Expectation Maximisation Algorithm for Detection of High Bandwidth Sonar Pulses

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ABSTRACT

The use of high bandwidth pulses in active sonar systems can reduce interference from reverberation and variability due to channel fading. However, the matched filter detection performance is degraded more by acoustic multipath when the bandwidth is increased. The performance degradation due to multipath distortion results when the destructive interference of the paths cancels some frequencies reducing the similarity between the echo and the transmitted pulse required for matched filter operation. Increased bandwidth means that more frequencies are cancelled so distortion and degradation increases. If the acoustic properties of the channel are known well enough to predict the acoustic multipath characteristics, detection can be improved by matched filtering for the distorted signal rather than the transmitted signal. In cases where the channel properties are unknown, acoustic path delays and amplitudes must be estimated from the data itself. In this paper the acoustic path delays and amplitudes are first estimated using the Expectation Maximisation (EM) algorithm. The estimates are then used to integrate the return from each path recovering part of the loss caused by multipath distortion.

INTRODUCTION

For active sonar targets with small relative velocity it has been shown (Van Trees 1965) that detection of signals in reverberation is enhanced when large bandwidths are used. This conclusion assumes a known signal in reverberation with known statistical properties. For active sonar the statistical properties of the echo signal are usually unknown. Acoustic multipath distortion results in a signal that is not a single replica of the transmission but several superimposed replicas with unknown amplitudes and delays. This distortion causes cancellation of the signal at some frequencies so that it is more serious when signals have wider bandwidth, as more frequencies will be cancelled.

A variety of strategies (Van Trees 1971) have been suggested to compensate for the loss caused by the deviation of the real signal from the assumption. The strategies generally attempt to integrate the return from some or all paths to achieve a better detection performance with broadband signals.

In this paper a detector will be presented that estimates the amplitudes of the paths and the combines the results to provide improved detection performance. Strategies for estimating the paths include exhaustive search (Carpenter 1995), Gibbs sampling (Michalopoulou 2005) and adding the largest outputs of the matched filter over a period (Abraham 2001). The first two strategies have an extremely high computational cost while the third could have degraded performance because it makes no effort to avoid double counting of a single strong signal.

In this paper the Expectation Maximisation (EM) algorithm (Laird 1977) is used to make the estimates in a computationally efficient manner. The T-square test (Anderson 1958) is used in an attempt to cancel mutual interference between different paths.

DISTORTION TOLERANT DETECTION

The Problem

Acoustic multipath propagation can be modelled as resulting in the return to the sonar receiver being composed of a sum of replicas of the transmitted pulse with different amplitudes and delays. The signal model used in this paper is

$$y[t] = \sum_{k=1}^{M} a_k s[t - \tau_k] + n[t] \quad \textbf{(1)}$$

where $y[t]$ is the received signal,

M is the number of paths,

$s[t]$ is the transmitted signal,

a_k is the path amplitude,

τ_k is the path delay and

$n[t]$ is the noise.

The noise is assumed to have a normal distribution but it is not assumed that it is ‘white’ or uncorrelated although only with noise considered in this paper.

The detection algorithm is applied to a block of data so the problem is best represented in vector form.

$$\mathbf{y} = \sum_{k=1}^{M} a_k \mathbf{s}[\tau_k] + \mathbf{n} \quad \textbf{(2)}$$

where $\mathbf{y} = \{y[t] \cdots y[t+N-1]\}^T$

$$\mathbf{s}[\tau_k] = \{s[t-\tau_k]\cdots s[t+N-\tau_k-1]\}^T$$

$$\mathbf{n} = \{n[t]\cdots n[t+N-1]\}^T$$

Under this formulation the noise process has a multivariate normal distribution with mean zero and covariance matrix **C**. The detection problem becomes the decision between two hypotheses about the amplitudes.

$$\begin{aligned} \mathrm{H}_0 &: a_k = 0 \quad k = 1,\ldots,M \\ \mathrm{H}_1 &: a_k \neq 0 \quad \text{for any } k \end{aligned} \tag{3}$$

The approach to solving the detection problem is to estimate the unknown amplitudes from the model in Equation 2 and then declare a detection if they are large enough. Precise criteria for 'large enough' will be defined below.

Expectation Maximisation

The estimation of the parameters for the model in Equations 1 and 2 is a non-linear optimisation problem. For this work the EM method was chosen to perform this optimisation. The version of EM used to solve this problem is due to Feder and Weinstein (1988).

The key idea in the EM approach is to devise a set of missing data that if known would make the problem much simpler. In this problem the solution is known when there is only one acoustic path so that the best choice for missing data is a decomposition of the problem into M single path problems.

$$\hat{\mathbf{n}}^{(i+1)} = \mathbf{y} - \sum_{k=1}^{M} \hat{a}_k^{(i)} \mathbf{s}[\hat{\tau}_k^{(i)}] \tag{4}$$

$$\hat{\mathbf{y}}_k^{(i+1)} = \hat{a}_k^{(i)} \mathbf{s}[\hat{\tau}_k^{(i)}] + \beta_k \hat{\mathbf{n}}^{(i+1)} \tag{5}$$

The hat symbol (^) is used to indicate estimated quantities while the superscript in parentheses records the iteration number. For the decomposed problem to sum correctly to the true problem it is necessary for

$$\sum_{k=1}^{M} \beta_k = 1 . \tag{6}$$

Equations 4 and 5 represent the expectation step of the algorithm. The maximisation step that completes the iteration involves correlation or matched filtering to find the delay and amplitude of the path for each individual problem

$$\hat{a}_k^{(i+1)} = \max_{\tau_k} \left(\mathbf{s}[\tau_k]^T \hat{\mathbf{y}}_k^{(i+1)} \right) \Big/ E_S , \tag{7}$$

$$\hat{\tau}_k^{(i+1)} = \arg_{\tau_k} \max\left(\mathbf{s}[\tau_k]^T \hat{\mathbf{y}}_k^{(i+1)} \right), \tag{8}$$

$$E_S = \sum_{t=1}^{N} \left| s[t] \right|^2 . \tag{9}$$

The iterations are repeated until the change in the mean square error between iterations is negligible. It is observed in practice that convergence is obtained after only four or five cycles.

There are two problems with this algorithm. The first is the issue of local maxima common to all multivariable non-linear optimisation problems. The algorithm can find a local maximum but it is not guaranteed to find the global maximum as desired.

The other problem is a tendency to over estimate the amplitudes particularly when the signal is weak (or non-existent) compared to the noise. When there is no signal the amplitude estimate in Equation 7 will have the correct value of zero only when all the noise generated samples from which we are taking the maximum are zero. This is very unlikely. Consideration of the properties of order statistics (see Freund and Walpole (1988)) reveals that the values will be biased higher than reality.

The Detector

Once the parameters of the model have been estimated the next step is to apply the estimates to signal detection. An issue with the estimates is that they may not be independent. If two paths have arrival times separated by less than the resolving power of the transmitted pulse the estimates of the path parameters will be degraded from what would be obtained if only one path was present.

Inaccuracy and correlation between the amplitude estimates is introduced by interference of other paths in estimation of the parameters of a particular path. A better detector is possible if the detector attempts to decorrelate the estimates allowing the value of some amplitude estimates to be discounted for inaccuracy and correlations caused by mutual interference. To do this, an indication of the correlation that exists between the estimates is needed. The Fisher information matrix will provide this indication. The distribution of the estimates from an unbiased estimator will asymptotically approach a multivariate normal distribution with the covariance matrix being the inverse of the Fisher information matrix (Lehmann 1998). There are two problems with using this result. Asymptotically means for large samples, which may not be available, and the argument in the previous section suggests that the estimates are not unbiased. In spite of these problems the Fisher Information is used as an approximation in the absence of a better solution.

Each term in the Fisher information matrix is given by

$$[\mathbf{I}_{\theta\theta}]_{ij} = -\mathrm{E}\left[\frac{\partial^2 \log_e \mathrm{f}(\mathbf{x};\boldsymbol{\theta})}{\partial\theta_{\mathrm{i}}\partial\theta_j} \right]. \tag{10}$$

The E in this equation denotes the expectation, the vector $\boldsymbol{\theta}$ has the estimated parameters as elements and f is the density function of the data. For the detection problem from Equation 3, the parameter vector is provided by the amplitudes estimated using EM. Under these circumstances the equation for the matrix elements has been found in Lourey (2004)

$$[\mathbf{I}_{aa}]_{ij} = \mathbf{s}[\tau_i]^T \mathbf{C}^{-1} \mathbf{s}[\tau_j] . \tag{11}$$

For multivariate normal parameters the Hotelling T-square test is used to test for a non-zero mean (Anderson 1958). It has been assumed that the amplitude parameters estimated by the EM algorithm are multivariate normal so that consideration of the T-square test suggests that the following statistic can be used for this problem

$$\mathrm{T}(\mathbf{a}) = \mathbf{a}^T \mathbf{I}_{aa} \mathbf{a} . \tag{12}$$

In summary, the detection procedure proposed in this paper consists of iterating Equations 4 to 8 until convergence and then evaluating Equations 11 and 12 to provide a statistic. If the statistic exceeds a threshold then a detection is declared.

RESULTS

A comparison between the algorithm described in this paper and traditional methods was carried out using simulations. An optimum "clairvoyant" detector designed on the assumption the channel is known was also tested to indicate the performance loss due to imperfect estimation of the acoustic parameters. The first simulation is concerned with the performance of the algorithms when the signal model of superimposed signals in Gaussian noise is correct and the number of paths is known correctly. The case where model order is not known is considered subsequently.

The signal was generated for a simple environment that has six acoustic paths with fixed delays and identical amplitudes. The transmitted signal was a linear FM chirp modulated from 200Hz to 400Hz over 0.5 seconds duration and sampled at 1000Hz. Detection of this signal was tested against ten thousand realisations of a white noise background with signal energy normalised to give a desired signal to noise ratio (-20dB). A test consisted of generating a sample of white Gaussian noise and then applying each detector to the noise and then to noise plus signal. Figure 1 shows the detection probability against false alarm probability for the three detectors. Detection performance of the T-square test is uniformly better than the simple matched filter for all false alarm values. The clairvoyant detector is also shown but nearly perfect performance hides this curve under the axes of the graph. The dashed line that represents this data can be seen to follow the unity probability of detection value closely.

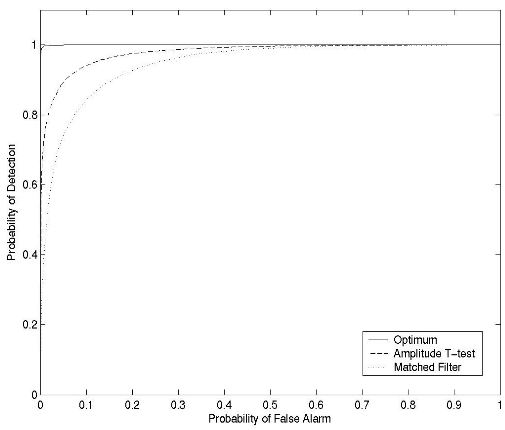

Figure 1 Receiver Operating Characteristics comparison for detection of six-path multipath signal in white noise (SNR -20dB)

These results show that the proposed detector improves on the matched filter at least for one particular SNR value. Figure 2 shows the results when the simulations were repeated for a range of SNR values. Probability of detection is estimated for the threshold giving a false alarm probability of 0.001. For this simple multipath environment results indicate that, regardless of SNR, the performance of the matched filter is improved on by both the performance of the T-square test on amplitude estimates and the "clairvoyant" detector.

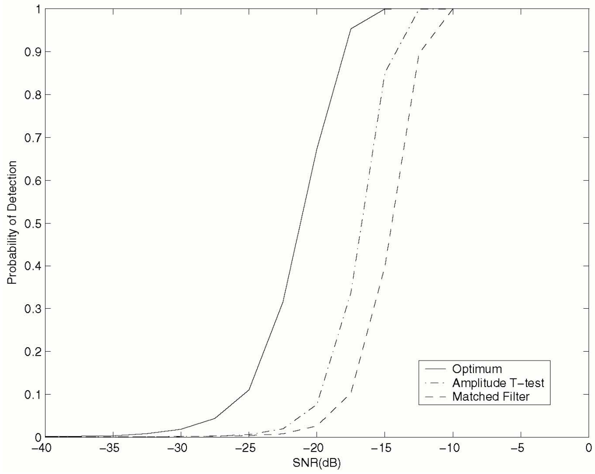

Figure 2 Performance comparisons for detection of six-path multipath signal in white noise with probability of false alarm 0.001.

A signal formed from six identical acoustic paths does not provide a very realistic representation of the true channel in the ocean. A more realistic simulation can be obtained from the assumption of constant velocity with fixed losses from each reflection from the channel boundaries (surface or bottom). This model, called the image model is described in Brekhovskikh and Lysanov (1982). This simulation used one-way propagation at a range of 4,000 metres in 100 metres deep water with 3dB loss on reflection from the bottom. Surface loss was assumed negligible but a 180 degrees phase change was applied for each surface reflection. Signal amplitude was normalised so that under the assumption of spherical spreading (power decreases inversely with range squared) the SNR would be -20dB for a single direct path. The results of ten thousand runs of this simulation are shown in Figure 3.

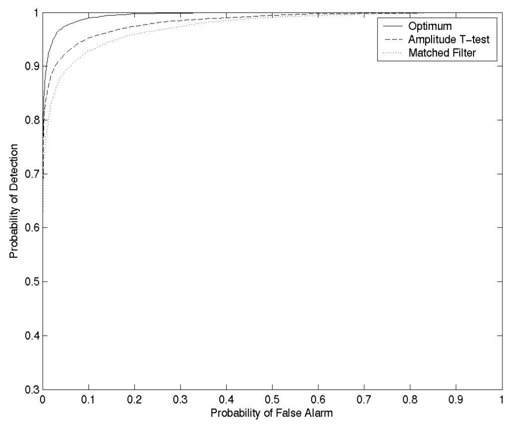

Figure 3 Receiver Operating Characteristics comparison for detection of image model in white noise (SNR -20dB)

Making the comparison between detectors for different SNR values using the image model for a signal is more complicated. The statistic for the new test shows more variability under this model. As a result at low false alarm rates it is not possible to resolve the performance of the T-square test and the "clairvoyant" detector. At a higher false alarm rate the determination of the threshold is easier and resolution of the curves is possible. For this reason the simulation result shown in Figure 4 is for a one percent false alarm rate.

The result in Figure 4 requires some explanation. The T-square test and the matched filter converge for low SNR values. This is due to the bias of the amplitude estimates to higher values that has been mentioned above. Low amplitude paths provide estimated amplitudes indistinguishable from estimates resulting only from noise. Eventually only one path is clearly different from the noise and the Matched filter and T-square test results converge for low SNR. This behaviour is less apparent in Figure 2 because all the signal components have the same amplitude and the estimates degrade consistently.

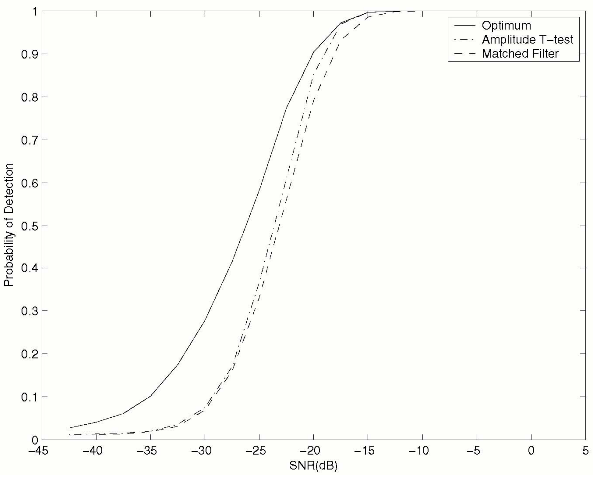

Figure 4 Performance comparisons for detection of Image Model in white noise with probability of false alarm 0.01.

CONCLUSIONS

In this paper detection of sonar returns in the presence of multipath distortion was discussed. The degradation of detection performance by this phenomenon is more severe when using broadband pulses that are predicted to have superior reverberation rejection properties. Navies are increasingly operating in littoral waters with high reverberation levels so the use of high bandwidth pulses with increased resistance to this interference is desirable.

The detector proposed in this paper used the EM algorithm to estimate the path amplitudes and delays from the data. It then used the amplitudes to perform detection. The path delays were used to derive the Fisher information of the amplitudes, which was used to approximate the correlation between estimated path amplitude due to interference between paths. Theoretically allowing for this correlation will give a better measure of the value of the data and hopefully better detection performance.

Simulations indicate that when multipath propagation occurs the T-test detector can give better performance than the matched filter that is usually used for sonar detection. Issues remain to be investigated. The approach has been compared only to the traditional matched filter and the “clairvoyant” detector designed on the assumption of a perfectly known environment. Comparison with the latter suggests that there is significant room for improvements. A performance comparison with other multipath tolerant and environmentally adaptive detectors would be valuable. The EM algorithm is only one of several algorithms that solve the superimposed signal problem, related to multipath propagation, and it is not clear which is most suitable for this application.

An important point is that the simulations discussed in this paper are for a signal in white Gaussian noise. Investigation of the results when reverberation is the background will be required before operational applications are considered.

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