Learning acoustics through the boundary element method: an inexpensive graphical interface and associated tutorials

Laura A. Brooks, Rick C. Morgans, Colin H. Hansen

School of Mechanical Engineering, The University of Adelaide, Australia

ABSTRACT

The Boundary Element Method (BEM) is a powerful tool which has become an important and useful numerical technique applied to problems in acoustics. It is particularly useful for analysing sound radiation and acoustic scattering problems. Numerous commercial BEM codes with graphical user interfaces (GUIs) and mesh generators exist; however these are relatively expensive, which discourages their use by academic institutions and smaller companies. Helm3D is a three-dimensional BEM code available with purchase of a relatively inexpensive book, but the command file driven interface is difficult to learn and some mechanism to generate the mesh is required. In addition, there is a limited availability of suitable tutorial material, so the uptake of BEM throughout the acoustics community has so far been limited. In this paper, the development of both a mesh generator / GUI interface for the Helm3D code and an associated tutorial are described. The interface links the Helm3D code to a freely available numerical simulation pre/post processor. The tutorial demonstrates the capability of BEM in two application areas: interior acoustics and external acoustic radiation. It is envisaged that the availability of the interface and tutorial will accelerate the uptake of BEM by the wider acoustics community.

NOMENCLATURE

ρ	density
ω	angular frequency
a	source radius
c	speed of sound
$c(x)$	position dependent constant
$g(x_s\|x)$	free space Green's function
k	wavenumber
l	duct length
n	mode number
n_e	number of elements
n_n	number of nodes
p	pressure
r	radial distance
t	time
$v_n(x_s)$	normal surface velocity
x	position of the field point
x_{chief}	location of CHIEF point
x_s	position of the source
Z_s	specific acoustic impedance

INTRODUCTION

The acoustic Boundary Element Method (BEM) has been used to solve a wide range of practical problems in acoustics, such as the modelling of sound generated by loudspeakers (Pederson and Munch 2002, and Hodgson and Underwood 1997) or received by microphones (Juhl 1993), the sound power radiated by a particular structure such as an engine valve cover (Ciskowski and Brebbia 1991) or a fan (von Estorff 2000), and the sound scattered by hard structures (Morgans 2000).

Numerous commercial codes that implement acoustic BEM exist; however the licensing costs are prohibitively expensive for casual users, limiting the uptake of this technology by the wider acoustics community. There exist numerous non-commercial acoustic BEM codes, such as those associated with the book edited by Wu (2000). These source codes exist as pedagogical examples for teaching the basics of BEM at an advanced undergraduate or postgraduate level. They are written in Fortran 77 and are available on the CD accompanying the book. They are fully featured and capable of solving practical problems (Morgans *et al.* 2004).

These non-commercial codes, whilst readily available with the purchase of the book, have not gained widespread use for a number of reasons: the interface is command file driven and requires access to some form of pre and postprocessor, and there is a limited availability of suitable tutorial material.

Thus there is a need for:

- an easy to use, freely available interface to an acoustic BEM code, and
- a well written, step by step tutorial on the use of BEM to solve simple relevant acoustic problems.

In this paper, brief outlines of direct BEM theory, the Helm3D BEM code and the GiD pre and postprocessor are presented. An outline of the Graphical User Interface (GUI), developed with GiD to solve direct BEM problems using the Helm3D code, is given. Finally, the tutorial material and how it will be used to teach the user fundamental acoustic and BEM concepts are described.

DIRECT BEM

The boundary element method is a general numerical method for solving the Helmholtz harmonic wave equation. The traditional (direct) approach to BEM is to numerically approximate the Kirchoff-Helmholtz (K-H) integral equation (Juhl 1993, Morgans *et al.* 2004, Koopmann and Fahnline 1997, and Pierce 1994):

$$c(x)p(x) = -\int_s i\rho\omega v_n(x_s)g(x_s \mid x) + p(x_s)\frac{\partial g(x_s \mid x)}{\partial n}\,ds \qquad (1)$$

where $c(x)$ is a position dependent constant (unity outside the volume of interest, ½ on the surface of the volume and zero inside the volume), $p(x)$ is the complex pressure amplitude (with $e^{-i\omega t}$ time dependence) at location x, $i=\sqrt{-1}$, ρ is the fluid density, ω is the angular frequency, $v_n(x_s)$ is the normal surface velocity at location x_s and $g(x_s \mid x)$ is the free space Green's function relating locations x and x_s. The K-H equation can be derived from the Helmholtz equation using either physical arguments using monopoles and dipoles (Fahy 2001) or using vector calculus and Green's theorem (Koopmann and Fahnline 1997, and Fahy 2001). Equation (1) is the fundamental equation of direct BEM, and shows that the pressure at any point can be represented by the surface integral of a combination of monopoles (first term in the integral of Equation (1)) and dipoles (second term in the integral of Equation (1)) aligned with the surface normal. The monopole source strength is weighted by the product of density and surface acceleration and the dipole source strength is weighted by the surface pressure. Given a distribution of surface normal velocity (which is the boundary condition usually prescribed), once the surface pressure is found, the pressure field anywhere in the domain can be calculated.

Direct BEM can be used to solve the Helmholtz equation in either a bounded interior domain (interior problem) or an unbounded exterior domain (exterior problem). The surface pressure is found by discretising Equation (1) with n_n nodes and n_e elements similar to those used in FEA. If the field point is positioned at each surface node (or "collocated") then a series of n_n equations for the n_n surface pressures can be found for a given velocity distribution. The equations are generated by numerical integration over each element, and the integration technique used must be capable of dealing with the singularities found at the locations of the monopoles and dipoles. The equations can be formed into a matrix and inverted using standard linear algebra techniques. Once the matrix is inverted, and the surface pressures known, the field pressures can be calculated.

There are a number of disadvantages to the direct BEM approach. If the K-H integral equation is used to represent the sound field on the exterior of a finite volume, at the natural frequencies of the interior of the finite volume, the exterior problem breaks down and the matrix becomes ill-conditioned. This is well documented (Copley 1968) and many solutions have been attempted (Shenck 1968, and Burton and Miller 1971). The CHIEF method (Shenck 1968) is commonly used to overcome the interior natural frequency problem because of its simplicity. This technique solves an overdetermined system of equations formed by placing extra points (x_{chief}) inside the volume of interest. Provided the CHIEF points are not placed at a nodal line of the interior solution, this will improve the matrix condition number and allow the matrix to be solved using least-squares methods.

Another problem occurs when the two surfaces of interest are brought close together, resulting in "thin-shape breakdown" (Martinez 1991). This means that although some geometries are probably best represented with a thin surface, a direct BEM simulation may either be not possible, or the geometry must be enclosed in a larger volume.

Although the BEM is mathematically complex, once it has been implemented in a computer code the user is somewhat removed from this complexity. The BEM formulation can be verified by comparison with analytical solutions, ensuring that the equations are being solved correctly, and validated against experimental data, ensuring that the equations are correct. The user can then concentrate on generating the geometry and applying boundary conditions.

HELM 3D

The direct BEM code used in this research is Helm 3D, a Fortran 77 implementation using linear triangular or quadrilateral elements. It is able to solve interior or exterior problems with a wide variety of applied boundary conditions. The code is available with the purchase of the accompanying book (Wu 2000). The code reads in the geometry, boundary conditions, field points and CHIEF points from a text based input file, forms the BEM matrix equations and solves the matrix for the boundary unknowns using least-squares routines. The sound pressure at user-specified points and the sound power and radiation efficiency for radiation problems are evaluated.

GUI

GiD (http://gid.cimne.upc.es) is a general-purpose, fully featured finite element pre and post processor developed over a number of years by the International Centre for Numerical Methods in Engineering (CIMNE) in Barcelona, Spain. It has extensive geometry creation features as well as CAD import (IGES and others), supports the meshing of many different element types and the application of boundary conditions, and has a postprocessing capability for viewing results. Figure 1 shows a representative car interior meshed in GiD.

The academic version of this program is freely downloadable, the only restriction being limited to 700 3D elements. Fortunately for BEM, this is a reasonable size and many useful acoustic problems can be solved.

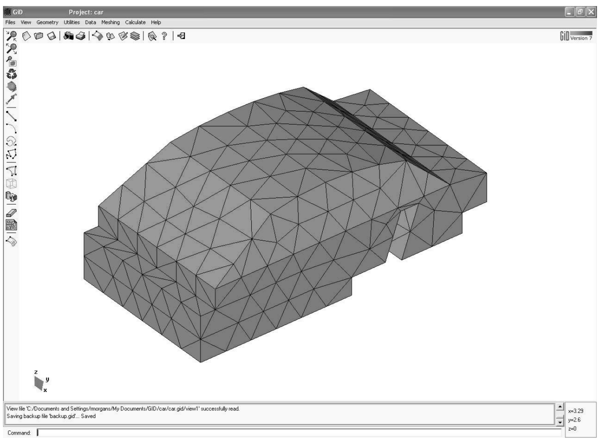

Figure 1. Car interior meshed using GiD.

GiD is designed to be easily customised and exchange data with a variety of numerical analysis codes. There are mechanisms available to apply custom boundary conditions, material properties and solution controls to the model. Most of these solvers, including Helm3d, require some form of text file as input. GiD completely wraps the creation of the text file, execution of the solver and interpretation of the post-processing data, making the operation transparent to the user.

The Helm3d GUI (graphical user interface) developed for this project is straightforward to install (installation instructions are included in the tutorial). Figure 2 shows the problem data dialogue box, which allows the user to specify most of the required inputs that control the simulation. These include the project title, the frequency range of interest, whether the problem is an internal or external problem, material properties such as density and speed of sound, the position of a field point (a "microphone" that can be placed anywhere in the domain), and the position of any required CHIEF point.

The boundary conditions that can be applied in Helm3d are a surface pressure (rarely used), a surface normal velocity or a surface normal impedance. These can be applied using the boundary conditions dialogue box, either to model surfaces, or directly to the surface mesh.

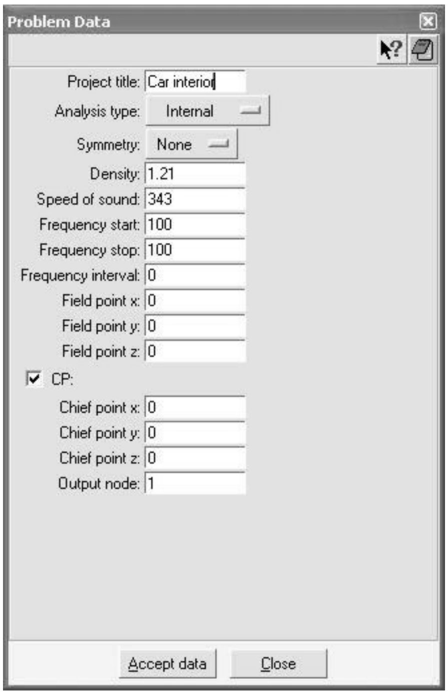

Figure 2. Problem data dialogue box.

An important requirement for a BEM code is control over surface normals. Each surface element has a positive "side", and it is imperative that the side is facing outwards for internal problems (cavities) and inwards for external problems. GiD has a mechanism of visualisation of surface normals, and it is easy to modify normal directions until all surfaces are pointing in the required direction. Figure 3 shows the car surfaces with dark grey positive and light grey negative. For this simulation the 4 light grey surfaces must be flipped in order to solve the internal BEM problem.

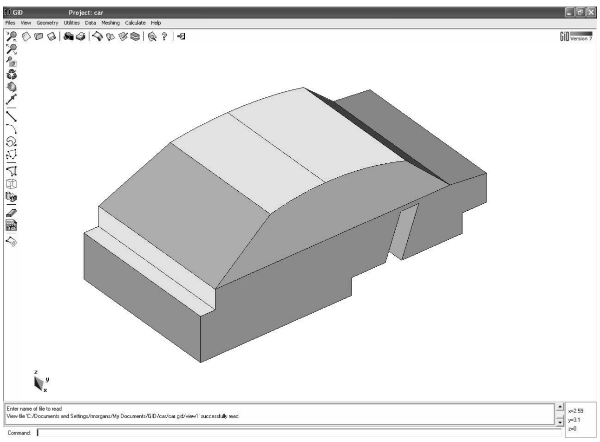

Figure 3. Surface normal visualisation.

The postprocessing capabilities of GiD are extensive, and the results of a Helm3d calculation can be read and displayed easily. Figure 4 shows an example of a plot of pressure magnitude at 100 Hz over the interior of the 3 m long, 1.2 m high and 1.8 m wide car. A velocity excitation represents sound transmission through the engine firewall.

The GUI interface to Helm3d developed for this project is somewhat rudimentary, although it is sufficient to learn the BEM and acoustics. Future developments of the GUI might allow: multiple CHIEF points; multiple field points or even a field mesh that allows visualisation of the sound field away from the surface; or the inclusion of acoustic scattering from within the GUI.

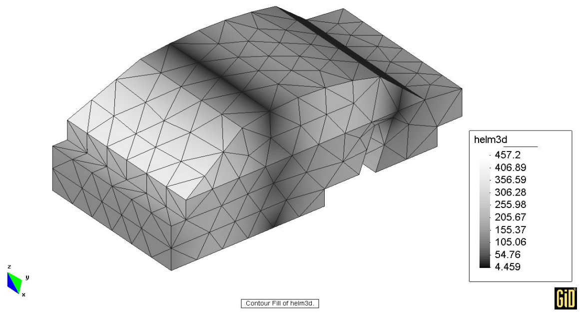

Figure 4. Pressure magnitude in a car interior (pressure in Pascals).

TUTORIALS

The tutorial guides the user through BEM modelling with eight problems, each introducing different aspects of:

- fundamental concepts in acoustics,
- BEM specific concepts, and
- using the GiD-Helm3d interface.

The tutorial material comprises step-by-step instructions which explain how to input each model, apply boundary conditions and postprocess the results. Comparisons with analytical solutions are given when possible.

By the end of the tutorial, the user should have had an introduction to these fundamental concepts in acoustics:

- one-dimensional standing waves,
- one-dimensional travelling waves,
- impedance (sound absorbing) boundary conditions,
- modes in a rectangular room,
- modes in more complex spaces,
- one-dimensional spherical waves,
- sound radiation from a sphere, and
- sound radiation from more complex shapes.

The user should understand these BEM specific concepts:

- advantages and disadvantages when compared to other techniques,
- interior versus exterior problems,
- element types,
- mesh size (6 elements per wavelength),
- non-uniqueness difficulty (CHIEF points),
- symmetry, and
- direction of normals.

The user should also have a working knowledge of these GiD-Helm3d interface concepts:

- inputting the geometry into GiD directly,
- importing CAD data into GiD for meshing,
- flipping surface normals,
- meshing the geometry,
- applying boundary conditions,
- solving the problem through the GiD interface to Helm3d, and
- post-processing results through GiD.

Figure 5 shows the breakdown of the tutorials. Two application areas are addressed: interior acoustics and external acoustic radiation. Simple problems with analytical solutions are introduced. The power of BEM is then demonstrated through application to more realistic problems.

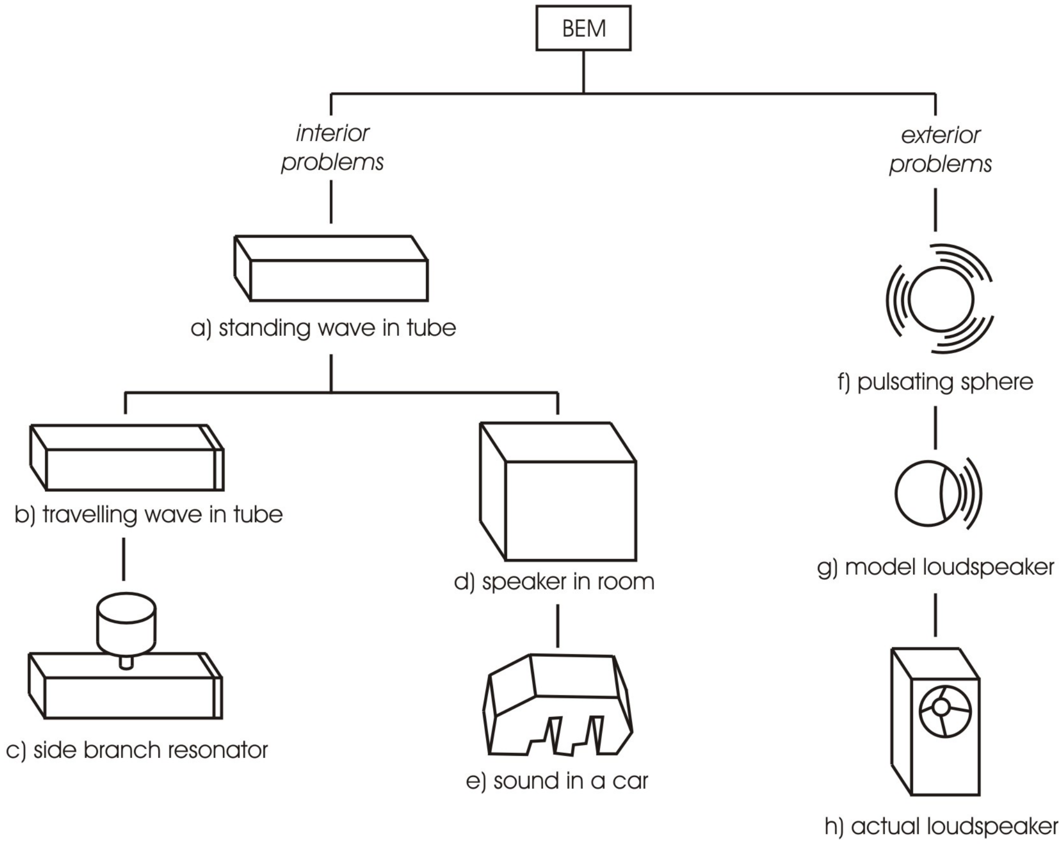

Figure 5. Breakdown of the tutorial problems.

Interior problems

A simple model of a 1D standing wave in a rigid walled duct (Figure 5.*a*) introduces the user to BEM through the very simple geometry of a long rectangle. Velocity boundary conditions, the required direction of normals and meshing are introduced. How the accuracy of results can be affected by mesh resolution is also demonstrated. Results obtained from the numerical model are then compared to the analytical solution. An example of sound with a wavelength identical to the duct length, resonating in a hard walled duct, is shown in Figure 6. A unit input velocity at the left end and zero velocity conditions elsewhere are assumed.

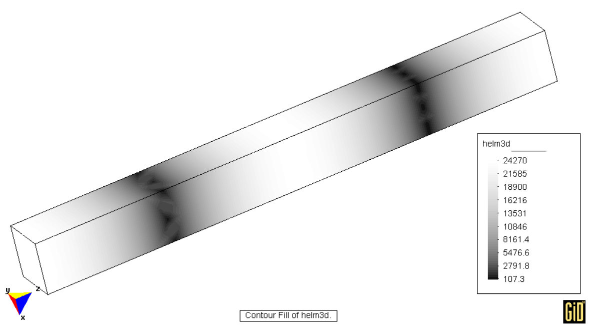

Figure 6. Standing wave in a duct at the second theoretical resonance frequency of the duct (pressure in Pascals).

The resonance frequencies of the system are simply the resonances of an open-closed duct and are given by:

$$f_n = \frac{nc}{2l} \tag{2}$$

where n is the mode number, c is the speed of sound and l is the length of the duct.

The analytical specific acoustic impedance at the excitation location is:

$$Z_s = 0 - i\rho c \cot(kl) \tag{3}$$

where $i = \sqrt{-1}$, ρ is the density of the medium and k is the wavenumber. The theoretical specific acoustic impedance and the BEM specific acoustic impedance (the ratio between the acoustic pressure and particle velocity) at the point of excitation are compared in Figure 7 as a function of frequency. The BEM is shown to be in good agreement with the theoretically determined values.

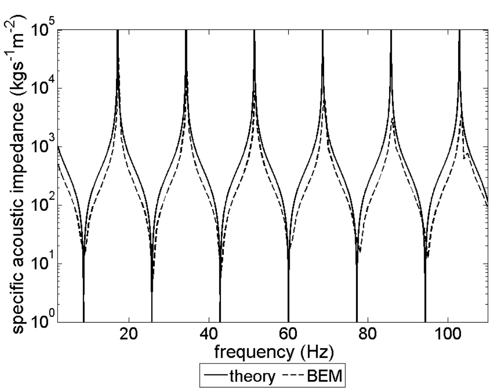

Figure 7. Harmonic response of an open-closed acoustic duct at the point of excitation.

The concept of impedance is introduced by the addition of absorption to the downstream end of the duct (Figures 5.*b* and 8), yielding a travelling plane wave, which is shown to have a very simple analytical solution.

The analytical pressure at any point in the duct is given by the equation:

$$p(x) = \rho c e^{-ikx} \tag{4}$$

where x is the distance from the point of excitation along the duct. As can be seen from Figure 9, the real and complex pressures of the travelling wave estimated using BEM agree well with the analytical solution.

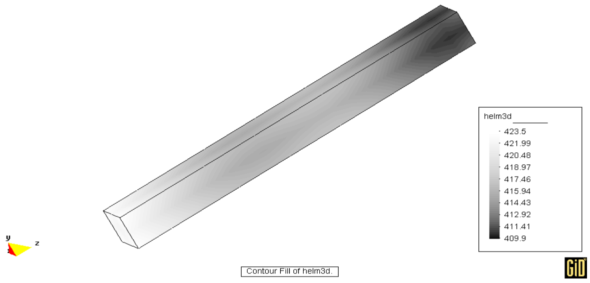

Figure 8. Travelling wave in a duct (pressure in Pascals).

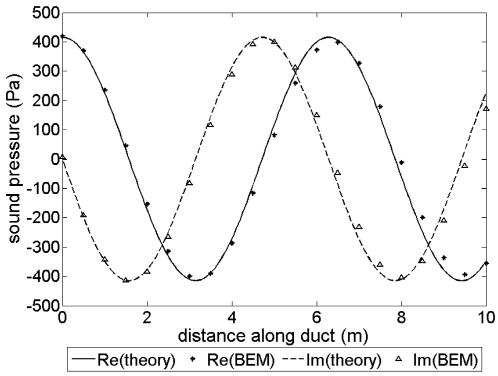

Figure 9. Sound pressure along the centre of one side of the duct.

A side branch resonator is then added (Figure 5.*c*), and the analysis frequency is swept through resonance. The results of the analysis are used to show how the resonator adds impedance in parallel with that of the pipe, resulting in a suppression of tones close to the resonance frequency. An example of a meshed boundary element model of a duct with a side branch Helmholtz resonator is shown in Figure 10.

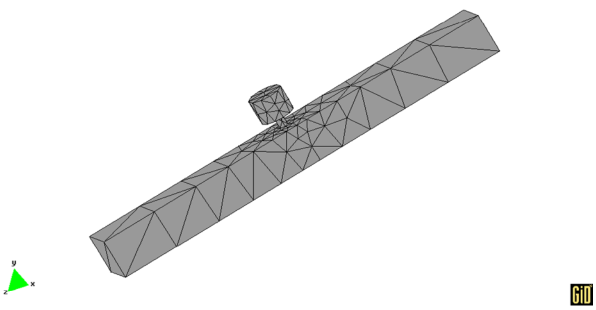

Figure 10. Mesh view of duct with side branch resonator.

A model of a speaker in the corner of a rigid walled room (Figure 5.*d*) introduces the user to the excitation of modes in a 3D environment. Rectangular rooms with three different axial dimensions are compared to those that have two or more identical dimensions. Various source shapes (of identical volume velocity) are also investigated, extending the user's understanding of room acoustics and BEM source modelling. An example of the excitation of room modes in two directions is shown in Figure 11. The room dimensions are 2.5 m × 5 m × 3 m. The source is a 0.04 m^2 sound source, located near the bottom left corner of the wall with the longest dimension, and operates at a frequency of 68.5 Hz, corresponding to a wavelength of 5 m.

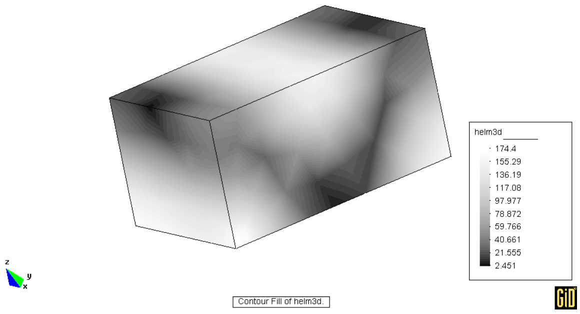

Figure 11. Pressure on wall of room containing sound source near one corner.

The final internal problem, the sound pressure in the interior of a car (Figures 4 and 5.*e*), is an example of how BEM can be applied to a practical 3D problem. Figure 4 shows the response within the interior of the car. Rigid wall boundary conditions are assumed. In practice the flexibility of the enclosing structure would need to be accounted for; however, coupled problems such as this would require BEM codes far more complicated than the Helm3D code.

Exterior problems

The first exterior problem presented is the classical fundamental radiation problem of a pulsating sphere (Figure 5.*f*). Key concepts covered are modelling symmetry and how this affects computational efficiency, appropriate direction of normals for an external problem and the use of CHIEF points in the interior to improve the condition number of the governing matrix. A meshed model of the half sphere (a symmetry boundary condition is used) is shown in Figure 12.

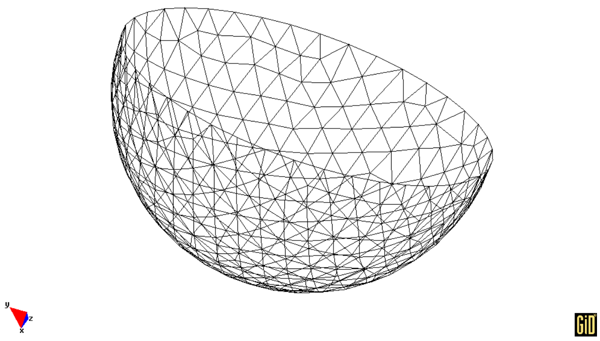

Figure 12. Mesh view of a pulsating sphere.

The analytical solution for the pressure produced by a pulsating sphere, which can be derived from the spherical wave equation, is:

$$p(r) = \left(\frac{a^2}{r}\right)\frac{i\rho\omega}{1+ika}e^{-ik(r-a)} \tag{5}$$

where a is the sphere radius, r is the radius at which the pressure is being calculated and ω is the angular frequency. The characteristic eigenfrequencies of the sphere, which are the eigenfrequencies of the interior Dirichlet problem, are given by the equation:

$$\sin ka = 0 \ . \qquad (6)$$

Figure 13 shows the variation in surface pressure (*r*=*a*) with frequency for the pulsating sphere for an analytical solution, and BEM calculations with no CHIEF point, a CHIEF point at the centre of the sphere and a CHIEF point at half the radius.

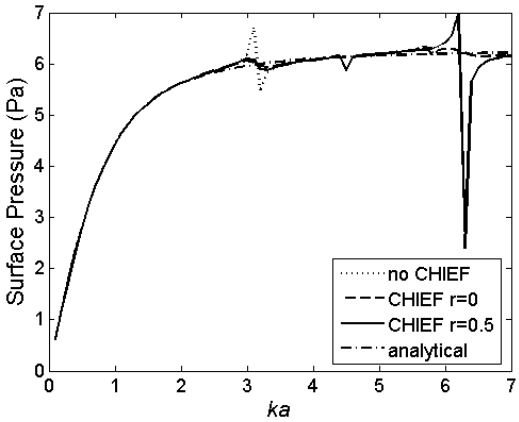

Figure 13. Surface pressure of a pulsating sphere

The BEM solution with no CHIEF point shows poor agreement with the analytical solution at *ka*=π and *ka*=2π, where *k* is the wavenumber and *a* is the radius of the source. This is due to poor conditioning of the matrix. The placement of a CHIEF point at *r*/*a* = 0.5, where r is the radial location from the centre of the sphere, ameliorates the problem at *ka*=π; however, poor agreement at *ka*=2π still occurs due to the CHIEF point being on the interior nodal surface corresponding to the characteristic eigenfrequency *ka*=2π, meaning that this resonance cannot be cancelled. Placing the CHIEF point at the sphere centre ensures that it does not lie on a nodal surface. The resulting solution is therefore in good agreement with the analytical solution. When using BEM to analyse more complex geometries, the user generally has no prior knowledge of the optimal CHIEF point location, and therefore multiple CHIEF points randomly distributed within the volume are used. The condition number of the matrix will also give an indication of whether there are any interior resonance problems.

A spherical volume with an external velocity over a proportion of its surface is presented as a simplified model of a loudspeaker in a rigid walled box (Figure 5.*g*). Comparison of results at different frequencies is used to show that radiation is inefficient at low frequencies. The example shows how a BEM of a problem with simplified geometry can be used to model a more complex problem, producing results which exhibit a similar pattern of behaviour. Application of the external BEM to a more realistic situation is presented as the analysis of radiation from a speaker of more realistic geometry (Figure 5.*h*).

AVAILABILITY

It is intended that the tutorial material (Brooks and Morgans 2005) is available at the time of this publication on the University of Adelaide Active Noise and Vibration Control (ANVC) Group homepage.

(http://www.mecheng.adelaide.edu.au/anvc/publications.php)

The program GiD can be downloaded from its homepage.
(http://gid.cimne.upc.es)

The book *Boundary Element Acoustics: Fundamentals and Computer Codes* (Wu 2000), including a CD containing a PC executable of Helm3d, as well as F77 source code, is available from WITPress.
(http://www.witpress.com/acatalog/5709.html)

CONCLUSIONS

This paper describes: a freely available interface that has been developed between GiD and Helm3d; and tutorial material describing some fundamental acoustic problems and how they would be solved with BEM using the newly developed interface. It is hoped that the resulting practical and freely available introduction to BEM will be the basis for both student projects within universities around Australia, as well as for a series of lectures in acoustics courses at some universities. The proposed greater availability of the code and tutorial will accelerate the uptake of BEM by the wider acoustics community, including members of the acoustical society as well as practicing acoustic engineers.

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REFERENCES

Brooks, L. A. and Morgans, R. C. (2005) '*Learning Acoustics and the Boundary Element Method using Helm3d and GiD'*, tutorial material, University of Adelaide, Australia.

Burton, A. J. and Miller, G. F. (1971) 'The application of integral equation methods to the numerical solutions of some exterior boundary-value problems', *Proceedings of the Royal Society of London* A, 323:201-210.

Ciskowski, R. D. and Brebbia, C. A., Editors (1991) *Boundary element methods in Acoustics,* Computational Mechanics Publications, Co-published with Elsevier Applied Science.

Copley, L. G. (July 1968) 'Fundamental results concerning integral representations in acoustic radiation', *Journal of the Acoustical Society of America*, 44(1):28-32.

Fahy, F. (2001) *Foundations of Engineering Acoustics*, Academic Press.

GiD Homepage, http://gid.cimne.upc.es.

Hodgson, T. H. and Underwood, R. L. (1997) 'BEM computations of a finite length acoustic horn and comparison with experiment', in *Computational Acoustics and Its Environmental Applications*, 213-222.

Juhl, P. M. (1993) *The boundary element method for sound field calculations*, PhD Thesis, Technical University of Denmark.

Koopmann, G. H. and Fahnline, J. B. (1997) *Designing quiet structures: a sound minimization approach*, Academic Press.

Martinez, R. (1991) 'The thin-shape breakdown (TSB) of the Helmholtz integral equation', *Journal of the Acoustical Society of America,* 90(5):2728-2738.

Morgans, R. C. (2000) *External Acoustic Analysis Using Comet*, Internal Report, The University of Adelaide.

Morgans, R. C., Zander, A. C., Hansen, C. H., and Murphy D. J. (2004) 'Fast Boundary Element Models For Far Field Pressure Prediction', *Australia Acoustical Society conference, Acoustics 2004*.

Pederson, J. A. and Munch, G. (October 2002) 'Driver directivity control by sound redistribution', in *113th Convention of the Audio Engineering Society*, Los Angeles, California.

Pierce, A. D. (1994) *Acoustics: an introduction to its physical principles and applications*, Acoustical Society of America, New York.

Schenck, H. A. (July 1968) 'Improved integral formulation for acoustic radiation problems', *Journal of the Acoustical Society of America*, 44(1):41-58.

von Estorff, O. (2000) *Boundary Elements in Acoustics*, WIT Press.

Wu, T. W. (2000) *Boundary Element Acoustics: Fundamentals and Computer Codes*, WITPress. (http://www.witpress.com/acatalog/5709.html)