Robust virtual acoustic sensing

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ABSTRACT

The aim of the paper is to develop a robust virtual sensing method where dynamic variations in the actual acoustic system would not degrade the sensing performance. The approach here is to consider the possible uncertainties in the systems and to take into account this information to develop a virtual sensing method that is robust against these uncertainties. A certain sensing performance can be enforced and the task is formulated as an optimal robust control problem that includes uncertainty modelling. Numerical studies are performed on an acoustic duct system with varying properties, which show a satisfactory performance of virtual sensing when the system varies within a particular range. The proposed approach guarantees a certain level of performance robustness for virtual sensing when the systems are expected to vary during operations. Therefore, the approach can be used for practical implementation in actual acoustic systems where it is possible that the systems might vary during the sensing and control operation.

INTRODUCTION

Active noise control generally requires the use of acoustic sensors to improve its control performance since such sensors can detect the actual acoustic parameters that are of interest. For instance, microphones can be used in an acoustic enclosure to detect the sound pressure level at a number of particular locations. Signals from the microphones are then fedback to a controller/compensator, whose purpose is to generate control actuation to the acoustic enclosure to minimise these 'error' signals that consequently minimises the sound pressure level at those locations. The microphones, therefore, as any other acoustic sensors, generally need to be placed at the locations in which acoustic parameters such as sound pressures are to be controlled.

The previous requirement for sensor placement may not be easy to implement since in some instances, it is not practical to place the sensors directly at locations where the sound pressure needs to be minimised. A virtual acoustic sensing method can be used for predicting sound pressure at a 'virtual' location away from the actual microphone or a set of microphones (Elliot and David 1992; Roure and Albarrazin 1999; Munn 2003). This type of sensing is useful for active noise control purposes where it is not possible to place a microphone directly at the location where the sound pressure needs to be controlled. The current approach in virtual sensing is to obtain measurements that relate the sound pressure at a virtual location to that at the actual microphone(s) (Munn 2003; Munn et al. 2002; Roure and Albarrazin 1999; Garcia-Bonito et al. 1996). The general acoustic systems, where the measurements are obtained, are assumed to experience no significant variations during the virtual sensing operations. However, when the actual systems experience some changes, it may reduce the performance effectiveness of the active noise control strategy. The changes in the system may even cause the system to be unstable which is obviously not desirable for practical applications.

Motivated by the problem in improving the virtual sensing and control performance, this work considers determining a control design method for the virtual sensing strategy to guarantee a certain level of performance.

ACOUSTIC VIRTUAL SENSING METHOD

The general approach of acoustic virtual sensing method is to estimate the acoustic parameters, such as sound pressure, at a virtual location based on the acoustic parameters measured by sensors at other locations. There are several virtual sensing methods that have been discussed by previous researchers. One of the methods requires the use of a set of microphones with forward projection method (Cazzolato 2002). The other method is based on measuring the transfer function from the pressure at the actual microphone to the pressure at the virtual microphone, i.e. the virtual sensing path (Yuan 2004). Both methods require off-line identification to estimate the virtual sensing path.

Estimating the virtual sensing path

In this work, the virtual sensing using the transfer function method is utilised for control design. The virtual sensing path can be estimated by performing system identification or using an adaptive algorithm to find the estimated transfer function that is 'optimally close' to the actual transfer function. Alternatively, finding the virtual sensing path can be shown to resemble to a feedforward compensator configuration as shown in the virtual sensor arrangement in Figure 1. Let P_v and P_a respectively to be the transfer functions from input h to the output sound pressures at the locations of virtual microphones p_v and actual microphones p_a. It can be simply shown that the estimated pressure at the virtual microphone is:

$$\tilde{p}_v = WP_a h \tag{1}$$

where W is the virtual sensing path.

From the figure, h is the disturbance or control input signals, depending whether the primary or secondary sound field is under consideration. P_v and P_a are the transfer functions from the input to the pressure at the virtual and actual microphone locations that can be obtained from system identification by placing a microphone at the virtual location. In this case, the task is to find an optimal filter W that minimise the error signal e described as:

$$e = p_v - \tilde{p}_v = P_v h - WP_a h\,. \tag{2}$$

This corresponds to the optimal Wiener filter problem, where the optimal filter W is the required virtual sensing path. The problem can be solved using the H_2 optimal control framework with feedforward configuration. Note that a small weight c from the input h shown in Figure 1 is necessary to avoid a numerical problem in the formulation.

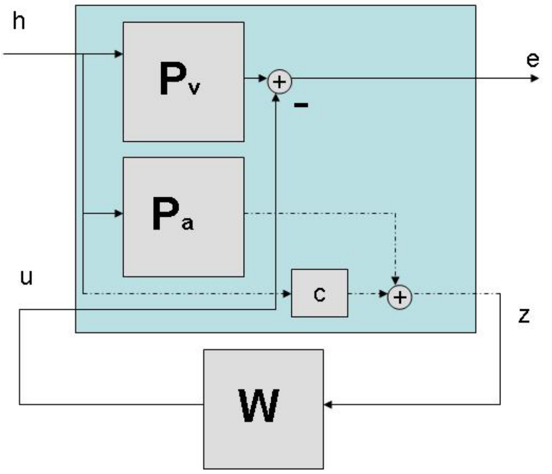

Figure 1. Estimating the transfer function from the actual microphone pressure signal to the virtual microphone pressure signal.

Let the state-space realisations of transfer functions P_v and P_a as:

$$\begin{aligned} P_v &\equiv (A, B, C_v, 0) \\ P_a &\equiv (A, B, C_a, 0). \end{aligned} \tag{3}$$

The two transfer functions share the same poles since the same acoustic system is considered, so they share the same state matrix A. Thus, the optimal feedforward control problem in Figure 1 can be described by the following state-space system that relates input h, u with the states x and outputs e and z:

$$\begin{aligned} \dot{x} &= Ax + Bh \\ \begin{bmatrix} e \\ z \end{bmatrix} &= \begin{bmatrix} C_v \\ C_a \end{bmatrix} x + \begin{bmatrix} 0 \\ c \end{bmatrix} h + \begin{bmatrix} -1 \\ 0 \end{bmatrix} u. \end{aligned} \tag{4}$$

The H_2 optimal control problem is to find an optimal filter W such that:

$$\|G\|_2^2 = \int_{-\infty}^{\infty} \text{trace}\{G(j\omega)^* G(j\omega)\} d\omega \tag{5}$$

is minimised where G is the transfer matrix of the generalised plant system in Equation (4), augmented with the optimal filter W. Here, G^* is the Hermitian transpose of a complex matrix G.

An acoustic duct system

In this work, an acoustic duct with closed ends is considered for the analysis of the virtual sensing design as shown in Figure 2. The dimensions of the duct are 4.2m x 0.205m x 0.205 m, similar to the model mentioned in (Cazzolato 2002) that has a cut-on frequency of 836 Hz. An acoustic model of the duct is obtained from modal analysis (Nelson and Elliot 1992) using a quality factor of 20, by taking into account the first 6 modes below the cut-on frequency. Table 1 describes the natural frequencies of the modes 1 to 5, recognising that there is a fundamental mode at 0 Hz. To avoid excessive gain at low frequency due to the fundamental mode, the pole of that mode is moved to s=-80, which corresponds to a frequency below the next lowest frequency mode at 40.8 Hz. Point disturbance and control sources are respectively located at x=0.5m and x=4.0m from one end of the duct. The microphone location is at x=2.0m, while the virtual location is at x=2.3m.

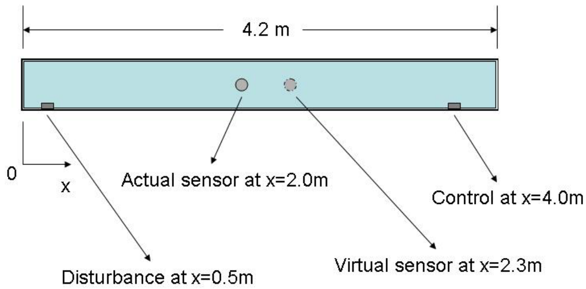

Figure 2. An acoustic duct with the locations for disturbance and control sources, and virtual and actual sensors.

Table 1. Natural frequencies of modes 1-5.

Mode	Frequency [Hz]
1	40.8
2	81.7
3	122.5
4	163.3
5	204.2

Virtual sensing path estimation for an acoustic duct

Matlab Robust Control Toolbox is used to solve for the optimal control problem based on Equations (4) and (5) for the acoustic duct system. Figure 3 illustrates the estimated virtual sensing path W_1 based on the 'primary' transfer functions from the disturbance source. The two transfer functions plotted are P_v and WP_a described in Equation (2).

Similar to the Figure 3, Figure 4 illustrates the estimated virtual sensing path W_2 based on the 'secondary' transfer functions from the control source. It can be seen that the estimated transfer functions are relatively close to the actual transfer functions, particularly at the resonances. Deviations from the actual transfer function are expected at some frequencies, especially at lower and higher frequency regions because of the lower energy contribution. In this work, however, an accurate virtual sensing path is only needed for frequencies up to around 220 Hz to capture up to the fifth acoustic mode of a duct system.

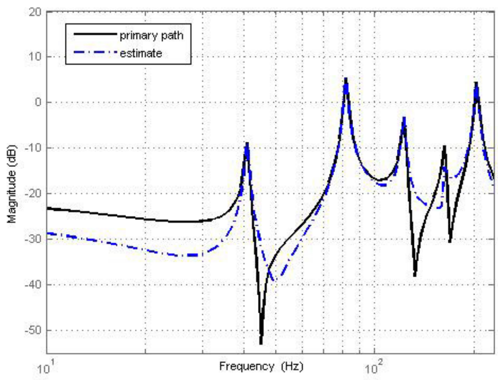

Figure 3. The primary transfer function P_v and its estimate WP_a.

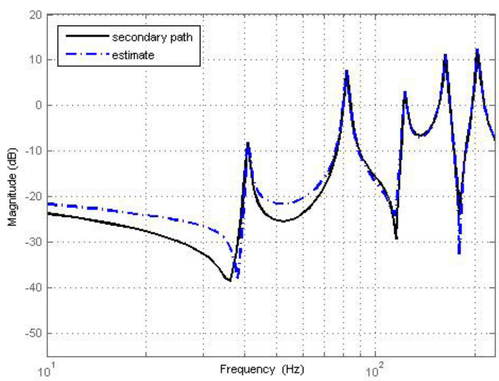

Figure 4. The secondary transfer function P_v with its estimate WP_a.

The difficulty in obtaining an accurate virtual sensing path generally arises because the accuracy depends on the non-minimum phase properties of transfer functions P_v and P_a. It can be seen from Equation (2) that the ideal virtual sensing path W can be found from (Yuan 2004):

$$W = P_v P_a^{-1}. \tag{6}$$

Since both transfer functions share the same poles, the ideal path W depends on the zeros of both transfer functions. In practice, the transfer functions are always non-minimum phase to some degree, where some zeros are 'unstable', hence the ideal virtual sensing path is generally unstable. Estimation of this ideal path may require a transfer function with a very high order, which is not practical. It is possible to judiciously place the locations of microphones to achieve better performance (Yuan 2004), but in some cases, the freedom of placing the microphones may be limited.

However, for the rest of the work here, it is assumed that the estimated pressure at virtual location is sufficiently accurate so $p_v \approx \tilde{p}_v$.

ROBUST VIRTUAL SENSING AND CONTROL DESIGN

In this section, it is considered that the model of the plant may not be accurate to some degree. Consider the virtual sensing and control configuration shown in Figure 5. The estimated virtual sensing paths from the disturbance input d and control input u are described by W_1 and W_2 respectively.

The pressure p_v at the virtual location can thus be expressed as:

$$p_v = W_1 P_{ad} d + W_2 P_{au} u \tag{7}$$

where P_{ad} and P_{au} are respectively the transfer functions from disturbance input d and control input u to the pressure at the actual microphone location. The controller K utilises the pressure p_v at the virtual location to generate the necessary control input u for minimising the pressure. A filter W_p is used for setting up the performance criterion for robust performance control design that will be explained later.

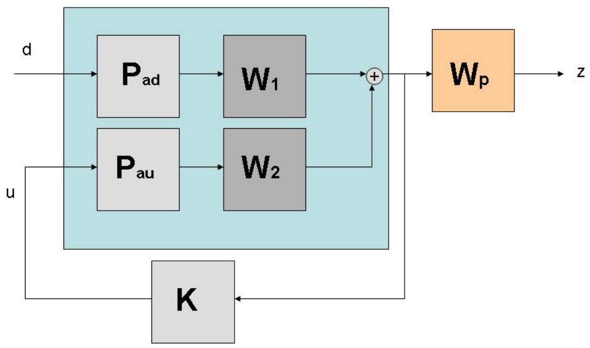

Figure 5. Virtual sensing and control diagram.

Uncertainty modelling of the virtual sensing system

It is noted that transfer functions P_{ad} and P_{au} in Equation (7) are assumed to be sufficiently accurate. However, in the case where there are changes in the actual plant, the control performance and control stability could suffer.

To deal with the above issues, this work considers modelling the uncertainties that may arise in the virtual sensing system with the objective to design a controller that has satisfactory control stability and performance. Figure 6 describes a particular multiplicative uncertainty modelling for a general plant denoted as P. The input multiplicative uncertainty is modelled by frequency dependent normalisation weight W_I and the uncertainty model Δ that could be real or complex.

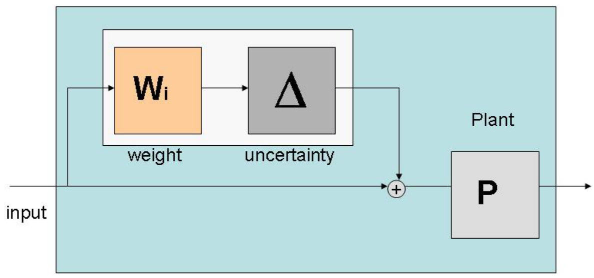

Figure 6. Description of the multiplicative input uncertainty in the plant.

Thus, based on the general virtual sensing problem in Equation (7), each of the transfer function can be modelled as a perturbed transfer function as follows:

$$\begin{aligned} P_{ad} &= \tilde{P}_{ad}(1+\Delta_{ad}W_{ad}) \\ P_{au} &= \tilde{P}_{au}(1+\Delta_{au}W_{au}) \end{aligned} \tag{8}$$

where the uncertainty 'size' is $\|\Delta_{au}\|_\infty = \|\Delta_{ad}\|_\infty = 1$ and $\tilde{P}_{ad}, \tilde{P}_{au}$ are the estimated or nominal transfer functions based on the off-line system identification used in virtual sensing approaches.

Equation (8) can be substituted into Equation (7), so the following can be obtained based on the control configuration in Figure 5:

$$\begin{aligned} p_v &= W_1 \widetilde{P}_{ad}(d + w_1) + W_2 \widetilde{P}_{au}(u + w_2) \\ z &= W_p p_v \end{aligned} \tag{9}$$

where

$$\begin{aligned} y_1 &= W_{ad} d \\ y_2 &= W_{au} u \\ w_1 &= \Delta_{ad} y_1 \\ w_2 &= \Delta_{au} y_2 . \end{aligned} \tag{10}$$

The robust control design is then can be constructed based on the generalised system in Figure 7. There are 3 system blocks, i.e. the generalised plant P block, the uncertainty block, and the controller K block to be designed.

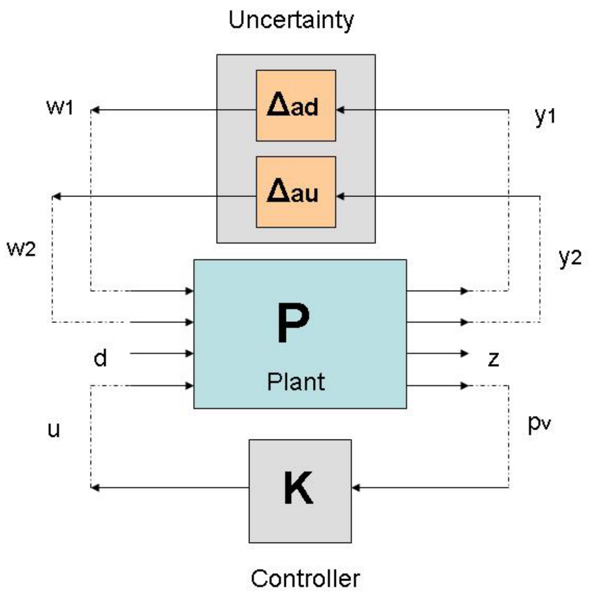

Figure 7. Robust control design for virtual sensing system.

The initial robust performance control design can be approximated by an optimal H_∞ control problem whose objective function J is the minimisation of the mixed sensitivity H_∞ conditions (Skogestad and Postlethwaite 1996):

$$J = \left\| \begin{matrix} W_p S \\ F_I \end{matrix} \right\|_\infty \tag{11}$$

where W_pS is the transfer function from disturbance d to performance output z, and S is the sensitivity function. Also, F_I is the transfer function from uncertainty output $[w_1\ w_2]^T$ to uncertainty input $[y_1\ y_2]^T$.

The optimal control results from the H_∞ norm minimisation in Equation (11) is then tested to check if it satisfies 3 criteria for nominal performance (NP), robust stability (RS) and robust performance (RP). NP criterion is based on the performance of the nominal plant without model uncertainty, while RS and RP criteria take into account the uncertainty that can occur in the plant. The measures for the criteria incorporate the use of a structured singular value μ that relates to the smallest perturbation that makes the associated closed-loop system singular (Skogestad and Postlethwaite 1996).

It is desired to achieve μ's for NP, RS, RP to be less than one. A smaller μ indicates that the closed-loop system can deal with a larger size perturbation/uncertainty while still satisfying the stability and performance requirements. If the controller does not satisfy all 3 criteria, a D-K iteration procedure can be done to obtain an updated controller for achieving μ-optimal control. Interested readers can find the detailed discussion about the control criteria, the structured singular value μ and the D-K iteration procedure in (Skogestad and Postlethwaite 1996).

Choice of performance and uncertainty weights

The weights used for control design is chosen to be first-order transfer functions as shown in Figure 8. The normalised weighting W_{ad} and W_{au} in Equation (8) relates to the expected uncertainty/perturbation for the actual virtual sensing transfer functions P_{ad} and P_{au}. The weights are chosen to be similar, i.e. $W_{ad}= W_{au}=W_i$ as shown in Figure 8. In this case, the 'worst' perturbation in the virtual sensing system is expected to vary from 20% at low frequencies to larger than 200% at high frequencies. The robust controller is then to be designed to be stable under this type of perturbation in the system.

The performance requirement for μ-optimal control is to ensure that the magnitude of the perturbed sensitivity function S is bounded by the inverse of performance weighting W_p. More specifically, it can be shown that:

$$\bar{\sigma}(S) < \frac{1}{\bar{\sigma}(W_p)} \tag{12}$$

where $\bar{\sigma}(G)$ denotes the largest singular value of G.

This performance requirement essentially defines the guaranteed reduction of sound pressure at the virtual location in the case where the plant experiences changes in its dynamics. The inverse of performance weighting W_p is shown in Figure 8. In this work, the performance criteria is defined such that it varies from 10% at low frequencies to about 60% at the highest frequency of interest of about 220 Hz, i.e. the variation of between 10% to 60% of controlled sound pressure at the virtual location with respect to the size of primary disturbance input.

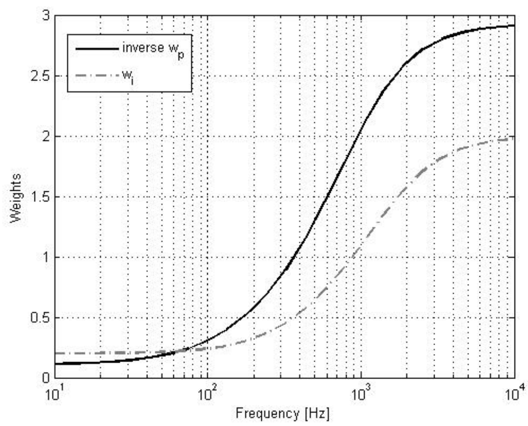

Figure 8. Frequency-dependent weights used for robust virtual sensing and control design.

The controller is designed by loop shaping the sensitivity and complementary sensitivity functions according to the prescribed weights. Matlab Mu-Analysis and Synthesis Toolbox is used to compute the optimal controller, initially based on the H_∞ control criterion in Equation (11). To improve the performance, a single D-K iteration for optimising the control performance is performed.

Results from the robust virtual sensing design

Figure 9 shows the results for the μ-optimal controller for frequencies of 60-1400 rad/sec (9.5-222 Hz). It can be seen that μ's for NP, RS and RP criteria are all well below 1 as desired. The results indicate that in the case where the virtual sensing system experiences changes within the uncertainty range, the closed-loop system can be guaranteed to be stable and the performance requirement is satisfied.

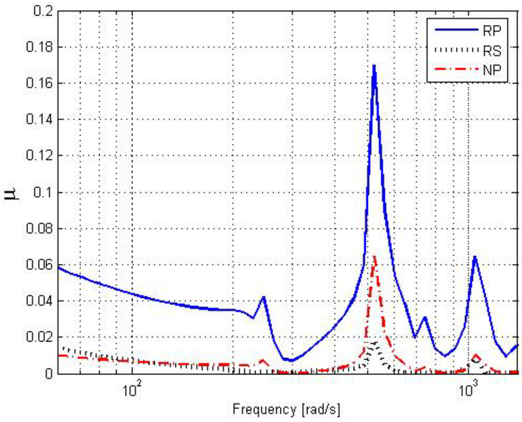

Figure 9. μ-plot for robust virtual sensing design. RP: Robust Performance criterion. RS: Robust Stability criterion. NP: Nominal Performance criterion.

The controller K is shown in Figure 10 where it can be seen that the control gain is high at low frequencies since it is required to compensate the low gain of the plant for achieving better disturbance rejection and control performance. The control gain rolls off at higher frequency to minimise the sensitivity to the higher frequency noise and uncertainties.

Figure 11 describes the sensitivity function of the plant that relates the disturbance input with the sound pressure at virtual location, which is the main objective of the virtual sensing approach. Note that the frequency range of interest is between 15 to 220 Hz where 5 acoustic modes in the acoustic duct system lie. The sensitivity function for 3 systems are shown: the open-loop system, the closed-loop system with the nominal plant and the closed-loop system with the worst-case perturbed plant, representing one of the possible worst-case scenarios for the control performance. Here, the performance specification plotted is defined by the inverse of weighting performance W_p, which is also shown in Figure 8. The inverse weighting function provides the permitted upper bound for the sensitivity function. It is clear that the controller manages to shape the sensitivity function such that it satisfies the upper bound provided by the performance specification, even in the case for the worst-case perturbed plant.

It can be observed that the controller manages to reduce the resonance responses of the duct system although responses at some other frequencies have actually increased. However, the responses still satisfy the performance criteria provided, so the results are still acceptable since the responses at these frequencies are smaller relative to the resonance responses that contribute mostly to the overall sound pressure at the virtual location.

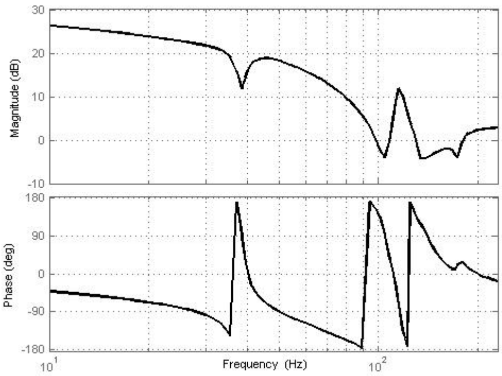

Figure 10. Frequency response of the controller K from the robust virtual sensing design.

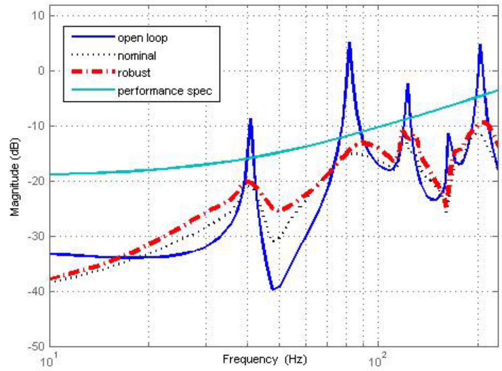

Figure 11. Sensitivity plot for open-loop and closed-loop systems. The closed-loop results based on the nominal plant and the worst-case perturbed plant are also shown.

Time response of pressure at virtual location

Finally, a low-pass filtered white noise disturbance input is added into the system and the estimated pressure at the virtual location is plotted in Figure 12. It can be seen that the controller manages to reduce broadband noise at the virtual location, even when the worst-case perturbed plant is considered. The results indicate that the robust virtual sensing and control strategy can be used to provide a guaranteed level of control stability and performance.

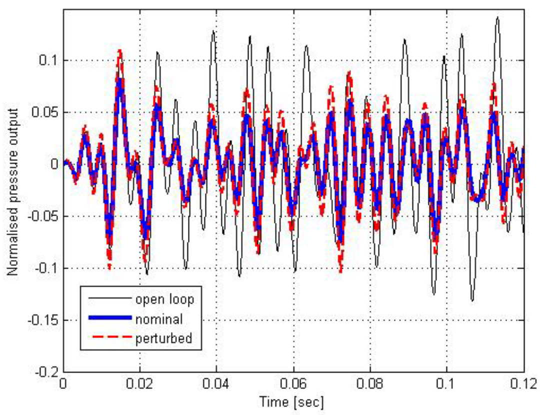

Figure 12. Time responses due to filtered white noise disturbance input. The results are for the open-loop system, and closed-loop for the case of the nominal plant and the worst-case perturbed plant.

CONCLUSIONS

The design and analysis of robust virtual acoustic sensing has been introduced. It is observed that the control stability and performance of the acoustic sensing strategy can be made robust against the possible perturbation in the virtual sensing system. Numerical analysis on the acoustic duct system showed that the control performance can be guaranteed to a certain level. The proposed approach can be used to make the virtual sensing and control system more robust against the expected changes in the system's dynamics.

ACKNOWLEDGEMENTS

The authors would like to thank the Australian Research Council for its financial support.

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