a distance of 1.5 m from the source along the source axis for three cases: sound source turned on with no input signal; sound source turned on with a 500 Hz input signal; and sound source turned on with a 3 kHz input signal. The frequency responses are depicted graphically in Figure 11 on the one set of axes. The noise floor is also indicated.

Figure 11. Frequency response of the parametric array.

The SPLs are similar for both the 500 Hz and 3 kHz modulation envelopes. The response for the 3 kHz input signal case shows a greater number of distinct peaks. These can be explained by the pre-processing technique which is applied within the device to the modulating signal to decrease harmonic distortion. A square root operator is applied to the signal before it is utilised to modulate the carrier in order to correct for harmonic distortion. This introduces multiple sidelobes and therefore vastly increases the bandwidth of the emitted signal. The higher the modulating frequency, the greater this bandwidth becomes.

According to Howard et al. (2005), for health and safety reasons, the level of ultrasound to which a person is exposed should be maintained at levels below 110 dB, regardless of the duration of exposure. From Figure 11, the ultrasonic levels can be seen to peak at approximately 130 dB, far greater than the recommended maximum. Even when no modulating signal was delivered to the source, the ultrasonic carrier was still produced at levels of almost 120 dB. This is of particular concern, as any person standing in the main beam produced by the source will only hear a low level hum from the source, mainly emanating from the cooling fan, and hence they may be completely unaware that they are being subjected to high ultrasonic SPLs and consequently may not take the hearing protection cautions which would be taken if such high SPLs were being emitted within the audible frequency range.

CONCLUSIONS

A theoretical analysis of a parametric array, which is commercially available from ATC, has shown that, although the nonlinear absorption is confined to the near field, the requirement of the quasilinear approximation that the nonlinear effects being relatively weak are not met. This indicates that Berktay’s theory may not accurately model the performance of the device. Because of the limited size of the anechoic chamber used for tests, experimental investigations of the device were undertaken within ranges of less than four metres of the parametric array. This distance is well within the region of nonlinear interaction, placing further doubt on the accuracy of using Berktay’s theory to model the results.

As had been anticipated, the parametric array was shown to be highly directive, which is an ideal source characteristic in a localised ANC system. However there exist a number of factors which may adversely affect the performance of the parametric array in such a system. The noise floor of the parametric array was seen to be high at low frequencies, resulting in a poor signal to noise ratio. The variations in the amplitude of the demodulated signal output by the source as a function of time were shown to be significant, resulting in a potential degradation of the performance of an ANC system.
It is possible that through better understanding of the non-linear phenomena and the electronic implementation used in the commercially available device, the parametric array design may be improved for ANC system applications, ameliorating this problem.

The parametric array produces high levels of harmonic distortion within the frequency range of interest, which have the potential to adversely affect the performance of an ANC system by increasing the noise level at frequencies other than the target control frequencies. Again it is possible that in the future, harmonic distortion may be reduced without compromising amplitude through improved parametric array understanding and design.

The high levels of ultrasound produced by the device, which may potentially have an adverse health effect upon exposed individuals, were also noted. These high levels are of particular concern as they are intrinsic in the governing physical sound generation mechanism and therefore cannot be reduced significantly through improved parametric array design.

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REFERENCES


